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Mean-stress dependent damping of seismic waves in sand

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ABSTRACT : This paper deals with the numerical modelling of seismic waves propagation in soils. Some essential aspects of numerical dispersion are firstly discussed. Classical numerical approaches for the modelling of damping are compared with physical approaches. A rheological model corresponding to classical Rayleigh damping is considered. The link with wave propagation centrifuge experiments is then made. From these tests, homogeneous damping does not appear to be a suitable assumption to analyse the whole experimental results. Many other experimental results on sand (cyclic, resonant...) also give evidence to the dependence of damping on mean stress. This paper then presents a numerical modelling of non-homogeneous damping of seismic waves in sand. Numerical computations involving mean-stress dependent damping are performed to model spherical wave propagation in the centrifuged medium. We consider different laws giving attenuation versus mean stress (Hardin, Saxena, Stewart). Finite element computations, involving these various influences of mean stress on damping, are performed. Numerical investigations give some interesting results on wave propagation through non-homogeneously damped medium. It leads to a good numerical quantification of damping phenomena in the whole medium by taking into account the mean-stress dependence.

1. DYNAMIC EXPERIMENTS ON SOILS

1.1 *Classification of various dynamic experiments*

Many different experimental methods are available to investigate the dynamic response of soils. From cyclic tests at very low frequencies to Hopkinson bar tests at very high strain rates (Semblat et al., 99), there is a wide range of experimental approaches. These testing methods consequently involve very different frequencies and strain magnitudes. Furthermore, as it is shown in figure 1, the ratio χ between the main wavelength λ and the reference length of the problem l_{ref} fastly changes from one experimental method to another.

For small values of χ, wave propagation phenomena strongly prevail on others (as for ultrasonic tests, see figure 1). For high values of χ, such parameters as stress or strain are nearly constant in the specimen and one can directly determine the behaviour of the medium (as for cyclic tests). In earthquake engineering problems, wave propagation phenomena are important but the spatial variations of the mechanical parameters are not very fast. In this paper, we consequently investigate wave propagation in soils considering reduced-scale experiments in the centrifuge and a finite element numerical analysis.

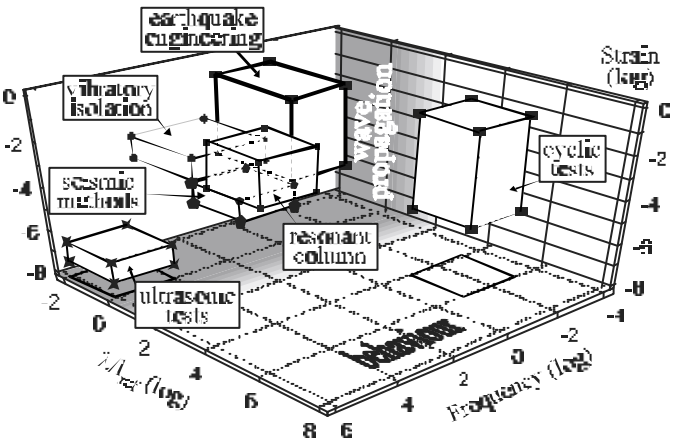


Figure 1 : Classification of different types of dynamic experiments on soils (Semblat et al., 99).

1.2 *Dynamic experiments in the centrifuge*

Dynamic centrifuge experiments have been performed to study wave propagation in soils (Semblat, 98a). It allows the analysis of wave propagation phenomena as well as that of the dynamic behaviour of soils. Acceleration measurements were made in the three directions of space in ten different points (sensors S1 to S10). Reduced-scale experiments are of great interest in the field of earthquake geotechnical engineering (Luong, 93,95). A complete analysis of dispersion and attenuation, characterizing the transient nature of the loading, is proposed in (Semblat, 98a)). Attenuation has been estimated by analytical models (homogeneous damping). It is acceptable for plane waves, but it is not possible, under this assumption, to characterize material attenuation of spherical waves in the whole medium.

2. NUMERICAL MODELLING OF WAVE PROPAGATION PHENOMENA

2.1 *Numerical dispersion of seismic waves*

To perform an acurate and complete numerical analysis, it is necessary to estimate the constitutive parameters as well as the geometry of the medium, the areas where the variations of these parameters are strong... In the field of dynamics, propagation phenomena could be difficult to model using finite difference or finite element methods since the numerical error may increase during propagation. Waves propagation in a specific numerical scheme depends for instance on the element size, the time step, the time integration scheme... This phenomenon is called *numerical dispersion* in reference to the physical dispersion making the wave velocity depend on frequency (Ihlenburg, 95, Semblat, 98b).

It is possible to analyze and quantify this numerical dispersion. As shown in figure 2, the relative period error for structural dynamics (Hughes, 87) is different from one integration scheme to another. It is related to the numerical dispersion of waves but does not take into account the influence of the element size. For Houbolt scheme, the error reaches 10% for a time step of 1/10th of the fundamental period.

To analyze the numerical error for wave propagation problems, we consider a simple one-dimensional case. Table 1 gives the size Δh and the number of elements generated in this 1D medium. In the two last column, the ratios $\Delta h/\lambda$ and $\Delta h/\Delta \ell$ characterize respectively the normalized size of the elements towards the wavelength λ ($\Delta h/\lambda$) and towards the elementary distance $\Delta \ell$ such as $\Delta \ell = C.\Delta t$ (where C is the wave velocity in the medium).

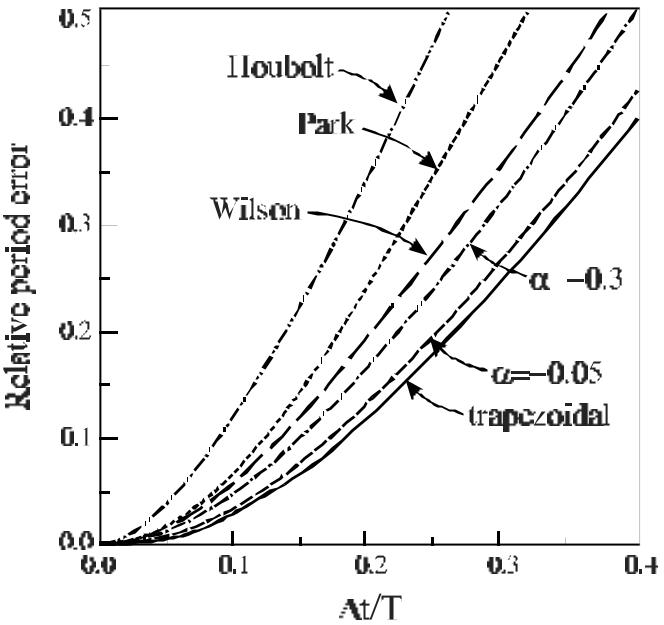


Figure 2. Relative period error for different time integration schemes (Hughes, 87).

The waveform variations during propagation in these different meshes are displayed in figure 3. These curves clearly show that the element size strongly influence the numerical error. In this example, the numerical error is large on both amplitude and wave velocity. Coarse meshes lead to numerical results underestimating amplitudes but overestimating velocities (phase or group). This is the pratical effect of numerical dispersion which can be overwhelmed by using an element size well-adapted to the wavelength of the problem. For 2D wave propagation, the wave type and incidence has a strong influence on numerical dispersion (Bamberger, 80, Semblat, 98b).

Table 1. Mesh refinement and ratio to wavelength for all numerical models.

Size of the elements	Number of elements	$\Delta h/\lambda$	$\Delta h/\Delta \ell$
Δh_1=1m	100	0.36	10
Δh_2=0.5m	200	0.18	5
Δh_3=0.333m	300	0.12	3.33
Δh_4=0.25m	400	0.09	2.5
Δh_5=0.125m	800	0.04	1.25

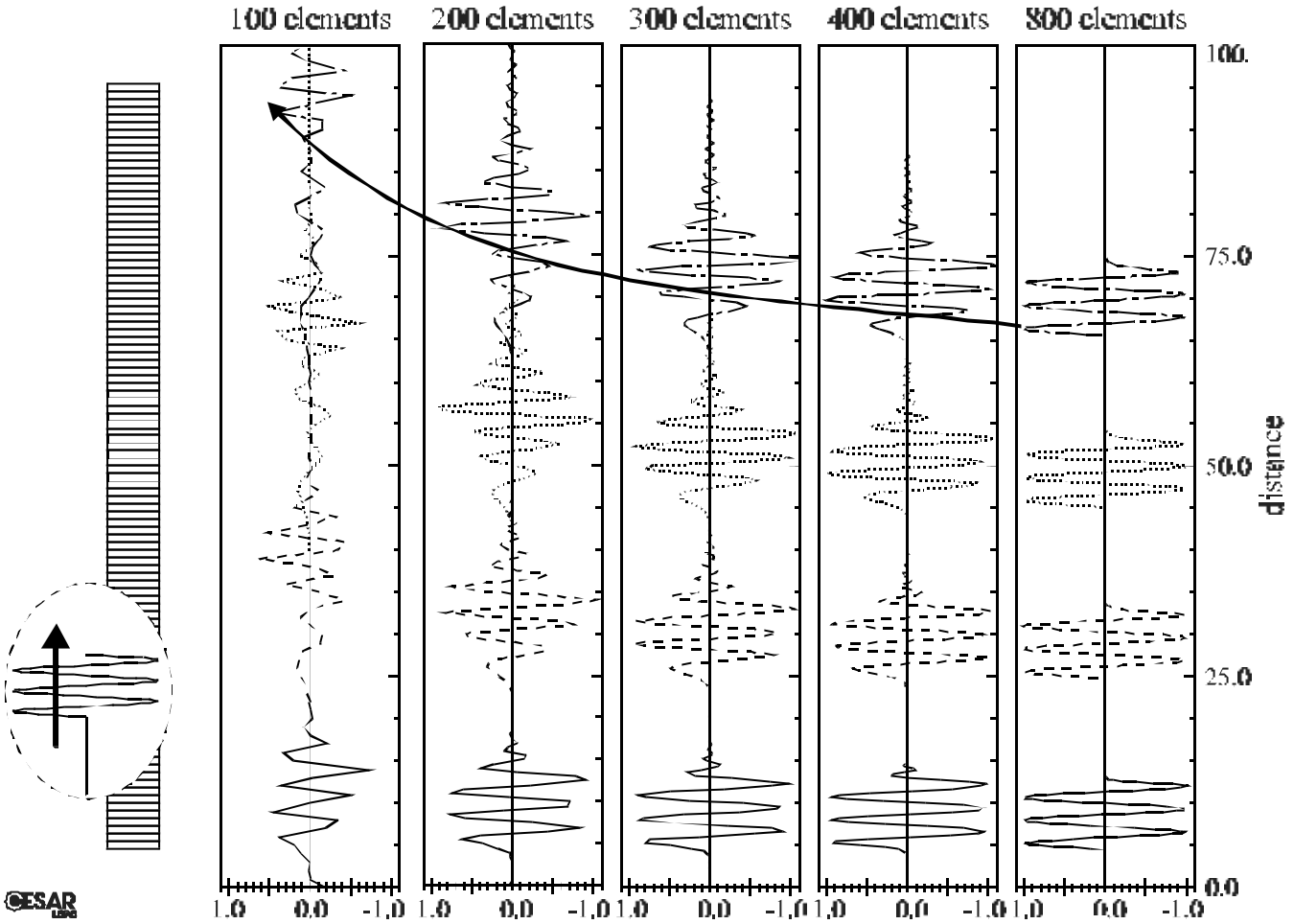


Figure 3. Analysis of 1D-numerical dispersion for wave propagation problems (snapshots at different times).

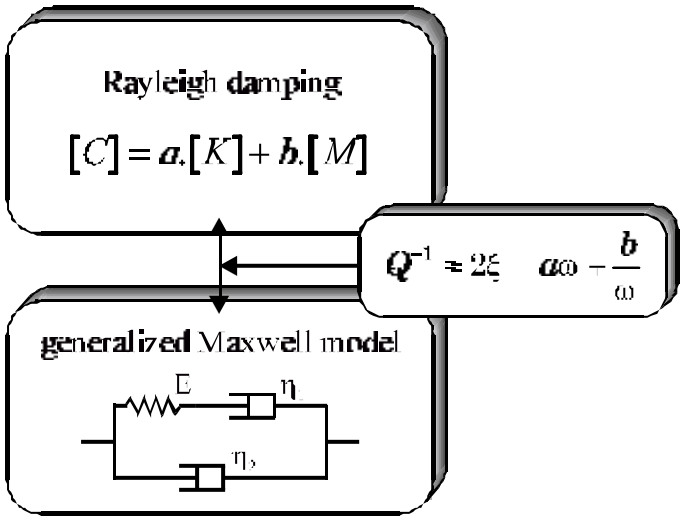


Figure 4 : Equivalence of Rayleigh damping and generalized Maxwell model towards attenuation (Semblat, 97).

2.2 *Rheological interpretation of Rayleigh damping*

For numerical purposes, damping is often considered as Rayleigh damping (Clough, 93). The damping matrix [C] is built using a linear combination of mass and stiffness matrices [M] and [K]. As it is shown in (Semblat, 97), for wave propagation problems, there is a direct coincidence between Rayleigh damping and a generalized Maxwell model in terms of attenuation properties. This rheological model is depicted in figure 4 and the related attenuation expression is given. For weak to moderate damping, Rayleigh formulation is equivalent to a generalized Maxwell model. The rheological interpretation of Rayleigh damping is used to compare analytical results (Semblat, 98a) and numerical results.

3. MEAN-STRESS DEPENDENT DAMPING

3.1 Need for inhomogeneous damping

For drop-ball experiments in the centrifuge (Semblat, 98a)., spherical waves are generated. Considering different rheological models, a complete analytical description of spherical wave propagation involving homogeneous damping was proposed. It gives a quantitative evaluation of damping parameters. But this approach is not sufficient to describe the attenuation

phenomena in the whole medium. In fact, material damping is found to be strong in some locations and very low in some others (Semblat, 98a). As the experiments were performed on dry sand, the influence of mean stress on damping is considered herein. This influence is obvious in laboratory tests on sand (cyclic, resonant). Inhomogeneous material properties for the numerical analysis of seismic wave propagation were for example considered in (Idriss, 74) and (Yeh, 98). In the following section, a finite element model involving different descriptions of non-homogeneous damping is then built to analyze the wave propagation phenomena in the centrifuged sand mass.

3.2 *Finite element model*

To have limited numerical dispersion, the size of the elements is generally chosen between a twentieth and a tenth of the shortest wavelength. For the present study, the mesh has 13624 quadrilateral elements with quadratic interpolation (41419 nodes, size $3.75.10^{-3}$ m). The numerical model is plane (length L=0.75m and height h=0.38m) and axisymetrical strain is considered. The transient loading is a uniform pressure applied on the upper left (round) corner of the model (figure 5). The numerical computation is made using the finite element code CESAR-LCPC (Humbert, 1989). Preliminary undamped numerical computations are given in (Semblat, 98a) showing the different wave types and the reflected waves at the bottom of the container.

3.3 *Damping to mean stress dependence*

Many differents authors have studied damping phenomena in soils and rocks (Hardin, 65, Saxena, 89, Stewart, 83). They have proposed various relations between attenuation Q^{-1} and such parameters as strain amplitude ε, confining pressure P, density...
Three of them are considered herein.
Hardin (65) proposed a relationship of the following form for sand :

$$Q^{-1} = 9\gamma_{\theta x}^{0.2}.P^{-0.5} \quad (1)$$

where $\gamma_{\theta x}$ is the distorsion and P the mean stress

Saxena and Reddy (89) proposed another kind of dependence between quality factor and mean stress :

$$Q^{-1} = K.P^{-0.13}.\varepsilon^{0.33} \quad (2)$$

where K is a constant, P the mean stress and ε the strain

Stewart et al. (83) assumes another type of relation for rocks :

$$Q^{-1} = \frac{K.\zeta.\varepsilon}{P^{4/3}} \quad (3)$$

where ζ is related to the crack density

3.4 *Modelling non-homogeneous damping*

The numerical model starts from these experimental results to consider damping depending on mean stress. As it is depicted in figure 5, we consider ten layers of different damping properties from the free surface down to the mid-depth sensors. Different Rayleigh damping parameters are considered in each layer (Clough, 93, Semblat, 97). Using the rheological interpretation of Rayleigh damping, reference values of damping (layer No.10) are estimated from analytical results on spherical wave propagation in homogenously damped medium (Semblat, 98a). A more complex formulation (e.g. Caughey) could be easily considered.

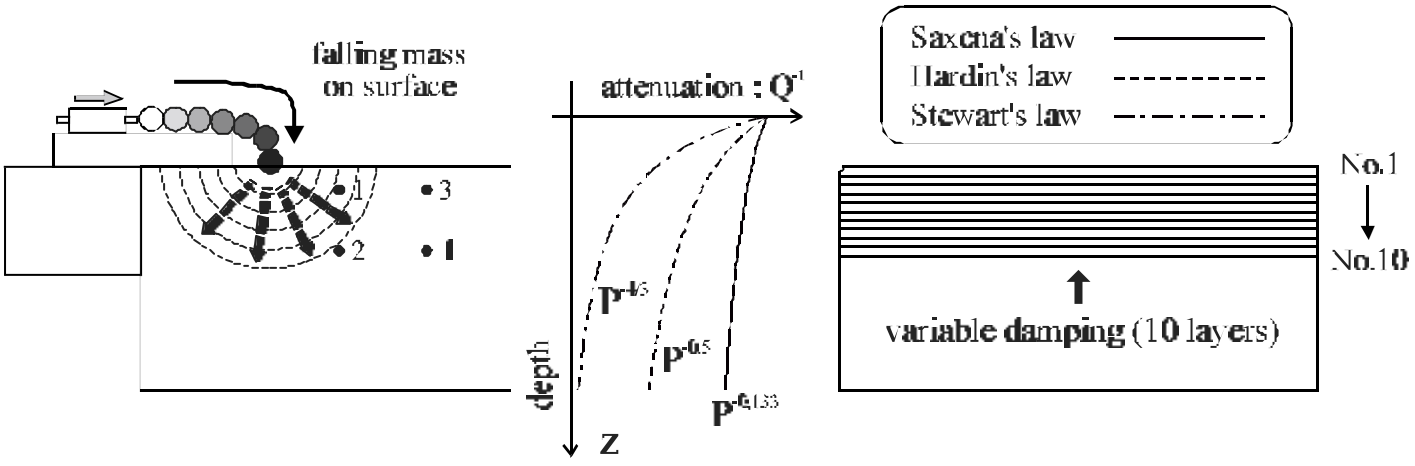


Figure 5 : Non-homogeneous damping depending on depth (or mean stress).

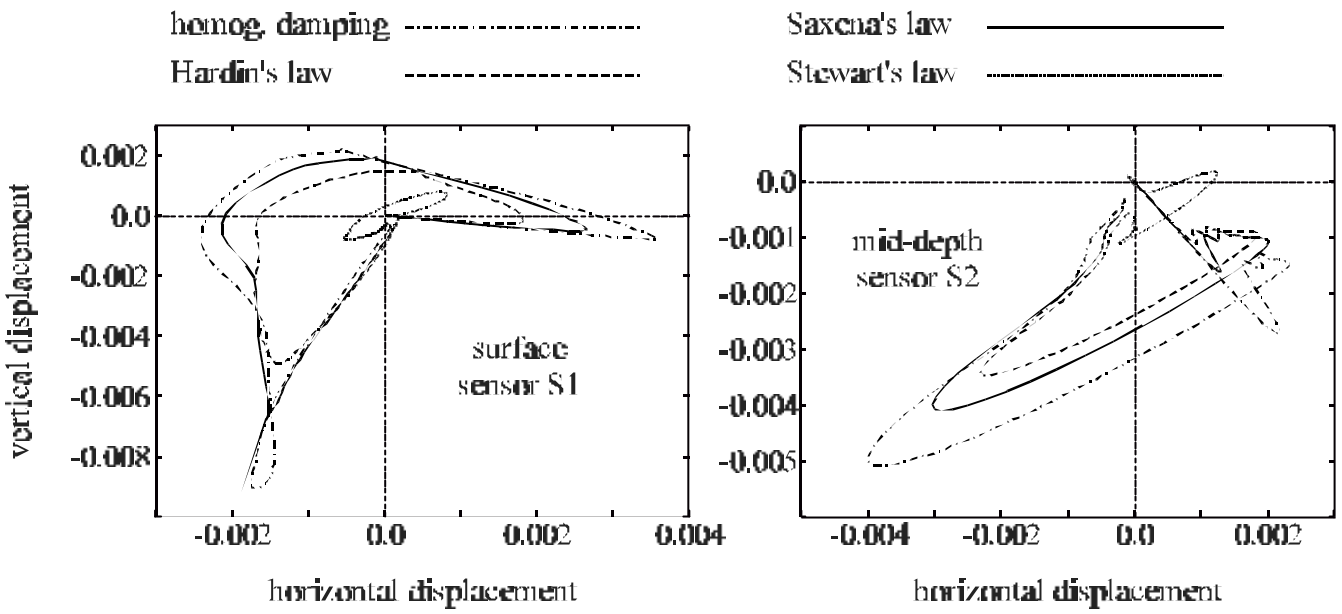


Figure 6 : Surface (left) and mid-depth (right) responses for different damping to mean stress laws.

Considering attenuation dependence on mean stress given by equations 1, 2 and 3, numerical viscoelastic computations involving non-homogeneous damping (depending on depth) are performed. Figure 5 gives the variations of attenuation versus depth using the three relations. The power of mean stress for each relation is respectively -0.5, -0.133 and -1.33 (for equations 1, 2 and 3). As it appears in figure 5, for Saxena's law, the damping properties of the medium are very close to the homogeneously damped case. For Hardin's and Stewart's laws, the inhomogeneity is stronger.

Considering damping properties depending on depth (figure 5), it is then possible to perform numerical computations of wave propagation in damped medium with more realistic assumptions.

4. NUMERICAL RESULTS

4.1 *Influence of damping variations*

Under non-homogeneous damping assumption, the responses computed for sensors 1 (surface) and 2 (mid-depth) are much lower than for the homogeneous case (figure 6). The wave form is slightly influenced by damping variations for surface response (left curves) but not at all for mid-depth response. For Stewart's law, the amplitude decrease is very strong and this law does not seem to be really convenient for soils (it was originally proposed for rocks).

4.2 *Comparison of amplitude decrease values*

Figure 7 gives the values of relative amplitude decrease for mid-depth sensors 2 to 10 (normalized to that of the first sensor S2). From these curves, it can be noticed that Saxena's law gives numerical results very close to the homogeneously damped case. For this law, the dependence on mean-stress is not very strong and the inhomogeneity of damping is very small. For Hardin's and Stewart's relations, the wave amplitude decrease in the medium is larger.

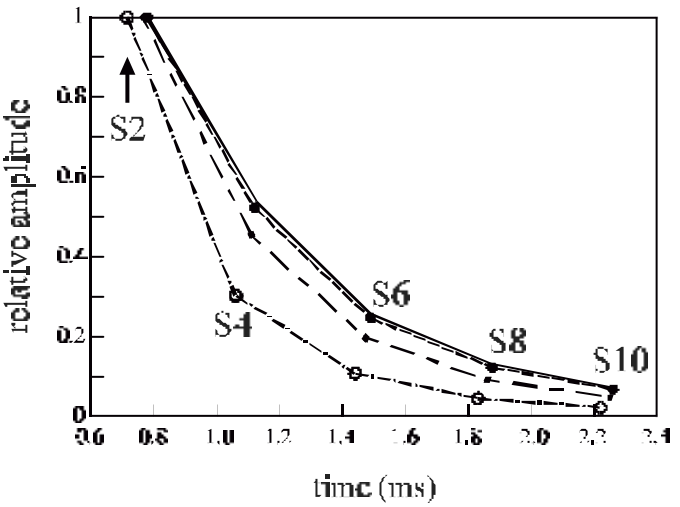


Figure 7 : Relative amplitude decrease for different damping vs mean stress laws.

As it is shown by experimental results, attenuation is much stronger between sensors 2 and 4 (for instance)

than between sensors 6 and 8 (Semblat, 98a). For Hardin's and Stewart's laws, the relative amplitude decrease is stronger for first sensors than for the others. In figure 7, the numerical results involve both geometrical and material damping. As the generated wave field is spherical, the geometrical damping is very large. The observed relative amplitude decrease is then contributing much more to the material attenuation process than it appears in figure 7. The relative amplitude decrease with Hardin's law is consequently very strong between sensors S2 and S4, much lower between sensors S4 and S6 and very small for last sensors (S6, S8, S10). These conclusions coincide with analytical estimation of damping parameters performed in (Semblat, 98a) from experimental centrifuge results on sand.

5. CONCLUSION

The experimental basis of the numerical analysis proposed in this paper is wave propagation tests in a centrifuged sand mass. Numerical investigations on wave propagation phenomena is possible when considering well-adapted models to avoid numerical dispersion. Numerical dispersion is depending on many parameters such as mesh refinement, time integration scheme, wave type...

Homogeneous damping assumption, considered in (Semblat, 98a) to express analytical solutions of the wave propagation problem, was not sufficient to recover the whole experimental results (except for plane waves). Mean-stress dependent damping was then involved in the finite element model. A rheological interpretation of classical Rayleigh damping was firstly proposed to link the analytical previous results to the finite element model with variable damping. Different attenuation to mean stress dependences were considered. The numerical results are promising since they clearly indicate that a variation of damping with mean-stress give a strong amplitude decrease for first sensors and a much lower decrease for further sensors. These numerical results are in good agreement with experimental conclusions given in (Semblat, 98a).

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