Collective behaviours of light and matter

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Abstract Coupling of light and matter can lead to the emergence of new collective phenomena, which render a separate description in terms of light or matter impossible. To understand and describe such cases, new composite light matter objects need to be introduced. In this chapter, we present theoretical studies of two examples of such systems. The first is an assembly of coupled Rabi cavities that shows coherent behaviour similar to Dicke superradiance. The second is a Bose-Einstein condensate coupled to the optical modes of a cavity, that mediate an effective long range interaction between the atoms of the condensate and drive it into a supersolid phase.

1 Introduction

In quantum and condensed matter physics, light has always been used as a tool to manipulate and observe objects and phenomena. For example, in cold atoms experiments, electromagnetic (EM) fields are used to trap the atoms, to modify their mutual interaction through Feshbach resonances, and to impose all kinds of optical lattices upon them [1]. In solid state physics,

Thibaut Flottat, Frédéric Hébert, and George Batrouni
Université Côte d'Azur, CNRS, INPHYNI, 1361 route des Lucioles, 06560, Valbonne, France, e-mail: frederic.hebert@inphyni.cnrs.fr.

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photoemission spectroscopy [2] allows to determine the electron dispersion relations.

Beyond this, the coupling of light and matter can bring forth new collective behaviour where the observed phenomena, and the relevant degrees of freedom used to describe them, intricately mix both. When the light-matter coupling is strong, a separate description is not relevant, and new quantum objects emerge. For example, polaritons are quantum superpositions of a photon and a dipolar excitation of a solid medium. They behave as bosonic quasi-particles and can undergo Bose-Einstein condensation [3]. Similar collective light-matter phases can be observed in cold atoms experiments placed in optical cavities [4], for example Dicke superradiance [5] or crystallization [6]. In this chapter, we will exemplify such light-matter collective behavior by studying two cases.

First, we will consider the collective behaviour of coupled cavities. Each cavity is composed of an artificial atom coupled to the electromagnetic (EM) modes of the cavity. If the coupling is strong enough, the degrees of freedom of the atom and the EM field cannot be separated and these cavities, once connected to each others, are then new bricks to study collective behaviour. Quantum electrodynamic circuits are experimental realisation of such systems.

Secondly, we study an ensemble of atoms placed in a single cavity. The coupling with the field mediates an effective long range interaction between the atoms, that drives them into a supersolid phase, which exhibits simultaneous Bose condensed and charge density orders.

Both these studies present theoretical approaches to these problems. We use exact numerical techniques, quantum Monte Carlo (QMC) simulations with the SGF algorithm [7], supplemented by mean field techniques. The SGF method allows the calculation of many physical quantities, including complex correlation functions, at finite temperature on finite clusters.

2 Coupled cavities

Recently, it became possible to build elementary cavity quantum electrodynamics systems on solid state chips. For example a Josephson junction can play the role of an artificial atom and can be coupled to microwave photons localized in a small wave guide [8, 9]. A simple description of such cavities is based on the Rabi model. In this description, the material system is described as a two level quantum system, that is a spin 1/2, of excitation energy ω_s. We neglect all of the EM modes but one, of energy ω_l, and introduce a coupling strength between light and matter g (**Fig. 1** (a)). Cavities

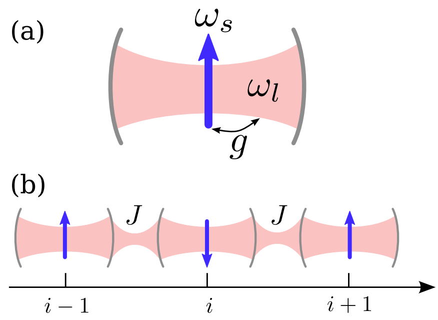


Fig. 1 (a) The Rabi model, where a two level quantum system (spin) is coupled to a single EM mode, is the simplest model used to describe a cavity. (b) Coupling different cavities by tunnel effect, we obtain a QED circuit described by the so-called Rabi-Hubbard model.

are coupled to each other by tunnel effect of strength J, which gives a so-called circuit quantum electrodynamics (QED) system (**Fig. 1** (b)). Written in second quantized form, the Hamiltonian of the model reads

$$H = \sum_i \Big(\omega_s \sigma_i^+ \sigma_i^- + \omega_l a_i^\dagger a_i + g(\sigma_i^+ + \sigma_i^-)(a_i + a_i^\dagger)\Big) \\ + J\sum_i (a_i a_{i+1}^\dagger + a_{i+1} a_i^\dagger) \tag{1}$$

i is the index of the cavity, operators $a_i^\dagger$ and a_i create and destroy a photon in cavity i, σ_i^+ and σ_i^- excite or de-excite the atom (spin) in cavity i. This model was dubbed Rabi-Hubbard model due to the presence of the tunnel effect term which is similar to those of Hubbard models in solid state physics. The tunnel effect term is diagonal in Fourier space. If the photons were decoupled from the atoms, the eigenstates would be plane waves and the eigenenergies would form a band, which is similar to what is observed for massive particles in lattices. The lowest energy state would be the state with wave vector $k = 0$. In the following we will concentrate on the resonant case where $\omega_s = \omega_l = \omega$.

This model has long been studied in the so-called rotating wave approximation (RW) (also known as the Jaynes-Cummings Hamiltonian). In this case, terms that do not conserve the total number of excitations N (N being the sum of the number of excited atoms and of the number of photons in the system) are neglected. Using this RW approximation, it was shown [10, 11] that a phenomenon similar to a photon blockade occurs, which was later observed experimentally [12]. In the photon blockade regime, the coupling between atoms and light is strong enough to stabilize a phase where, in each cavity, there is exactly one excitation, that is a quantum superposition of the excited atom and of a photon. The coupling g lowers the energy of such a state, which forbids other photons to come in the cavity. The different cavities are then decoupled, as photons are forbidden from tunneling from one cavity to the next, which is similar to the Mott insulating phase observed in condensed matter physics. When the system is driven out of this phase by varying the parameters, the tunnel effect is once again allowed and light propagation throughout the system will yield long range phase coherence where photons and atoms will settle in the same state akin to a Bose-Einstein condensation.

While the RW approximation is valid for small g, it seems not to be when g becomes of the order of ω, which is the regime that is now reached in circuit QED systems [9]. A mean-field study by Schiro *et al.* [13], taking into account the full Rabi-Hubbard Hamiltonian, showed that there would be no photon blockade. Taking into account the "counter rotating" (CR) terms (those which do not conserve the number of excitations) introduces fluctuations that destroy the blockade/Mott-like phase.

In our work [14], we studied the phase diagram of this system using exact SGF QMC simulations. It is challenging to treat exactly such systems as the number of particles changes and can become rather large. Other numerical techniques have difficulties tackling such problems. We confirmed the mean field predictions of [13] and derived the complete phase diagram of the Rabi-Hubbard model. The system adopts two phases (**Fig. 2**, left), depending on the Hamiltonian parameters: a phase where there is no coherence (but no blockade) dubbed a Rabi insulator and a phase where the systems becomes coherent, which is essentially the physics of the Dicke superradiant transition. In the incoherent phase, all correlation functions decay exponentially with distance between cavities, as in a photon blockade

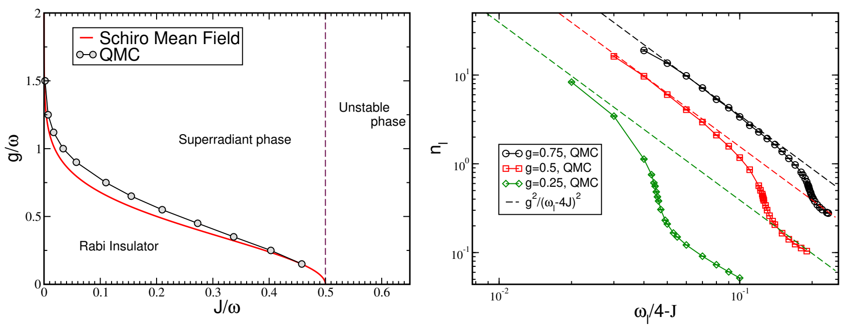


Fig. 2 (left) The QMC phase diagram of the Rabi Hubbard model, compared with mean field results. We observe two phases: an incoherent Rabi insulator and a coherent superradiant phase. There is also a region where the system is unstable for $J > \omega_l/2$. (right) Density of photons in the system, as a function of $-J$. As J approaches the unstable region ($J > \omega_l/4$ in this case), the density of photons diverges. As n_l becomes large, it reaches a mean field regime where $n_l \simeq g^2/(\omega_l - 4J)^2$. See [14] for details.

regime, but the density of excitations remains small and fluctuates, in other words the system is not gapped: It is compressible. As g and J are increased, one passes from the incoherent to the coherent phase. In the coherent phase, some correlation functions for both atoms and photons remain non zero at long distances, which means that this is collective phase where both matter and light become ordered at the same time. The density of photons becomes macroscopic in the $k = 0$ Fourier mode, which means that they condense in one state, the atoms are then "synchronised" by the collective $k = 0$ mode. Despite the fact that we have a band of photons mode, only the lowest energy one is relevant in that case, which is similar to the superradiant physics.

When the density of photons becomes large, the mean field predictions accurately describe the system (**Fig. 2**, right). For some parameters, the number of photons diverge and, beyond this limit, the Hamiltonian is not bounded (unstable region). We also studied variants of the Rabi Hubbard model to explore further the effect of the counter rotating terms on the physics of such systems [14].

In our equilibrium study, we did not find a blockade regime, except for some extreme parameters. With our Hamiltonian, the density of excitations grows with g but so do the fluctuations due to the counter rotating terms, which forbids the establishment of the blockade. On the contrary, in the RW approximation, the density of excitations is a conserved quantity, set independently of g, and we can have a large density with a value of g that is small to limit fluctuations. In experiments, that are made out of equilibrium, a similar effect happens as pumps set the density of excitations to the desired level, independently of g. However, when g is intrinsically large, we have shown that the RW description is not valid and that the full Rabi model must be used. This is the case for recent experiments that now reach the so-called ultrastrong coupling ($g \simeq \omega$) or deep strong coupling ($g > \omega$) regimes [9].

3 Bosons with cavity mediated interactions

The elusive supersolid phase has been proposed almost 60 years ago [15,16] as a phase that shows both spatial ordering and superfluid (Bose condensed) properties. This is a priori contradictory as, in a Bose condensed phase, the particles are delocalized, which generates the long range phase coherence that is typical for this phase but smears any density pattern.

Recently, one of the first observation of such a phase has been achieved for condensed bosons in a cavity [17], as a collective light matter phase. The system is a two-dimensional condensed gas of cold atoms (**Fig. 3**, (left)) placed in an optical cavity. A cavity mode of wave length λ is pumped and a laser

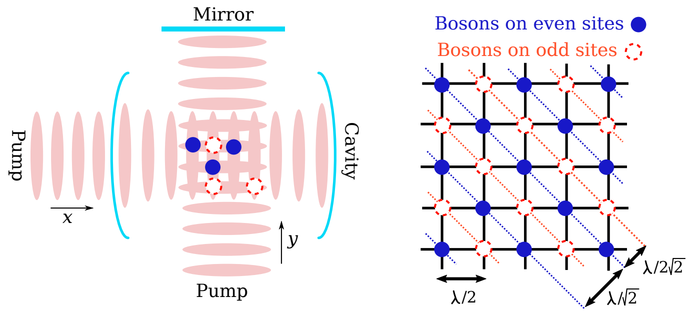


Fig. 3 (left) In the experiment [17], a 2D gas of cold atoms is subjected to two perpendicular modes in a cavity. (right) The two optical modes are coupled if the atoms adopt a chequerboard pattern that provides the right wave planes to couple modes by scattering.

of the same wave length creates, by reflection on a mirror, a perpendicular standing wave mode. This creates a square optical lattice for the atoms to move into. But, if the atoms are placed with a chequerboard arrangement in this lattice, they scatter one mode into the other, which lowers the energy. There are two such chequerboard arrangements on odd or even sites of the square lattice (**Fig. 3**, (right)).

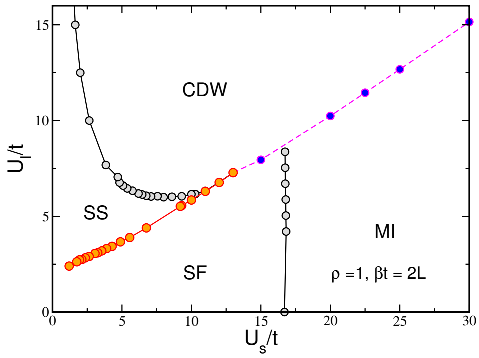
Integrating out the EM field yields an effective model for the atoms which reads

$$\hat{H} = -t\sum_{\langle i,j\rangle}\left(b_i^\dagger b_j + \mathrm{H.c.}\right) + U_s\sum_i \frac{n_i\,(n_i-1)}{2} - \frac{U_l}{L^2}\left(\sum_{i\in e} n_i - \sum_{i\in o} n_i\right)^2 . \quad (2)$$

The $b_i^\dagger$ and b_i operators create and destroy bosons on site i of a $L\times L$ optical lattice while n_i is the boson number operator on site i. The t term propagates the particles in the lattice by tunnel effect between neighbouring sites $\langle i,j\rangle$. U_s is the strength of the on site repulsion between bosons. The U_l term is the interaction mediated by the coupling to the cavity modes that favours having particles either on the even (e) sites or on the odd (o) sites of the lattice. This is an infinite range interaction as all the particles are globally coupled. In this work, we concentrate on the physics of bosons, as the light degrees of freedom are integrated out in the description, but the observed phenomenon is in fact a collective phase of matter and light and a complete description requires to take both into account.

In our work [18], we derive the phase diagram of this model by QMC techniques, especially concentrating on the case where there is, on average, a density of one particle per site $\rho = 1$. This is the experimental case [17] although, in cold atom experiments, the density varies depending on the

Fig. 4 The phase diagram for fixed density $\rho = 1$ of one particle per site as a function of U_l and U_s is qualitatively similar to the experiments' findings [17].



position in the system, which yields some differences between experimental results and our theoretical study. This model was also studied using mean-field techniques [19,20].

The superfluid nature of the system is signalled by a finite density of condensed bosons $n(\mathbf{k}=0)$ in the $\mathbf{k}=0$ mode while the chequerboard patterns give a non zero structure factor (Fourier transform of the density correlations) $S(\pi,\pi)$. The phase diagram (**Fig. 4**), as a function of U_s and U_l (t sets the energy scale), includes four phases, that were observed experimentally:

- A superfluid (SF) Bose condensed phase at low interactions ($n(\mathbf{k}=0) \neq 0$, $S(\pi,\pi) = 0$)
- An incompressible homogeneous Mott insulator (MI) phase with one particle per site when U_s dominates ($n(\mathbf{k}=0) = 0$, $S(\pi,\pi) = 0$)
- a charge density wave phase with 2 particles on even sites and 0 on odd sites (or the reverse), which breaks the translation symmetry (dubbed CDW(2,0) phase) when U_l dominates ($n(\mathbf{k}=0) = 0$, $S(\pi,\pi) \neq 0$)
- a supersolid phase between the SF and CDW(2,0) phases with both an alternating density between even and odd sites and a phase coherence ($n(\mathbf{k}=0) \neq 0$, $S(\pi,\pi) \neq 0$).

The same four phases were found in the experiment but the extent of the supersolid phase appears to be larger, compared to our data.

Numerically we explored other regimes, especially by varying the density. We found CDW phases with different patterns depending on the density : pattern (1,0) for $\rho = 1/2$ and patterns (2,1) or (3,0) for $\rho = 3/2$. For moderate U_l (**Fig. 5** (a)), varying the density, we observe an alternation of superfluid regions, CDW phases for $\rho = 1/2$ and $\rho = 3/2$ that are surrounded by supersolid phases, as expected, and a Mott phase for $\rho = 1$. On the contrary, for large U_l (**Fig. 5** (b)) the cavity mediated interaction always impose the

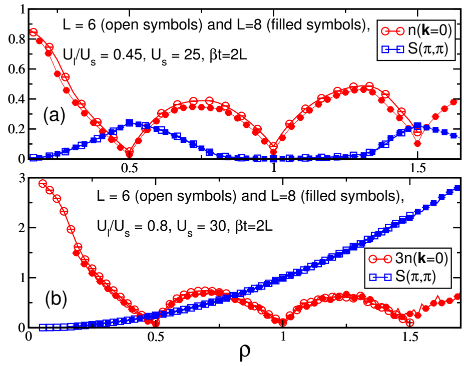


Fig. 5 Varying density, we find different charge density wave, superfluid and supersolid phases. (a) For moderate U_l we find an alternation of phases with ($S(\pi,\pi) \neq 0$) and without ($S(\pi,\pi) = 0$) density modulation. (b) For large U_l there is always density modulation.

presence of a density modulation and we only observe CDW and supersolid behaviour ($S(\pi,\pi)$ always $\neq 0$).

We also analysed in detail the nature of the phase transitions between these different phases. In some cases, we observe first order phase transitions, which shows that the system is unstable towards phase separation, for example for $\rho = 1/2$ (see [18] for details).

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