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Topological optimization of periodic materials to enhance anisotropic dispersive effects

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Abstract In the context of waves in periodic media, we propose an iterative algorithm that determines an optimal material distribution to reach target effective dispersive properties. It relies on an homogenized model of this medium, an update procedure based on the topological derivative concept, and on an efficient FFT-accelerated method to solve cell problems.

Keywords: Topological optimization, homogenization, dispersion

Introduction We are interested in standing waves in unbounded two-dimensional domains filled with a periodic medium, *e.g.* antiplane elastic shear waves. The material is made of cells $Y_\ell = \ell Y$, where Y is the reference unit cell and ℓ the characteristic periodicity length (Fig. 1). At circular frequency ω , the wave amplitude u_ℓ obeys the equation:

$$\nabla \cdot (\mu_\ell \nabla u_\ell) + \rho_\ell \omega^2 u_\ell = 0, \quad (1)$$

where μ_ℓ and ρ_ℓ are the Y_ℓ -periodic shear modulus and density of the medium.

In the long wavelength regime ($\lambda > \ell$), the motion can be described approximatively by the superposition of a slowly varying mean field U and higher-order (in $\varepsilon = \ell/\lambda$) oscillating correctors. Given an effective (homogenized) model satisfied by U , the effective dispersion of the medium is characterized by the variations of the phase velocity $c(k, \mathbf{d}) = \omega(k, \mathbf{d})/k$ of a plane wave $U(\mathbf{x}) = \exp(ik\mathbf{d} \cdot \mathbf{x})$ when the wavenumber k and direction \mathbf{d} vary. Here we adopt the following direction-dependant dispersion indicator:

$$\gamma(\mathbf{d}) := \left. \frac{\partial^2 c^2(k, \mathbf{d})}{\partial k^2} \right|_{k=0}. \quad (2)$$

In this work, the goal is to optimize the material distribution within the unit cell Y to reach some target effective dispersive properties, in particular to maximize their anisotropy. The proposed method relies on four main components: a second-order homogenized effective model, the topological derivatives of the model's coefficients, an iterative topological optimization algorithm for the unit cell, and an efficient

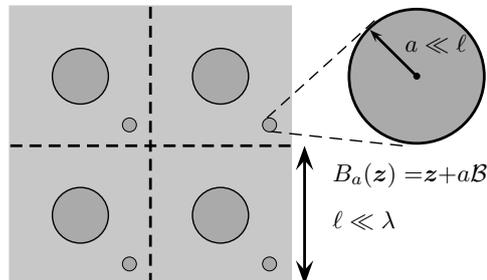


Figure 1: Periodic material and small phase change

fixed-point FFT method to evaluate these topological derivatives at each iteration. These components are now described in more details.

Second-order homogenized model The two-scale asymptotic homogenization method [1,3] is a popular way to derive an effective model from the equation (1). One obtains a fourth-order wave equation for the mean field U :

$$\begin{aligned} \mu^0 : \nabla^2 U + \omega^2 \varrho^0 U \\ + \ell^2 [\mu^2 :: \nabla^4 U + \omega^2 \varrho^2 : \nabla^2 U] = 0, \end{aligned} \quad (3)$$

where $(\mu^0, \varrho^0, \mu^2, \varrho^2)$ are constant tensors computed by solving cell problems over Y [3], “:” and “::” indicate inner products between second- and fourth-order tensors, and $\nabla^j = \nabla(\nabla^{j-1})$. Moreover, this model coincides with the one obtained by the Bloch-Floquet wave method (see [1, Sect. 3] for the case $\rho_\ell = 1$). The dispersion indicator γ defined by (2) is then simply given by:

$$\gamma(\mathbf{d}) = 2\ell^2 \left[\frac{\varrho^2 \otimes \mu^0 - \varrho^0 \mu^2}{(\varrho^0)^2} \right] :: (\mathbf{d} \otimes \mathbf{d} \otimes \mathbf{d} \otimes \mathbf{d}). \quad (4)$$

Topological optimization of the unit cell

To reach a target dispersive behavior, one first defines a cost functional to be minimized, *e.g.*

$$J(Y; \mathbf{d}_1, \mathbf{d}_2, \dots, \mathbf{d}_{N_d}) = \frac{1}{2} \left(\sum_{j=1}^{N_d} w_j [\gamma(\mathbf{d}_j)]^{\alpha_j} \right), \quad (5)$$

where each exponent α_j is fixed to 2 (resp -2) to minimize (resp. maximize) the dispersion in direction \mathbf{d}_j and the positive constants w_j are user-defined weights that balance the contribution of each term in the cost functional. This cost functional depends on the cell Y implicitly through the homogenized coefficients that intervene in the expression (4) of γ .

An iterative topological optimization algorithm, already used in microstructural optimization of static properties [2], is then adapted to the present situation. We restrain ourselves to two-phase cells, which are described thanks to a level-set function ψ . At each iteration, the update of this function is based on the concept of topological sensitivity (TS) of the cost functional to an infinitesimal phase change at point \mathbf{z} , supported by a small disc B_a of size $a \ll \ell$, as depicted in Figure 1. This TS, denoted $\mathcal{D}J$, is the leading-order coefficient of the following expansion of the perturbed cost functional J_a :

$$J_a = J + (a/\ell)^2 \mathcal{D}J(\mathbf{z}) + o((a/\ell)^2).$$

The negative minima of the map $\mathbf{z} \mapsto \mathcal{D}J(\mathbf{z})$ thus indicates the locations in Y where a small phase change would decrease J the most. Following [2], the level-set update $\psi_n \rightarrow \psi_{n+1}$ is done by partial projections of ψ_n onto the TS $\mathcal{D}J$. The algorithm stops when the optimality condition $\mathcal{D}J > 0$ is reached, indicating a local minimum of J .

Computational considerations At each iteration, the TS $\mathcal{D}J$ is computed by simple algebraic combinations of the TSs of the four coefficients $(\mu^0, \rho^0, \mu^2, \rho^2)$ whose evaluation requires the resolution of 12 direct and adjoint scalar potential problems on the current material configuration of the cell Y , as specified in [3].

To solve these problems, we adopted the FFT-accelerated algorithm proposed by Moulinec and Suquet in the 90's (see *e.g.* the introduction of [4]), that permits (i) a meshless representation of the unit cell as an image, (ii) a very simple implementation of the whole procedure and (iii) good computational performances.

An example of microstructure obtained with the proposed procedure is presented in Fig. 2. We also note that a similar study is conducted in [1], using the shape derivative of the homogenized coefficients rather than their topological derivative to update the level-set.

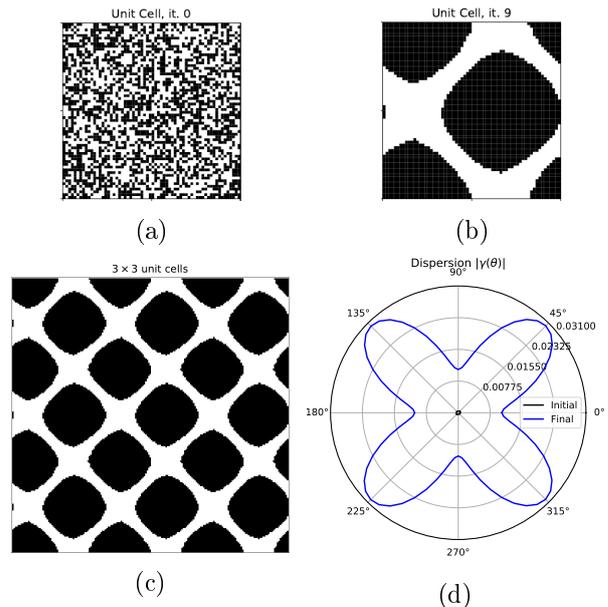


Figure 2: Microstructure optimization to maximize the dispersion in the directions $\theta = \pm\pi/4$ and minimize it in the directions $\theta = 0, \pi/2$ ($N_d = 4$ in the definition (5) of J). The unit cell is discretized into 128×128 pixels and initialized with a random distribution of materials 1 (white pixels) and 2 (black pixels). The material ratios are $\mu_2/\mu_1 = 6$ and $\rho_2/\rho_1 = 1.5$. The optimality condition was reached after 9 iterations. (a) Initial unit cell, (b) final unit cell, (c) resulting microstructure (3×3 unit cells) and (d) final dispersion indicator γ .

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