Inverse problems in fluid-structure interaction: Application to hemodynamics

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Abstract. *In this talk we deal with the simulation of fluid-structure interaction (FSI) problems in hemodynamics, with the emphasis in data assimilation and simulation in physiological regimes.*

Keywords: fluid-structure interaction, medical imaging, data assimilation, Kalman filtering, Luenberger observers, aortic coarctation.

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1 Introduction

In this talk we study sequential data assimilation techniques in fluid-structure interaction (FSI). We first apply a reduced-order Unscented Kalman Filter for the effective estimation of relevant physical parameters, like the stiffness distribution of the vessel wall and the proximal resistance in the fluid, from displacement measurements at the fluid-structure interface. We also analyze some Luenberger observers from solid mechanics in FSI, with the aim to construct tractable state estimators for large-scale FSI problems.
Then we apply some of the aforementioned methodologies to real physical problems. We perform the estimation of the wall stiffness (for linear and non-linear solid models) from data coming from MR-images of a silicone rubber aortic phantom. To finish, we deal with the forward analysis of a real aorta with repaired coarctation and we show some results with the patient's data.

2 Sequential approaches for data assimilation in fluid-structure interaction

2.1 Data assimilation background

A physical system like blood flowing in a compliant artery can be observed through various measurement modalities: artery wall movements obtained from three-dimensional or multi-slice dynamic medical imaging, pressure in few points provided by a catheter, *etc*. These measurements are usually limited to a few locations in space, and typically come from different cardiac cycles (in this

case they are resynchronized with the electrocardiogram). They are of course subject to noise and their postprocessing can introduce some further inaccuracies, and only a limited number of physical quantities can be simultaneously obtained.
The physical system of interest, in this case a blood vessel, can also be represented by mathematical models like the FSI-equations in moving domains [4]. In that case, many quantities – displacement, velocity, pressure, stress – are available in "all" locations and at "any" time instant. But of course, the model itself contains approximations – due for example to the modeling choices and the numerical solution – and relies on parameters that are not perfectly known.
The objective of data assimilation is to take advantage of both measurements and models. Measurements can be used to reduce the uncertainties of the model, and the model can be used to access some "hidden" physical quantities, for example the mechanical stress in the artery wall. Data assimilation can also be viewed as a way to reduce the measurements noise by means of a model that takes into account the underlying physical principles.
Data assimilation techniques usually consists of minimizing a cost function like:

$$J(\hat{X}(0), \theta) = \int_0^T \|Z - H(\hat{X})\|^2_{W^{-1}} \,\mathrm{d}t + \|\theta - \hat{\theta}_0\|^2_{(P_0^\theta)^{-1}} + \|\hat{X}(0) - \hat{X}_0\|^2_{(P_0^X)^{-1}}\ , \tag{1}$$

with $\hat{X}$ satisfying $\dot{X} = A(X, \theta)$ and $\hat{X}_0$ and $\hat{\theta}_0$ are given *a priori* values for the initial condition and parameters, and Z corresponds to the observations, assumed to be the result of the operator $H(\cdot)$ applied to the reference dynamical state X plus some noise.
This minimization problem can be addressed by many techniques that are classically divided in two groups: the variational and the sequential approaches. The variational approach consists of minimizing this cost function by an optimization algorithm that is usually based on the computation of its gradient, obtained by solving an adjoint model (see e.g. [2]). In this work we will focus on the sequential approaches, also known as *filtering*, which modify the forward dynamics with a correction term that takes into account the discrepancy between actual measurements and observations generated by the model:

$$\dot{\hat{X}} = A(\hat{X}, \theta) + K(Z - H(\hat{X})),\ \hat{X}(0) = \hat{X}_0, \tag{2}$$

where $\hat{X}$ is called *estimator* of X, and the operator K depends on the method. Note that Equation (2) considers only an uncertainty in the initial condition. In the case where θ has also to be included in the estimation procedure, the filtered dynamics can be written in a similar way by defining an *extended* estimator $\hat{X}_e = (\hat{X}, \hat{\theta})$ and its corresponding dynamics by

$$\dot{\hat{X}}_e = A_e(\hat{X}_e) + K_e(Z - H(\hat{X})),\ \hat{X}_e(0) = (\hat{X}_0, \hat{\theta}_0). \tag{3}$$

2.2 Nonlinear Kalman filtering in FSI

The most famous sequential approach is the Kalman filter [5]. On a given time interval $[0, T]$, and if all the operators are linear, the variational method (namely the minimization of functional (1)) and the Kalman filter algorithm turns out to give the same estimation at $t = T$. Concerning the computational complexity, whereas the variational method has to solve several forward and adjoint problems on the whole interval $[0, T]$, the estimation in the sequential algorithm is computed by solving only once the filtered dynamics (2). However, the optimal operator K is determined by operations (multiplications, inversions, *etc.*) involving full matrices of the size of the state and the observations, which makes Kalman-based filters prohibitive for discrete problems derived from partial differential equations (PDEs).
For these reasons, data assimilation of distributed mechanical systems are usually based on variational methods. Particularly in FSI for hemodynamics, this approach has been investigated for

simplified models in [6, 8, 13], and for three-dimensional problems in [3, 12] by modifying the minimization problem in order to avoid the resolution of the adjoint equations.
In this work, we deal with the application to FSI problems of the reduced order Unscented Kalman Filter presented in [9], intending to optimize the model parameters for reducing error between model and measurements for all times in a least squares sense, as for instance in Equation (1). It is based on the assumption of low rank of the matrices involved in the Kalman filtering procedure, leading to a factorized and therefore tractable form of the Kalman gain K_e. Its main features are that it does not require any adjoint or tangent problems and it can easily be run in parallel, which is of great interest in fluid-structure problems where the forward simulation is already a challenge in itself. We illustrate this technique through the estimation of the Young's modulus of the arterial wall and the proximal fluid resistance in a three-dimensional idealized abdominal aortic aneurysm. The measurements are assumed to be the nodal wall displacements. We also study the effect of a reasonable error in the initial condition, and propose a simple way to improve the estimation by including the boundary condition's pressure in the uncertainty space.

2.3 Luenberger observers in FSI

In the case that one has to deal with uncertainties also in $\hat{X}(0)$, data assimilation methods suffer from the "curse of dimensionality" as explained by Bellman in [1], which makes them intractable for systems coming from discretized partial differential equations. However, in [7] Luenberger introduced a new class of estimators, called *observers*, for which he relaxed the optimality condition for computing the gain matrix K in the estimator dynamics (2). There, he only bases the filter design on the requirement that the error system $\tilde{X} = X - \hat{X}$ be asymptotically stable.
We study analytically and numerically the performance of the observers presented in [10, 11] based on displacement and velocity measurements in the solid in the FSI framework. We first show that the straightforward usage of the these estimators in FSI lead to a considerably better performance of the displacement with respect to the velocity feedback, while in pure solid mechanics usually the opposite occurs. After a more detailed theoretical analysis, we conclude that the velocity feedback does not take into account (by construction) the added mass effect. Hence, we propose a way to improve its performance by including the added mass operator in the feedback. We also point out that only measurements in the solid are not enough to stabilize the error of the whole fluid-structure system. Hence, the design of a feedback using measurements in the fluid is needed, what can be subject of future developements.

3 In vitro validation and in vivo results

Finally, wee apply the reduced-order Unscented Kalman Filter with data measured from real physical systems using FSI models. We perform the estimation of the stiffness parameters from data coming from MR-images of a silicone rubber aortic phantom. The experimental setup, basically consists of MR-compatible emulator of the cardiovascular system with a flow pump, a valve, a silicon rubber tube, a compliance chamber and a venous return. The tube is imaged when the pump is enforcing flow through the system. The MR-images are then segmented obtaining a sequence of surfaces. The first one is used to build the geometry (meshes) of the FSI model. Moreover, the measured pressures are used as boundary conditions in the fluid. All estimation results are reasonable in the sense that: (a) the estimation algorithm always reduces the discrepancy between model and segmented surfaces (for both linear and nonlinear solid models), (b) for pressure ranges where the linear model is more adequate, the estimated Young's modulus matches with the one obtained from nondestructive mechanical tests, and (c) for large pressure ranges, the estimated constitutive parameter of the linear model seems not to converge, whereas for the nonlinear solid does.
Finally, we present the FSI analysis of a whole aorta with mild coarctation based on clinical data. Even though we are aware about the model limitations, the estimation algorithm is able to reduce

the discrepancy between model and measurements. At the same time, as for the phantom, the interpretation of the estimation results allow unmasking the model's weaknesses and confirming its strengths.

4 CONCLUSIONS

This work illustrates that the combination of detailed fluid-structure models automatic image segmentation tools, and robust data assimilation procedures provide a powerful toolchain for the in vivo analysis of arterial tissue properties. In particular, state-of-the-art sequential approaches provide a robust and efficient framework for data assimilation for complex physical systems, like coupled fluid-structure problems.

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