



Characterization of Poisson integrals on non-tube bounded symmetric domains

Abdelhamid Boussejra, Khalid Koufany

► To cite this version:

Abdelhamid Boussejra, Khalid Koufany. Characterization of Poisson integrals on non-tube bounded symmetric domains. *Journal de Mathématiques Pures et Appliquées*, 2007, 87, pp.438-451. hal-00095481

HAL Id: hal-00095481

<https://hal.science/hal-00095481>

Submitted on 15 Sep 2006

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

CHARACTERIZATION OF POISSON INTEGRALS FOR NON-TUBE BOUNDED SYMMETRIC DOMAINS

ABDELHAMID BOUSSEJRA AND KHALID KOUFANY

ABSTRACT. We characterize the L^p -range, $1 < p < +\infty$, of the Poisson transform on the Shilov boundary for non-tube bounded symmetric domains. We prove that this range is a Hua-Hardy type space for harmonic functions satisfying a Hua system.

1. INTRODUCTION

Let $\Omega = G/K$ be a Riemannian symmetric space of non-compact type. To each boundary G/P one can define a Poisson transform, which is an integral operator from hyperfunctions on G/P into the space of eigenfunctions on Ω of the algebra $\mathcal{D}(\Omega)^G$ of invariant differential operators. For the maximal boundary, G/P_{\min} , the most important result is the Helgason conjecture, proved by Kashiwara *et al.* [8] which states that a function is eigenfunction of all invariant differential operators on Ω if and only if it is Poisson integral

$$\mathcal{P}_\lambda f(gK) = \int_K f(k) e^{-\langle \lambda + \rho, H(g^{-1}k) \rangle} dk.$$

of a hyperfunction on the maximal boundary, for a generic $\lambda \in \mathfrak{a}_\mathbb{C}^*$. For other function spaces such as $L^p(G/P_{\min})$ the characterization is in connection with Fatou's theorems. We mention here the work of Helgason [5] and Michelson [14] for $p = \infty$, and Sjörgen [17] for $1 \leq p < \infty$ using weak L^p -spaces. Another characterization for $1 \leq p \leq \infty$, using Hardy-type spaces, was done by Stoll [18] in the harmonic case and by Ben Saïd *et al.* [1] in the general case.

If Ω is a bounded symmetric domain, one is interested in functions whose boundary values are supported on the Shilov boundary (minimal boundary) $S := G/P_{\max}$ rather than the maximal boundary G/P_{\min} .

1991 *Mathematics Subject Classification.* 43A85; 32A25; 32M15.

Key words and phrases. Bounded symmetric domains, Shilov boundary, Hua operator, Poisson transform, Fatou-type theorem.

For the Shilov boundary the Poisson transform is defined by

$$\mathcal{P}_s f(gK) = \int_K f(k) e^{-\langle s\rho_0 + \rho_1, H_1(g^{-1}k) \rangle} dk, \quad s \in \mathbb{C}.$$

In this case, Hua [6] had proved that the algebra of invariant differential operators is not necessarily the most appropriate for characterizing harmonic functions (i.e., annihilated by the algebra $\mathcal{D}(\Omega)^G$). Johnson and Korányi [7], generalizing the earlier work of Hua, Korányi and Stein [10], and Korányi and Malliavin [11], introduced an invariant second order ($\mathfrak{k}_{\mathbb{C}}$ -valued) operator \mathcal{H} , called since, second-order Hua operator (or Hua system). They showed, in the tube case, that a function is annihilated by the Hua operator if and only if it is the Poisson integral $\mathcal{P}_{s_0} f$ ($s_0 = n/r$) of a hyperfunction on the Shilov boundary. Thus, in the tube case, The Hua operator plays the same role with respect to the Shilov boundary as the algebra $\mathcal{D}(\Omega)^G$ does with respect to the maximal boundary. In his paper [13], Lassalle showed the existence of a smaller system (a projection of the Hua operator) with the same properties.

Later Shimeno [16] generalize the result of Johnson and Korányi; namely he proved that a function is eigenfunction of \mathcal{H} if and only if it is a Poisson transform $\mathcal{P}_s f$ of a hyperfunction on the Shilov boundary for generic $s \in \mathbb{C}$.

In [2], the first author gave a characterization of the Poisson transform \mathcal{P}_s on $L^p(S)$, which closes the tube type symmetric domains case characterization.

It thus arises the question of characterizing the range of the Poisson transform \mathcal{P}_s on $L^p(S)$, $1 < p < +\infty$, for non-tube bounded symmetric domains on $L^p(S)$. The purpose of this paper is to answer this question.

For general bounded symmetric domains the Poisson integrals are not eigenfunctions of the second-order Hua operator \mathcal{H} , see for instance [3] or [12]. However for type $\mathbf{I}_{r,r+b}$ domains of non-tube type, (see [3] and [12]) there is a variant of the second-order Hua operator, $\mathcal{H}^{(1)}$, by taking the first component of \mathcal{H} , since in this case $\mathfrak{k}_{\mathbb{C}}$ is a sum of two irreducible ideals $\mathfrak{k}_{\mathbb{C}} = \mathfrak{k}_{\mathbb{C}}^{(1)} \oplus \mathfrak{k}_{\mathbb{C}}^{(2)}$. It is proved, in [12] (and in [3] for the harmonic case, $s = (2r + b)/r$) that a smooth function f on $\mathbf{I}_{r,r+b}$ is a solution of the Hua system, $\mathcal{H}^{(1)} f = \frac{1}{4}(s^2 - (r + b)^2) f I_r$ if and only if it is the Poisson transform \mathcal{P}_s of a hyperfunction on the Shilov boundary.

For general non-tube domains, and for the harmonic case (i.e., for $s = n/r$ in our parametrization) the characterization of the image of the Poisson transform $\mathcal{P}_{\frac{n}{r}}$ on hyperfunctions over the Shilov boundary was done by Berline and Vergne [3] where certain third-order Hua operator was introduced. Recently, the second author and Zhang [12] generalize the result of of Berline and Vergne to any (generic) s . They introduce two third-order Hua operators \mathcal{U} and \mathcal{W} (different from the Berline and Vergne operator) and prove that an eigenfunction f of $\mathcal{D}(\Omega)^G$ is a solution the Hua system (6) if and only if it is a Poisson transform of a hyperfunction on the Shilov boundary.

Let $\mathcal{E}_s(\Omega)$ be the space of harmonic functions on Ω that are solutions of the Hua system (for type $\mathbf{I}_{r,r+b}$ domains, an eigenfunction of $\mathcal{H}^{(1)}$ is indeed harmonic). Then the image $\mathcal{P}_s(L^p(S))$ is a proper closed subspace of $\mathcal{E}_s(\Omega)$. For $1 < p < +\infty$, we introduce the Hua-Hardy type space, $\mathcal{E}_s^p(\Omega)$ of functions $f \in \mathcal{E}_s(\Omega)$ such that

$$\|f\|_{s,p} = \sup_{t>0} e^{-t(\Re(s)r-n)} \left(\int_K |f(ka_t)|^p dk \right)^{1/p} < +\infty.$$

Our main result (see Theorem 4.10) says that *if $s \in \mathbb{C}$ is such that $\Re(s) > \frac{a}{2}(r-1)$, a smooth function F on Ω is the Poisson transform $F = \mathcal{P}_s f$ of a function $f \in L^p(S)$ if and only if $f \in \mathcal{E}_s^p(\Omega)$* . Our method of proving this characterization uses an L^2 version of this theorem (see Theorem 4.8) and an inversion formula for the Poisson transform (see Proposition 4.9) which needs Fatou-type theorems (see Theorem 4.3 and Theorem 4.5).

2. PRELIMINARIES

Let Ω be an irreducible bounded symmetric domain in a complex n -dimensional space V . Let G be the identity component of the group of biholomorphic automorphisms of Ω , and K be the isotropy subgroup of G at the point $0 \in \Omega$. Then K is a maximal compact subgroup of G and as a Hermitian symmetric space, $\Omega = G/K$. Let \mathfrak{g} be the Lie algebra of G , and

$$\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$$

be its Cartan decomposition. The Lie algebra \mathfrak{k} of K has one dimensional center \mathfrak{z} . Then there exists an element $Z_0 \in \mathfrak{z}$ such that $\text{ad} Z_0$ defines the complex structure of \mathfrak{p} . Let

$$\mathfrak{g}_{\mathbb{C}} = \mathfrak{p}^+ \oplus \mathfrak{k}_{\mathbb{C}} \oplus \mathfrak{p}^-$$

be the corresponding eigenspaces decomposition of $\mathfrak{g}_{\mathbb{C}}$, the complexification of \mathfrak{g} . Let $G_{\mathbb{C}}$ be a connected Lie group with Lie algebra $\mathfrak{g}_{\mathbb{C}}$ and P^+ , $K_{\mathbb{C}}$, P^- be the analytic subgroups of $G_{\mathbb{C}}$ corresponding to \mathfrak{p}^+ , $\mathfrak{k}_{\mathbb{C}}$, \mathfrak{p}^- . Denote by σ the conjugaison of $G_{\mathbb{C}}$ with respect to G . Then we have, $\sigma(P^{\pm}) \subset P^{\mp}$ and $\sigma(K_{\mathbb{C}}) \subset K_{\mathbb{C}}$.

Let \mathfrak{h} be a maximal Abelian subalgebra of \mathfrak{k} , and let $\Delta(\mathfrak{g}_{\mathbb{C}}, \mathfrak{h}_{\mathbb{C}})$ be the corresponding set of roots. As Z_0 belongs to \mathfrak{h} , the space \mathfrak{p}^+ is stable by $\text{ad}\mathfrak{h}$. The roots $\gamma \in \Delta(\mathfrak{g}_{\mathbb{C}}, \mathfrak{h}_{\mathbb{C}})$ such that $\mathfrak{g}^{\gamma} \subset \mathfrak{p}^+$ are said to be positive non-compact, and we denote by Φ the set of such roots. Let $\gamma \in \Phi$, then one may choose elements $H_{\gamma} \in i\mathfrak{h}$, $E_{\gamma} \in \mathfrak{g}^{\gamma}$, $E_{-\gamma} \in \mathfrak{g}^{-\gamma}$ such that $[E_{\gamma}, E_{-\gamma}] = H_{\gamma}$ and $\sigma(E_{\gamma}) = -E_{-\gamma}$. Let $X_{\gamma} = E_{\gamma} + E_{-\gamma}$ and $Y_{\gamma} = i(E_{\gamma} - E_{-\gamma})$. Then, by a classical Harish-Chandra construction, there exists a maximal set $\Gamma = \{\gamma_1, \dots, \gamma_r\}$ of strongly orthogonal roots in Φ . For simplicity, let us set for, $1 \leq j \leq r$,

$$E_j = E_{\gamma_j}, \quad X_j = X_{\gamma_j}, \quad Y_j = Y_{\gamma_j}.$$

Then,

$$\mathfrak{a} = \sum_{j=1}^r \mathbb{R}X_j,$$

is a Cartan subspace of the pair $(\mathfrak{g}, \mathfrak{k})$. Let \mathfrak{a}^* denote the dual of \mathfrak{a} and let $\{\beta_1, \beta_2, \dots, \beta_r\}$ be a basis of \mathfrak{a}^* determined by

$$\beta_j(X_k) = 2\delta_{j,k}, \quad 1 \leq j, k \leq r.$$

The restricted root system $\Sigma = \Sigma(\mathfrak{g}, \mathfrak{a})$ of \mathfrak{g} relative to \mathfrak{a} is (of type C_r or BC_r) given by

$$\pm\beta_j \quad (1 \leq j \leq r) \quad \text{each with multiplicity } 1,$$

$$\pm\frac{1}{2}(\beta_j \pm \beta_k) \quad (1 \leq j \neq k \leq r) \quad \text{each with multiplicity } a,$$

and possibly

$$\pm\frac{1}{2}\beta_j \quad (1 \leq j \leq r) \quad \text{each with multiplicity } 2b.$$

Let $\Sigma^+ = \{\beta_j, \frac{1}{2}\beta_j, \frac{1}{2}(\beta_{\ell} \pm \beta_k); \quad 1 \leq j \leq r, \quad 1 \leq \ell \neq k \leq r\}$ the set of positive restricted roots. Then the set $\Lambda = \{\alpha_1, \dots, \alpha_{r-1}, \alpha_r\}$ of simple roots in Σ^+ is such that

$$\alpha_j = \frac{1}{2}(\beta_{r-j+1} - \beta_{r-j}), \quad 1 \leq j \leq r-1$$

and

$$\alpha_r = \begin{cases} \beta_1 & \text{for tube case} \\ \frac{1}{2}\beta_1 & \text{for non-tube case.} \end{cases}$$

Let $\Lambda_1 = \{\alpha_1, \dots, \alpha_{r-1}\}$ and write $\Sigma_1 = \Sigma \cap \mathbb{Z} \cdot \Lambda_1$. Define

$$\mathfrak{m}_{1,1} = \mathfrak{m} + \mathfrak{a} + \sum_{\gamma \in \Sigma_1} \mathfrak{g}^\gamma, \quad \mathfrak{n}_1^+ = \sum_{\gamma \in \Sigma^+ \setminus \Sigma_1} \mathfrak{g}^\gamma.$$

Let

$$\mathfrak{a}_1 = \{H \in \mathfrak{a} : \gamma(H) = 0 \ \forall \gamma \in \Lambda_1\},$$

then $\mathfrak{m}_{1,1}$ is the centralizer of \mathfrak{a}_1 in \mathfrak{g} and $\mathfrak{p}_1 = \mathfrak{m}_{1,1} + \mathfrak{n}_1^+$ is a standard parabolic subalgebra of \mathfrak{g} with Langlands decomposition $\mathfrak{m}_1 + \mathfrak{a}_1 + \mathfrak{n}_1^+$, where \mathfrak{m}_1 is the orthocomplement of \mathfrak{a}_1 in $\mathfrak{m}_{1,1}$ with respect to the Killing form. Note that $\theta(\mathfrak{n}_1^+) = \sum_{\gamma \in \Sigma^+ \setminus \Sigma_1} \mathfrak{g}^{-\gamma}$. Let P_1 be the corresponding parabolic subgroup and $P_1 = M_1 A_1 N_1^+$ its Langlands decomposition. Obviously, P_1 is a maximal parabolic subgroup of G , thus the Shilov boundary S can be viewed as $S = G/P_1 = K/K_1$, where $K_1 = M_1 \cap K$.

If we define the element $X_0 = \sum_{j=1}^r X_j$, Then $\mathfrak{a}_1 = \mathbb{R}X_0$. Let

$$\mathfrak{a}(1) = \sum_{j=1}^{r-1} \mathbb{R}(X_j - X_{j+1})$$

be the orthocomplement of \mathfrak{a}_1 in \mathfrak{a} with respect to the Killing form,

$$(1) \quad \mathfrak{a} = \mathfrak{a}_1 \oplus^\perp \mathfrak{a}(1) = \mathbb{R}X_0 \oplus^\perp \sum_{j=1}^{r-1} \mathbb{R}(X_j - X_{j+1}).$$

We denote ρ_0 the linear form on \mathfrak{a}_1 such that, $\rho_0(X_0) = r$. We extend ρ_0 to \mathfrak{a} via the orthogonal projection (1). If ρ_1 is the restriction of ρ to \mathfrak{a}_1 , then it is clear that

$$\rho_1(X_0) = rb + r + a \frac{r(r-1)}{2} = n.$$

Again, we extend ρ_1 to \mathfrak{a} via the orthogonal projection (1). Then

$$\rho_1 = (b + 1 + a \frac{(r-1)}{2}) \rho_0 = \frac{n}{r} \rho_0.$$

For $g \in G$, define $H(g) \in \mathfrak{a}$ as the unique element such that

$$g \in K \exp(H(g))N \subset KAN = G.$$

We also denote by $\kappa(g) \in K$ and $H_1(g) \in \mathfrak{a}_1$ the unique elements such that

$$g \in \kappa(g)M_1 \exp(H_1(g))N_1 \subset KM_1A_1N_1 = G.$$

The following lemma will be useful for the sequel.

Lemma 2.1 ([15, Lemma 6.1.6]). (i) Let $x, y \in G$, $\bar{n} \in \bar{N}_1$ and $a \in A_1$. Then

$$(2) \quad H_1(x\kappa(y)) = H_1(xy) - H_1(y)$$

$$(3) \quad H_1(\bar{n}a^{-1}) = H_1(\bar{n}) - H_1(a)$$

(ii) Let $t > 0$ and $\bar{n} \in \bar{N}_1$. Then

$$(4) \quad \rho_0(H_1(a_t \bar{n} a_{-t})) \leq \rho_0(H_1(\bar{n})).$$

3. THE POISSON TRANSFORM AND THE HUA OPERATORS

For any real analytic manifold X , we denote by $\mathcal{B}(X)$ the space of all hyperfunctions on X . We will view a function on the Shilov boundary $S = G/P_1$ as a P_1 -invariant function on G . For $s \in \mathbb{C}$, we denote by $\mathcal{B}(G/P_1; s)$ the space of hyperfunctions f on G satisfying

$$f(gman) = e^{(s\rho_0 - \rho_1)\log a} f(g), \quad \forall g \in G, m \in M_1, a \in A_1, n \in N_1^+,$$

The Poisson transform of a function $f \in \mathcal{B}(G/P_1; s)$, is defined by

$$\mathcal{P}_s f(gK) = \int_K e^{-\langle s\rho_0 + \rho_1, H_1(g^{-1}k) \rangle} f(k) dk.$$

Since $G = KP_1$, the restriction from G to K defines a G -isomorphism from $\mathcal{B}(G/P_1, s)$ onto the space $\mathcal{B}(K/K_1)$ of all hyperfunctions f on K such that $f(kh) = f(k)$ for all $h \in K_1$.

We review the construction of Hua operators of the second order (see [7]) and the third order (see [3], [12]).

Let $\{v_j\}$ be a basis of \mathfrak{p}^+ and $\{v_j^*\}$ be the dual basis of \mathfrak{p}^- with respect to the Killing form. Let $\mathcal{U}(\mathfrak{g}_{\mathbb{C}})$ denote the universal enveloping algebra of $\mathfrak{g}_{\mathbb{C}}$. The second-order Hua operator, is the element of $\mathcal{U}(\mathfrak{g}_{\mathbb{C}}) \otimes \mathfrak{k}_{\mathbb{C}}$ defined by

$$\mathcal{H} = \sum_{i,j} v_i v_j^* \otimes [v_j, v_i^*]$$

It is known that the Hua operator does not depend on basis, therefore, for computations one can choose the root vectors basis $\{E_j\}_{j=1}^r$.

For tube domains the Hua operator \mathcal{H} maps the Poisson kernels

$$P_s(gk) = e^{-\langle s\rho_0 + \rho_1, H_1(g^{-1}) \rangle}$$

into the center of $\mathfrak{k}_{\mathbb{C}}$, namely the Poisson kernels are its eigenfunctions up to an element in the center, but it is not true for non-tube domains, see [12, Theorem 5.3]. However for non-tube type **I** domains, $\mathbf{I}_{r,r+b} \simeq SU(r, r+b)/S(U(r) \times U(r+b))$, the situation is not quite different

from the tube case. In fact, $\mathfrak{k}_{\mathbb{C}}$ is a sum of two irreducible ideals, $\mathfrak{k}_{\mathbb{C}} = \mathfrak{k}_{\mathbb{C}}^{(1)} + \mathfrak{k}_{\mathbb{C}}^{(2)}$ where

$$\begin{aligned}\mathfrak{k}_{\mathbb{C}}^{(1)} &= \left\{ \begin{pmatrix} A & 0 \\ 0 & \frac{\text{tr}(A)}{r+b} I_{r+b} \end{pmatrix}, A \in \mathfrak{gl}(r+b, \mathbb{C}) \right\}, \\ \mathfrak{k}_{\mathbb{C}}^{(2)} &= \left\{ \begin{pmatrix} 0 & 0 \\ 0 & D \end{pmatrix}, D \in \mathfrak{sl}(r+b, \mathbb{C}) \right\}.\end{aligned}$$

There is a variant of the Hua operator, $\mathcal{H}^{(1)}$ see [3], [12], by taking the projection of \mathcal{H} onto $\mathfrak{k}_{\mathbb{C}}^{(1)}$. In [12], the second author and Zhang showed that the operator $\mathcal{H}^{(1)}$ has the Poisson kernels as its eigenfunctions and they found the eigenvalues. They proved further that the eigenfunctions of the Hua operator $\mathcal{H}^{(1)}$ are harmonic functions (i.e., eigenfunctions of all invariant differential operators on Ω), and gave the following characterization of the range of the Poisson transform for $\mathbf{I}_{r,r+b}$.

Theorem 3.1 ([12, Theorem 6.1]). *Suppose $s \in \mathbb{C}$ satisfies the following condition*

$$-4[b+1+j+\frac{1}{2}(s-r-b)] \notin \{1, 2, 3, \dots\}, \text{ for } j = 0 \text{ and } 1.$$

Then the Poisson transform \mathcal{P}_s is a G -isomorphism of $\mathcal{B}(S)$ onto the space of smooth functions f on Ω that satisfy

$$(5) \quad \mathcal{H}^{(1)} f = \frac{1}{4}(s^2 - (r+b)^2) f I_r.$$

For the characterization of range of the Poisson transform for general non-tube domains the second author and Zhang [12] introduced new third-order Hua operators \mathcal{U} and \mathcal{W} :

$$\mathcal{U} = \sum_{i,j,k} v_i^* v_j^* v_k \otimes [v_i, [v_j, v_k^*]],$$

$$\mathcal{W} = \sum_{i,j,k} v_k v_i^* v_j \otimes [[v_k^*, v_i], v_j],$$

Similarly to \mathcal{H} , the operators \mathcal{U} and \mathcal{W} do not depend on the basis.

Denote

$$c = 2(n+1) + \frac{1}{n}(a^2 - 4) \dim(\mathcal{P}^{(1,1)}),$$

where $\mathcal{P}^{(1,1)}$ is the dimension of the irreducible subspaces of holomorphic polynomials on \mathfrak{p}^+ with lowest weight $-\gamma_1 - \gamma_2$. For any $s \in \mathbb{C}$, put $\sigma = \frac{1}{2}(s + \frac{n}{r})$. For general non-tube domains we have the following

Theorem 3.2 ([12, Theorem 7.2]). *Let Ω be a bounded symmetric non-tube domain of rank r in \mathbb{C}^n . Suppose $s \in \mathbb{C}$ satisfies*

$$-4[b + 1 + j\frac{a}{2} + \frac{1}{2}(s - \frac{r}{n})] \notin \{1, 2, 3, \dots\}, \text{ for } j = 0 \text{ and } 1.$$

Then the Poisson transform \mathcal{P}_s is a G -isomorphism of $\mathcal{B}(S)$ onto the space of harmonic functions f on Ω that satisfy

$$(6) \quad \left(\mathcal{U} - \frac{-2\sigma^2 + 2p\sigma + c}{\sigma(2\sigma - p - b)} \mathcal{W} \right) f = 0,$$

4. THE L^p -RANGE OF THE POISSON TRANSFORM

For $1 < p < +\infty$, we will consider the space $L^p(S) = L^p(K/K_1)$ as the space of all complex valued measurable (classes) functions f on K that are K_1 -invariant and satisfying

$$\|f\|_p = \left(\int_K |f(k)|^p dk \right)^{1/p} < +\infty,$$

where dk is the Haar measure of K . Let $d\bar{n}$ be the invariant measure on $\bar{N}_1 = \theta(N_1)$ with the normalization

$$(7) \quad \int_{\bar{N}_1} e^{\langle -2\rho_1, H_1(\bar{n}) \rangle} d\bar{n} = 1.$$

Then for a continuous function f on S we have

$$(8) \quad \int_K f(k) dk = \int_{\bar{N}_1} f(\kappa(\bar{n})) e^{-2\langle \rho_1, H_1(\bar{n}) \rangle} d\bar{n}.$$

The space $L^p(S)$ can be viewed as a subspace of $\mathcal{B}(S)$, thus its image $\mathcal{P}_s(L^p(S))$ is a proper closed subspace of $\mathcal{E}_s(\Omega)$. We will now, for specific s , characterize this image. For this we need some information on the integral \mathbf{c}_s in following proposition.

Proposition 4.1. *For $s \in \mathbb{C}$ such that $\Re(s) > \frac{a}{2}(r - 1)$, the integral*

$$\mathbf{c}_s = \int_{\bar{N}_1} e^{-\langle s\rho_0 + \rho_1, H_1(\bar{n}) \rangle} d\bar{n}$$

converges absolutely to a constant $\mathbf{c}_s \neq 0$.

Proof. For $s \in \mathbb{C}$ let $\lambda_s \in \mathfrak{a}_{\mathbb{C}}^*$ be the linear form defined by

$$\lambda_s(H) = (s\rho_0 - \rho_1)(H_1) + \rho(H), \quad H \in \mathfrak{a}$$

where H_1 is the projection of H onto \mathfrak{a}_1 . Then the condition $\Re(s) > \frac{a}{2}(r - 1)$ is equivalent to

$$(9) \quad \Re(\langle \lambda_s, \alpha \rangle) > 0 \quad \forall \alpha \in \Sigma^+ \setminus \Sigma_1.$$

Moreover, we can choose (see for example [15, Lemma 6.1.4]) ω in the Weyl group W of Σ such that

$$\begin{aligned} (i) \quad & \omega \cdot H = H, \quad \forall H \in \mathfrak{a}_1, \\ (ii) \quad & \omega(\Sigma^+ \cap \Sigma_1) = -\Sigma^+ \cap \Sigma_1, \\ (iii) \quad & \omega(\Sigma^+ \setminus \Sigma_1) = \Sigma^+ \setminus \Sigma_1. \end{aligned}$$

Since $\langle \omega \lambda_s, \alpha \rangle = \langle \lambda_s, \omega^{-1} \alpha \rangle$, the condition (9) is equivalent to

$$\Re(\langle \omega \lambda_s, \alpha \rangle) > 0, \quad \forall \alpha \in \Sigma^+$$

Furthermore

$$\langle s\rho_0 + \rho_1, H_1(g) \rangle = \langle \omega \lambda_s + \rho, H(g) \rangle$$

so that

$$\int_{\bar{N}_1} e^{-\langle s\rho_0 + \rho_1, H_1(\bar{n}) \rangle} d\bar{n} = \int_{\bar{N}_1} e^{-\langle \omega \lambda_s + \rho, H(\bar{n}) \rangle} d\bar{n}$$

and the right hand side is the Harish-Chandra c function, $c(\omega \lambda_s)$ associated with the maximal parabolic subgroup, which converges absolutely, see [15]. □

Let $s \in \mathbb{C}$. Let $\mathcal{E}_s(\Omega)$ be the space of harmonic functions on Ω that satisfy (5) in type I domains or (6) in general domains. It is clear that the image $\mathcal{P}_s(L^p(S))$ is a proper closed subspace of the eigenspace $\mathcal{E}_s(\Omega)$. Hence, it is natural to look for a characterization of those $F \in \mathcal{E}_s(\Omega)$ that are Poisson transform of some $f \in L^p(S)$.

For any $1 < p < \infty$, let $\mathcal{E}_s^p(\Omega)$ denote the Hua-Hardy type space of functions $f \in \mathcal{E}_s(\Omega)$ such that

$$\|f\|_{s,p} = \sup_{a \in A_1} e^{-\langle \Re(s)\rho_0 - \rho_1, \log a \rangle} \left(\int_K |f(ka)|^p dk \right)^{1/p} < +\infty.$$

Since $\mathfrak{a}_1 = \mathbb{R}X_0$, the above integral becomes

$$\|f\|_{s,p} = \sup_{t>0} e^{-t(\Re(s)r-n)} \left(\int_K |f(ka_t)|^p dk \right)^{1/p},$$

where $a_t = \exp(tX_0)$.

4.1. Fatou type theorems. As a preparation to Fatou-type theorems we prove the following

Proposition 4.2. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{a}{2}(r-1)$. Let Ψ_t be the function defined on \bar{N}_1 by*

$$\Psi_t(\bar{n}) = e^{-\langle s\rho_0 + \rho_1, H_1(\bar{n}) \rangle + \langle s\rho_0 - \rho_1, H_1(a_t \bar{n} a_{-t}) \rangle}.$$

Then there exists a non-negative function $\Phi \in L^1(\bar{N}_1)$ such that $\Psi_t \leq \Phi$ for each t .

Proof. It follows from (4), that for any $t > 0$ and for any $\bar{n} \in \bar{N}_1$,

$$0 \leq \rho_0(H_1(a_t \bar{n} a_{-t})) \leq \rho_0(H_1(\bar{n})).$$

Therefore,

$$|\Psi_t(\bar{n})| \leq \begin{cases} e^{-\langle \Re(s)\rho_0 + \rho_1, H_1(\bar{n}) \rangle} & \text{if } \frac{a}{2}(r-1) < \Re(s) \leq \frac{a}{2}(r-1) + b + 1 \\ e^{-2\langle \rho_1, H_1(\bar{n}) \rangle} & \text{if } \Re(s) > \frac{a}{2}(r-1) + b + 1 \end{cases}$$

and the second hand is an integrable function on \bar{N}_1 by (7) and Proposition 4.1 . □

Let $\mathcal{C}(S)$ be the space of complex-valued continuous functions on S with the topology of uniform convergence.

Theorem 4.3. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{a}{2}(r-1)$. Then*

$$f(k) = \mathbf{c}_s^{-1} \lim_{t \rightarrow +\infty} e^{-(rs-n)t} \mathcal{P}_s f(ka_t)$$

uniformly, for $f \in \mathcal{C}(S)$.

Proof. Let $f \in \mathcal{C}(S)$, then

$$\mathcal{P}_s f(ka_t) = \int_K e^{-\langle s\rho_0 + \rho_1, H_1(a_{-t}h) \rangle} f(kh) dh.$$

We transform this integral using the formula (8) to an integral over \bar{N}_1 ,

$$\mathcal{P}_s f(ka_t) = \int_{\bar{N}_1} e^{-\langle s\rho_0 + \rho_1, H_1(a_{-t}\kappa(\bar{n})) \rangle} f(k\kappa(\bar{n})) e^{-2\langle \rho_1, H_1(\bar{n}) \rangle} d\bar{n},$$

and by (2) we get

$$\mathcal{P}_s f(ka_t) = \int_{\bar{N}_1} e^{-\langle s\rho_0 + \rho_1, H_1(a_{-t}\bar{n}) \rangle} e^{\langle s\rho_0 - \rho_1, H_1(\bar{n}) \rangle} f(k\kappa(\bar{n})) d\bar{n}.$$

which by the substitution $\bar{n} \mapsto a_{-t}\bar{n}a_t$ and (3), becomes

$$\begin{aligned} \mathcal{P}_s f(ka_t) &= e^{\langle s\rho_0 - \rho_1, H_1(a_t) \rangle} \times \\ &\int_{\bar{N}_1} e^{-\langle s\rho_0 + \rho_1, H_1(\bar{n}) \rangle + \langle s\rho_0 - \rho_1, H_1(a_t \bar{n} a_{-t}) \rangle} f(k\kappa(a_t \bar{n} a_{-t})) d\bar{n}. \end{aligned}$$

But $\rho_1 = \frac{n}{r}\rho_0$, and $a_t \bar{n} a_{-t} \rightarrow e$ when $t \rightarrow +\infty$, thus, by Proposition 4.2,

$$\lim_{t \rightarrow +\infty} e^{-(rs-n)t} \mathcal{P}_s f(ka_t) = \mathbf{c}_s f(k).$$

□

Let

$$\varphi_s(a_t) := \int_K e^{-\langle s\rho_0 + \rho_1, H(a_{-t}k) \rangle} dk.$$

then, it follows from the above theorem that

$$(10) \quad \lim_{t \rightarrow \infty} e^{-(rs-n)t} \varphi_s(a_t) = \mathbf{c}_s, \quad \text{if } \Re(s) > \frac{a}{2}(r-1).$$

As a consequence we can prove the following

Proposition 4.4. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{a}{2}(r-1)$. Then there exists a positive constant γ_s such that, for $1 < p < \infty$ and $f \in L^p(S)$, we have*

$$\left(\int_K |\mathcal{P}_s f(ka_t)|^p dk \right)^{1/p} \leq \gamma_s e^{(rs-n)t} \|f\|_p.$$

Proof. For $t > 0$, we define the function p_s^t by

$$p_s^t(k) = e^{-\langle s\rho_0 + \rho_1, H_1(a_{-t}k^{-1}) \rangle}, \quad k \in K.$$

Then the Poisson transform can be written as the convolution

$$\mathcal{P}_s f(ka_t) = (f * p_s^t)(k).$$

Hence, to prove the proposition we use the Hausdorff-Young inequality,

$$\left(\int_K |\mathcal{P}_s f(ka_t)|^p dk \right)^{1/p} \leq \|p_s^t\|_1 \|f\|_p \quad (p > 1),$$

and (10). □

Let, as usual, \hat{K} , be the set of equivalence classes of finite dimensional irreducible representations of K . For $\delta \in \hat{K}$, let $\mathcal{C}(S)_\delta$ be the linear span of all K -finite vectors on S of type δ . It is well known that the space $\mathcal{C}^K(S) := \oplus_{\delta \in \hat{K}} \mathcal{C}(S)_\delta$ is dense in $\mathcal{C}(S)$. Recall also, that the space $\mathcal{C}(S)$ is dense in $L^p(S)$ for $1 < p < \infty$.

Theorem 4.5. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{a}{2}(r-1)$. Then*

$$f(k) = \mathbf{c}_s^{-1} \lim_{t \rightarrow +\infty} e^{-(rs-n)t} \mathcal{P}_s f(ka_t)$$

in $L^p(S)$, for $1 < p < \infty$.

Proof. Let $f \in L^p(S)$. By the above density arguments, for any $\epsilon > 0$, there exists $\varphi \in \mathcal{C}^K(S)$ such that $\|f - \varphi\|_p < \epsilon$. Then we have

$$\begin{aligned} \|\mathbf{c}_s^{-1} e^{-(rs-n)t} P_s^t f - f\|_p &\leq \|\mathbf{c}_s^{-1} e^{-(rs-n)t} P_s^t (f - \varphi)\|_p + \\ &\quad + \|\mathbf{c}_s^{-1} e^{-(rs-n)t} P_s^t \varphi - \varphi\|_p + \|\varphi - f\|_p \end{aligned}$$

where the function $P_s^t f$ is defined by

$$(11) \quad P_s^t f(k) = \mathcal{P}_s f(ka_t).$$

By Proposition 4.4,

$$\|\mathbf{c}_s^{-1} e^{-(rs-n)t} P_s^t(f - \varphi)\|_p \leq \gamma_s |\mathbf{c}_s^{-1}| \|f - \varphi\|_p,$$

and by Theorem 4.3

$$\lim_{t \rightarrow +\infty} \|\mathbf{c}_s^{-1} e^{-(rs-n)t} P_s^t \varphi - \varphi\|_p = 0.$$

Thus, $\lim_{t \rightarrow +\infty} \|\mathbf{c}_s^{-1} e^{-(rs-n)t} P_s^t f - f\|_p \leq \epsilon(\gamma_s + 1)$ and this proves the theorem. \square

We can now prove the following estimates

Proposition 4.6. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{a}{2}(r-1)$. Then there exists a positive constant γ_s such that for $1 < p < +\infty$ and $f \in L^p(S)$,*

$$(12) \quad |\mathbf{c}_s| \|f\|_p \leq \|\mathcal{P}_s f\|_{s,p} \leq \gamma_s \|f\|_p.$$

Proof. In fact, the right hand side of the estimate (12) follows from Proposition 4.4. On the other hand, by Theorem 4.5 we have

$$\lim_{t \rightarrow \infty} e^{(n-rs)t} \mathcal{P}_s f(ka_t) = \mathbf{c}_s f(k)$$

in $L^p(S)$. Hence, there exists a sequence $(t_j)_j$, with $t_j \rightarrow +\infty$ when $j \rightarrow +\infty$ such that $\lim_{j \rightarrow +\infty} e^{(n-rs)t_j} \mathcal{P}_s f(ka_{t_j}) = \mathbf{c}_s f(k)$, almost every where in K . Therefore, by the classical Fatou lemma,

$$|\mathbf{c}_s| \|f\|_p \leq \sup_j e^{(n-r\Re(s))t_j} \|P_s^{t_j} f\|_p$$

and this is how we prove the left hand side of (12). \square

4.2. The L^2 -Poisson transform range. Recall that

$$L^2(S) = \oplus_{\delta \in \hat{K}} V_\delta$$

where V_δ is the finite linear span of $\{\varphi_\delta \circ k, \ k \in K\}$, where φ_δ is the zonal spherical function corresponding to δ .

For $s \in \mathbb{C}$ and $\delta \in \hat{K}$, define the *generalized spherical function* $\Phi_{s,\delta}$ on A_1 by

$$\Phi_{s,\delta}(a_t) = (\mathcal{P}_s \varphi_\delta)(a_t).$$

Proposition 4.7. *Let $s \in \mathbb{C}$, $\delta \in \hat{K}$ and $f \in V_\delta$. Then for any $k \in K$ and $a_t \in A_1$,*

$$(\mathcal{P}_s f)(ka_t) = \Phi_{s,\delta}(a_t) f(k).$$

Proof. Since M_1 centralize A_1 , we can view the operator (11) as a bounded operator on $L^2(S)$. Moreover, P_s^t commutes with the left regular representation of K in $L^2(S)$. Hence, by Schur's lemma, $P_s^t = \Phi_{s,\delta}(a_t) \cdot I$ on each V_δ and the proposition follows. \square

The first main theorem of this section can now be stated as follows :

Theorem 4.8. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{a}{2}(r-1)$. A smooth function F on Ω is the Poisson transform $F = \mathcal{P}_s f$ of a function $f \in L^2(S)$ if and only if $F \in \mathcal{E}_s^2(\Omega)$.*

Proof. The necessary condition follows from Proposition 4.6 and [12, Theorem 6.1 and Theorem 7.2]. On the other hand, let $F \in \mathcal{E}_s^p(\Omega)$. We apply again [12, Theorem 6.1 and Theorem 7.2]. Then, there exists a hyperfunction $f \in \mathcal{B}(S)$ such that $F = \mathcal{P}_s f$. Let $f = \sum_{\delta \in \hat{K}} f_\delta$ be its K -type decomposition. By Proposition 4.7 we can write

$$F(ka_t) = \sum_{\delta \in \hat{K}} \Phi_{s,\delta}(a_t) f_\delta(k)$$

in $\mathcal{C}^\infty(K \times [0, +\infty[)$.

Now observe that

$$\|F\|_{s,2}^2 = \sup_{t>0} e^{2(n-r\Re(s))t} \sum_{\delta \in \hat{K}} |\Phi_{s,\delta}(a_t)|^2 \|f_\delta\|_2^2 < \infty.$$

Then, if Λ is an arbitrary finite subset of \hat{K} , we get

$$e^{2(n-r\Re(s))t} \sum_{\delta \in \Lambda} |\Phi_{s,\delta}(a_t)|^2 \|f_\delta\|_2^2 \leq \|F\|_{s,2}^2$$

for every $t > 0$ and hence from Theorem 4.3 it follows immediately that

$$|\mathbf{c}_s|^2 \sum_{\delta \in \Lambda} \|f_\delta\|_2^2 \leq \|F\|_{s,2}^2$$

which implies that $f = \sum_{\delta \in K} f_\delta$ in $L^2(S)$ and that

$$|\mathbf{c}_s|^2 \|f\|_2 \leq \|F\|_{s,2}.$$

This ends the proof of the theorem. \square

In the following proposition we show how to recover a function $f \in L^2(S)$ from its Poisson transform $\mathcal{P}_s f$.

Proposition 4.9. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{a}{2}(r-1)$. Let $F \in \mathcal{E}_s^2(\Omega)$ and $f \in L^2(S)$ its boundary value. Then the following inversion formula*

$$(13) \quad f(k) = |\mathbf{c}_s|^{-2} \lim_{t \rightarrow \infty} e^{2(n-r\Re(s))t} \int_K \overline{e^{-\langle s\rho_0 + \rho_1, H_1(a_{-t}k^{-1}h) \rangle}} F(ha_t) dh$$

holds in $L^2(S)$.

Proof. Let $F \in \mathcal{E}_s^2(\Omega)$, then it follows from Theorem 4.8 that there exists a unique $f \in L^2(S)$ such that $F = \mathcal{P}_s f$. Let $f = \sum_{\delta \in \hat{K}} f_\delta$ be its K -type expansion, then similarly to the preceding proof, we get

$$(14) \quad F(ka_t) = \sum_{\delta \in \hat{K}} \Phi_{s,\delta}(a_t) f_\delta(k).$$

For any $t > 0$, define the complex-valued function on K by

$$g_t(ka_t) = |\mathbf{c}_s|^{-2} e^{2(n-rs)t} \int_K \overline{e^{-\langle s\rho_0 + \rho_1, H_1(a_{-t}k^{-1}h) \rangle}} F(ha_t) dh.$$

Next, using the above series expansion we can write according to Theorem 4.5,

$$g_t(h) = |\mathbf{c}_s|^{-2} e^{2(n-rs)t} \sum_{\delta \in \hat{K}} |\Phi_{s,\delta}(a_t)|^2 f_\delta(h).$$

Thus,

$$\|g_t - f\|_2^2 = \sum_{\delta \in \hat{K}} \left| |\mathbf{c}_s|^{-2} e^{2(n-rs)t} |\Phi_{s,\delta}(a_t)|^2 - 1 \right|^2 \|f_\delta\|_2^2,$$

which shows that $\|g_t - f\|_2 \rightarrow 0$, since $\lim_{t \rightarrow \infty} e^{(n-rs)t} \Phi_{s,\delta}(a_t) = \mathbf{c}_s$. \square

4.3. The L^p -Poisson transform range, $p \neq 2$. We shall now prove the second main result of this paper, more precisely, we shall characterize the L^p -range of the Poisson transform. We will need the following notation. For each function f on Ω , define the function f^t , $t > 0$, on K by

$$f^t(k) = f(ka_t).$$

Theorem 4.10. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{a}{2}(r-1)$. A smooth function F on Ω is the Poisson transform $F = \mathcal{P}_s f$ of a function $f \in L^p(S)$ if and only if $F \in \mathcal{E}_s^p(\Omega)$.*

Proof. We will follow the technique used by Korányi [9]. Let $(\chi_n)_n$ be an approximation of the identity in $\mathcal{C}(K)$. That is $\chi_n \geq 0$, $\int_K \chi_n(k) dk = 1$ and $\lim_{n \rightarrow +\infty} \int_{K \setminus U} \chi_n(k) dk = 0$ for every neighborhood U of e in K . Let $F \in \mathcal{E}_s^p(\Omega)$. For each n , define the function F_n on Ω by

$$F_n(gK) = \int_K \chi_n(k) F(k^{-1}g) dk.$$

Then $(F_n)_n$ converges point-wise to F , and since the set $\mathcal{E}_s(\Omega)$ of harmonic functions satisfying the Hua system is G -invariant, $F_n \in \mathcal{E}_s(\Omega)$, for each n . Furthermore,

$$F_n^t(ka_tK) = (\chi_n * F^t)(k)$$

and this shows

$$(15) \quad \|F_n^t\|_2 \leq \|\chi_n\|_2 \|F^t\|_p,$$

and

$$(16) \quad \|F_n^t\|_p \leq \|F^t\|_p.$$

It follows from (15)

$$\sup_{t>0} e^{(n-rs)t} \left(\int_K |F_n(ka_t)|^2 dk \right)^{1/2} \leq \|\chi_n\|_2 \|F\|_{s,p}.$$

Thus $F_n \in \mathcal{E}_{s,2}(\Omega)$ and by Theorem 4.8, there exists $f_n \in L^2(S)$ such that $F_n = \mathcal{P}_s f_n$. Now, our goal is to prove that f_n belongs to $L^p(S)$. Using the inversion formula (13) we can write in $L^2(S)$,

$$f_n(k) = \lim_{t \rightarrow +\infty} g_n^t(k)$$

where

$$g_n^t(h) = g_n(ha_t) = |\mathbf{c}_s|^{-2} e^{2(n-r\Re(s))t} \int_K \overline{e^{-\langle s\rho_0 + \rho_1, H_1(a_{-t}k^{-1}h) \rangle}} F_n(ka_t) dk.$$

Let $\varphi \in \mathcal{C}(S)$ be a continuous function on S , then

$$\int_K f_n(h) \varphi(h) dh = \lim_{t \rightarrow \infty} \int_K g_n^t(h) \varphi(h) dh.$$

Moreover,

$$\begin{aligned} \int_K g_n^t(h) \varphi(h) dh &= |\mathbf{c}_s|^{-2} e^{2(n-r\Re(s))t} \times \\ &\times \int_K \int_K F_n(ka_t) \varphi(h) \overline{e^{-\langle s\rho_0 + \rho_1, H_1(a_{-t}k^{-1}h) \rangle}} dk dh \\ &= |\mathbf{c}_s|^{-2} e^{2(n-r\Re(s))t} \int_K \overline{\mathcal{P}_s \varphi(ka_t)} F_n(ka_t) dk. \end{aligned}$$

By the Holder inequality, if q is such that $1/p + 1/q = 1$, we get

$$\begin{aligned} \left| \int_K g_n^t(h) \varphi(h) dh \right| &\leq |\mathbf{c}_s|^{-2} e^{2(n-r\Re(s))t} \|\mathcal{P}_s \varphi\|_q \|F_n^t\|_p, \\ &\leq |\mathbf{c}_s|^{-2} e^{2(n-r\Re(s))t} \|\mathcal{P}_s \varphi\|_q \|F^t\|_p, \end{aligned}$$

where the second inequality follows from (16). But $F \in \mathcal{E}_{s,p}(\omega)$, then

$$\left| \int_K g_n^t(h) \varphi(h) dh \right| \leq |\mathbf{c}_s|^{-2} e^{2(n-r\Re(s))t} \|\mathcal{P}_s \varphi\|_q \|F\|_{s,p}.$$

Therefore, by Theorem 4.3,

$$\left| \int_K f_n(h) \varphi(h) dh \right| \leq |\mathbf{c}_s|^{-1} \|\varphi\|_q \|F\|_{s,p},$$

and by taking the supremum over $\varphi \in \mathcal{C}(S)$ with $\|\varphi\|_q = 1$, we get

$$\|f_n\|_p \leq |\mathbf{c}_s|^{-1} \|F\|_{s,p}.$$

Now, for each $\varphi \in L^q(S)$, define the functional

$$T_n(\varphi) = \int f_n(h) \varphi(h) dk.$$

Then it is obvious by (4.3) that

$$|T_n(\varphi)| \leq |\mathbf{c}_s|^{-1} \|\varphi\|_q \|F\|_{s,p}$$

hence, T_n is uniformly bounded operator in $L^q(S)$ with $\sup_n \|T_n\| \leq |\mathbf{c}_s|^{-1} \|F\|_{s,p}$. Thanks to Banach-Alaouglu-Bourbaki's theorem, there exists a subsequence of bounded operators $(T_{n_j})_j$ which converges as $n_j \rightarrow +\infty$ to a bounded operator T in $L^q(S)$, under the $*$ -weak topology, with $\|T\| \leq |\mathbf{c}_s|^{-1} \|F\|_{s,p}$. Then, by the Riesz representation theorem, there exists a unique function $f \in L^p(S)$ such that

$$T(\varphi) = \int_K f(h) \varphi(h) dh, \quad \forall \varphi \in L^q(S)$$

with

$$(17) \quad \|f\|_p \leq \|T_n\| \leq |\mathbf{c}_s|^{-1} \|F\|_{s,p}.$$

Now, observe that

$$F_{n_j}(g) = T_{n_j}(e^{-\langle s\rho_0 + \rho_1, H_1(g^{-1}k) \rangle}),$$

thus, by taking the limit as $n \rightarrow +\infty$ we get

$$F(g) = T(e^{-\langle s\rho_0 + \rho_1, H_1(g^{-1}k) \rangle}) = \mathcal{P}_s f(g)$$

with $|\mathbf{c}_s| \|f\|_p \leq \|F\|_{s,p}$, by (17), and this finishes the proof of the theorem. \square

REFERENCES

1. Ben Saïd, S.; Oshima, T.; Shimeno, N. Fatou's theorems and Hardy-type spaces for eigenfunctions of the invariant differential operators on symmetric spaces. *Int. Math. Res. Not.* **16** (2003) 915–931.
2. Boussejra, A. L^p -Poisson integral representation of solutions of the Hua system on Hermitian symmetric spaces of tube type. To appear in *J. Funct. Anal.* 2006.
3. Berline, N.; Vergne, M. Équations de Hua et noyau de Poisson. *Noncommutative harmonic analysis and Lie groups (Marseille, 1980)*, 1–51, Lecture Notes in Math., **880**, Springer, Berlin-New York, 1981.

4. Faraut, J.; Korányi, A. Analysis on Symmetric Cones, *Oxford Mathematical Monographs*, Clarendon Press, Oxford, 1994.
5. Helgason, S. A duality for symmetric spaces with applications to group representations. *Advances in Math.* **5** (1970), 1–154.
6. Hua, L. K. Harmonic analysis of functions of several complex variables in the classical domains. *American Mathematical Society, Providence, R.I.* 1963 iv+164 pp
7. Johnson, K.; Korányi, A. The Hua operators on bounded symmetric domains of tube type. *Ann. of Math. (2)* **111** (1980), no. 3, 589–608.
8. Kashiwara, M.; Kowata, A.; Minemura, K.; Okamoto, K.; Oshima, T.; Tanaka, M. Eigenfunctions of invariant differential operators on a symmetric space. *Ann. of Math.* **107** (1978), 1–39.
9. Korányi, A. The Poisson integral for generalized half-planes and bounded symmetric domains. *Ann. of Math.* **82** (1965), 332–350.
10. Korányi, A.; Stein, E. M. Fatou’s theorem for generalized halfplanes. *Ann. Scuola Norm. Sup. Pisa* **22** (1968) 107–112.
11. Korányi, A.; Malliavin, P. Poisson formula and compound diffusion associated to an overdetermined elliptic system on the Siegel halfplane of rank two. *Acta Math.* **134** (1975), 185–209.
12. Koufany, K.; Zhang G., Hua operators and Poisson transform for bounded symmetric domains. To appear in *J. Funct. Anal.* 2006.
13. Lassalle, M. Les équations de Hua d’un domaine borné symétrique du type tube. *Invent. Math.* **77** (1984), no. 1, 129–161.
14. Michelson, H. L. Fatou theorems for eigenfunctions of the invariant differential operators on symmetric spaces. *Trans. Amer. Math. Soc.* **177** (1973), 257–274.
15. Schlichtkrull, H. Hyperfunctions and harmonic analysis on symmetric spaces. Progress in Mathematics, 49. *Birkhäuser Boston, Inc., Boston, MA*, 1984.
16. Shimeno, N. Boundary value problems for the Shilov boundary of a bounded symmetric domain of tube type. *J. Funct. Anal.* **140** (1996), no. 1, 124–141.
17. Sjögren, P. Characterizations of Poisson integrals on symmetric spaces. *Math. Scand.* **49** (1981), 229–249
18. Stoll, M. Hardy-type spaces of harmonic functions on symmetric spaces of noncompact type. *J. Reine Angew. Math.* **271** (1974), 63–76.

ABDELHAMID BOUSSEJRA – DÉPARTEMENT DE MATHÉMATIQUES, FACULTÉ
DES SCIENCES, UNIVERSITÉ IBN TOFAIL, KÉNITRA, MAROC
E-mail address: Boussejra@mailcity.com

KHALID KOUFANY – INSTITUT ÉLIE CARTAN, UMR 7502, UNIVERSITÉ HENRI
POINCARÉ (NANCY 1) B.P. 239, F-54506 VANDŒUVRE-LÈS-NANCY, FRANCE
E-mail address: khalid.koufany@iecn.u-nancy.fr