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18 Complex Action Functioning as Cutoff and De Broglie-Bohm Particle

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Abstract. The purpose of this discussion contribution is to suggest the possibility that the imaginary action model could function as a cut off in loop diagrams. We argue also that the complex action model of M. Ninomiya and H.B. Nielsen has the DeBroglie-Bohm-particle appearing by itself, which is in a way already present in the contribution to this conference [1].

18.1 Introduction

In the contribution by M. Ninomiya and H.B. Nielsen to this workshop [1] it were suggested that the model of Ninomiya and Nielsen [3] would lead to improvement in the sense of interpretation of quantum mechanics. In the present discussion contribution we shall estimate how does this model with a complex action lead to that a very narrow range of paths come to dominate the Wentzel-Dirac-Feynman-path integral, since we use a path integral with integration over the whole phase space and not only over the configuration space alone as can also be chosen. This may at first looks as to be in contradiction with the Heisenberg uncertainty relation, but it should be stressed that the metaphysical way in which the result of Ninomiya and Nielsen about the dominance of some narrow classes or some discrete sets of classes of paths is not in contradiction with what one can achieve with wave functions or rather cannot achieve. In fact, Heisenberg's result is the information *we can have about the quantum system and still be able to use it*, whereas the information one can obtain out of the model with the extremely narrow region dominating the dynamics in the phase space according to the pathway dominance range is completely useless to work with. In fact this information is what one can claim one has about a particle in the time in between preparation and observation, say if one prepares its momentum and measures its position. Then one could metaphysically claim that in the period between the preparation and the measurement one has *both* the prepared momentum and the observed position at the same time. This claim is however totally useless and could fundamentally not be tested by further experiments because it would, if such an experiment is performed, lead to a disturbance of conditions and thus would spoil the correctness of the claim. In spite of being useless one could however still with good metaphysical right uphold that indeed in such a time interval a particle has both momentum and position at much more accurate values than allowed by the Heisenberg uncertainty principle. If one really takes seriously that in the complex action model everything is calculable just from the expression for the action, i.e. mainly the coupling constants and the form of the action, then one could in principle calculate (but it would be exceedingly hard and in practise totally impossible) this narrow range of dominating paths, meaning essentially an up to a very little uncertainty classical path. So if we could use such unrealistic but possible calculations we would indeed have the Heisenberg uncertainty violating prediction! In this sense we must say that in principle in our model there is no Heisenberg uncertainty principle at the metaphysical level. Such a metaphysical classical state of the system is extremely reminiscent of the Bohm-DeBroglie interpretation of quantum mechanics about which G. Moultaka has talked at this workshop [4].

Can one find, using the metaphysical way of treating our universe (or any system with extremely many degrees of freedom), with the complex action assuming the phase space of coordinates and momenta, the way for cutting away most of the space in a consistent way? Does the narrow range of dominating paths (meaning almost a classical path) help to make the theories of the Kaluza-Klein-kind renormalizable or at least trustable?

One of the open problems of the Kaluza-Klein[like] theories is, namely, the renormalizability of these theories. Even if one studies properties of a system of fermions interacting with the gauge gravity fields far bellow the quantum gravity regime, yet is the consistent treatment of the cutoff and correspondingly the renormalizability of the approach questionable. We suggest that the complex action as proposed by Ninomia and Nielsen [3] play a role of a cut of in loops diagrams for the Kaluza-Klein[like] theories. The *spin-charges-family-theory* [5], proposed by N.S. Mankoč Borštnik and presented and discussed in this workshop as a promising theory for explaining open questions of the *standard model*, is besides proposing the mechanism for generating families (and consequently possibly explaining the appearance of the masses and mixing matrices of fermions), unifying the spin and the charges into only the spin. This *spin-charges-family-theory* is namely sharing many a difficulty with the Kaluza-Klein[like] theories. One of these difficulties is also the cut off problem. We propose in this contribution that the imaginary action might help to make a choice of a trustable cut off.

What are conditions which the system must fulfil that the complex action model start to be efficient or usable in the sense, that it helps to make a choice of a very narrow part of the phase space of momenta and coordinates at least metaphysically? And how could one use it when describing systems, like quarks, hadrons, nuclei, atoms, molecules, scattering of particles on slits, and so on? How such cases come along with both, complex action model and the Bohm-DeBroglie interpretation of quantum mechanics.

18.2 A typical shape of the phase space distributions corresponding to the $|A(t)>$ and $<B(t)|$ states in the complex action model.

In this section we argue using the Lyapunov-exponent or better the Lyapunov-matrix when discussing properties of the universe existing for a very long time that the two states $|A(t)>$ and $<B(t)|$, defined in the contribution by H.B. Nielsen and M. Ninomiya [1] or in [3], that the first, $|A(t)>$, may be considered as a sort of wave functions describing the state of the universe which is favoured by having low action S_I up to the time t from the beginning of the time, and the second, $<B(t)|$, a sort of hidden variable wave function expressing a similar favourite state with respect to S_I coming from the time interval between t and the end of the time. In fact we define these two wave functions from the complex action model as a fundamental formulation from the functional path integral

$$\int \exp(\frac{i}{\hbar} * S[path])\, \mathcal{D}path, \tag{18.1}$$

by splitting it up into two factors

$$\begin{aligned} <q|A(t)> &= \int_{with\ \ path(t)=q} \exp(\frac{i}{\hbar} * S_{-\infty\to t}[path])\, \mathcal{D}path, \\ <B(t)|q> &= \int_{with\ \ path(t)=q} \exp(\frac{i}{\hbar} * S_{t\to\infty}[path])\, \mathcal{D}path. \end{aligned} \tag{18.2}$$

Here

$$\begin{aligned} S_{-\infty\to t}[path] &= \int_{-\infty}^{t} L(path(t'))\, dt, \\ S_{t\to\infty}[path] &= \int_{t}^{\infty} L(path(t'))\, dt \end{aligned} \tag{18.3}$$

and the subscript "$with\ \ path(t) = q$" means that we only include those paths which end at time t with representing the configuration point q in the path way integration. In the case of $<q|A(t)>$ we only use half paths from the beginning of time - symbolized by $-\infty$ to the finite time t, while in the definition of $<B(t)|q>$ we similarly only use half paths from t to the end of time, symbolized by ∞. We say that we split up the original functional integral (18.1), because we immediately see that

$$<B(t)|A(t)> = \int \exp(\frac{i}{\hbar} * S[path])\mathcal{D}path, \tag{18.4}$$

where here the time integration region is from the beginning of time to the end of time, although we have delete the index telling this so that we have put indeed,

$$S[path] = S_{-\infty\ \mathrm{to}\ \infty}[path]. \tag{18.5}$$

The idea is to seek to estimate the shape of the distribution in phase space describing in the best way the wave packet corresponding to the states $|A(t)>$

and $< B(t)|$. In a classical approximation one should get the state $|A(t) >$ by developing forward to time t a state determined roughly in some time prior to t by "optimising'(minimizing) S_I. In thinking of such a development during long times we have to have in mind how does the development of a series of very close (infinitesimally close) classical starting states in phase space develop as time goes on, and this is given by a matrix which is a generalization of the Lyapunov exponent. In fact if one phase space point P_2 deviates from another infinitesimally close one P_1 by an infinitesimal vector in phase space $\mathbf{l}$, this distance vector $\mathbf{l}(t)$ will develop with time exponentially in the sense that

$$\mathbf{l}(t) = exp(\lambda * t)\, \mathbf{l}(0) \tag{18.6}$$

where λ is a matrix with the order being equal to the dimension of the phase space. If the vector $\mathbf{l}(t_{start})$ at the starting time t_{start} has components along the subspace of positive eigenvalues for the matrix λ, the components in this space will grow up very drastically during sufficiently long time, while on the other hand the components in the subspace of negative eigenvalues will grow smaller and smaller as time passes. If thus at some time the starting state was selected by the S_I to be in some not especially elongated region and essentially just one quantum state (we speculate that this is the selection at some close to Big Bang time), then as time goes on this region will be more and more contracted in the λ negative eigenvalue subspace directions, while it will be expanded in the positive eigenvalue directions. After a long time - i.e. when t has become long after the era of the strongest influence of S_I - the region representing the most favourite state at time t, that is just $|A(t) >$, becomes very contracted in the directions corresponding to the negative eigenvalue subspace and very elongated in the directions corresponding to the positive eigenvalue subspace. This means that approximately this region corresponding to the state $|A(t) >$ becomes a surface of dimension as the number of positive eigenvalues of λ lying in the phase space, probably not a flat surface but a curved smooth one. Similarly - but now we can say time reversed - we obtain that the phase space region corresponding to the state $< B(t)|$ will be a very extended surface while strongly contracted in other directions. For $< B(t)|$ we must imagine that the dependence on the minimization on S_I on what goes on in the future - of the time t - determines in some presumably far future which state would be most favourable and then we must imagine how to develop backwardly (backward to time t) this most favoured state. The development under such a backward development is again exponential and given by a metric similar to the λ from before. Now however we develop a negative time namely from the presumably far future time back to the time t. Again some eigenvalues shrink under this backward development while others expand drastically. We therefore again obtain that the region in the phase space roughly describing $< B(t)|$ has the shape of a very extended surface. Both surfaces, the surface for $|A(t) >$ and that for $< B(t)|$, have dimensions presumably about the half dimension of the phase space, since they had their dimension given by number of respectively negative and positive eigenvalues of matrices of order of the dimension of phase space. In case they have really this half dimension of phase space both, their intersection will be point wise. That is to say they will intersect in a discrete set of points typically (if they intersect at all).

Such an intersection in one or a few points would mean that our whole model predicts *essentially one or a few classical solutions with very little uncertainty to represent the dominant part of the functional integral*! If we - metaphysically may be - take this dominant region - the overlap of the $|A(t)>$ region with the $< B(t)|$ one - in the space of paths to represent the realized history of the universe, then we have reached a picture in which the universe runs through a development in which the conjugate variables (i.e. momentum and position) are much more accurately determined than (formally) the Heisenberg uncertainty principle allows for. That is to say: The metaphysical picture put forward in our model turns out to deliver a classical picture in the sense that there is approximately a totally classically development, so that our complex action model makes it approximately as if there really were a true classical development as one would have imagined before quantum mechanics were invented. Well, we have the tiny deviation from this picture that there will typically not be only one such classical development, but rather several although still a discrete set of them.

18.3 A proposal for cutting off by means of the complex action model

It is the main purpose of the present discussion and contribution to point out that we have a hope that the complex action of H.B. Nielsen and M. Ninomiya is offering a "physical" mechanism for a cutoff and correspondingly find the "philosophical" support for the higher dimensional Kaluza-Klein-like theories which are not renormalizable. One could namely claim: we know a mechanism that in principle will cut off the divergences and replace them by finite expressions depending on the support of S_I effects of the complex action model.

Let us show how does such a principal cut off mechanism appear in the complex action model!

Let us therefore very shortly remind ourselves how can one get in the complex action the "usual" quantum mechanics. The basic approximation to reproduce the (usual) quantum mechanics in the complex action model is that we approximate the projection operator on the future-determined state $|B(t)>$, the hidden variable state we could call it, by a unit operator

$$|B(t)>< B(t)| \approx N * I \tag{18.7}$$

where N is an unimportant normalization factor and I is the unit operator. The argumentation for statistically justifying this approximation to be used for making the Born-probability distribution so as to obtain the usual expectation value formula from the one suggested at first in the complex action model goes with an ergodicity-like approximation. The hidden variable state from the future $< B(t)|$ affected by S_I is, as we mentioned in the previous section, essentially given by some favourable state in a presumably far future extrapolated backward in time through a large amount of time. Then if this time is long and the system, the universe, is roughly an ergodic system, we will argue that all states have almost the same practical chance for being the state $< B(t)|$. In this way we count that all states in some basis for the Hilbert space are equally likely to be in the state $< B(t)|$.

$$
\begin{aligned}
< O >_t &= \frac{\int exp(\frac{i}{\hbar} * S[path])\, O(path(t))\, \mathcal{D}path}{\int exp(\frac{i}{\hbar} * S[path])\, \mathcal{D}path} \\
&= \frac{< B(t)|O|A(t) >}{< B(t)|A(t) >} \\
&= \frac{< A(t)|B(t) >< B(t)|O|A(t) >}{< A(t)|B(t) >< B(t)|A(t) >} \\
&\approx \frac{< A(t)|N * 1O|A(t) >}{< A(t)|N * 1|A(t) >} \\
&= \frac{< A(t)|O|A(t) >}{< A(t)|A(t) >}.
\end{aligned} \tag{18.8}
$$

Thus we have justified the approximation (18.7). We hope that since the two states (as a function of a phase space) of a system, one describing the developing of the system from the very beginning up to the time t ($A(t)$) and the second describing the system from the very end backward up to time t ($B(t)$), define as an overlap a very tiny part of the phase space, the idea is that knowing this phase space, that is some almost classical solutions, would help us to make a choice of an appropriate cut off.

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