

# Image reconstruction from in-line holograms using inverse problems recipes: tutorial

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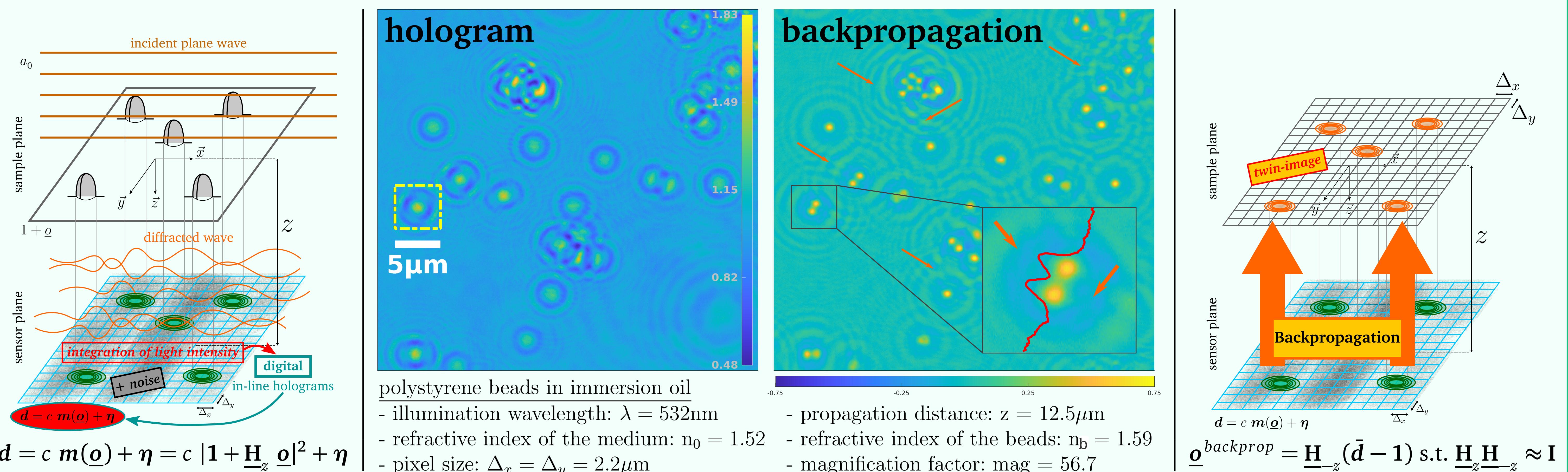
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## Abstract

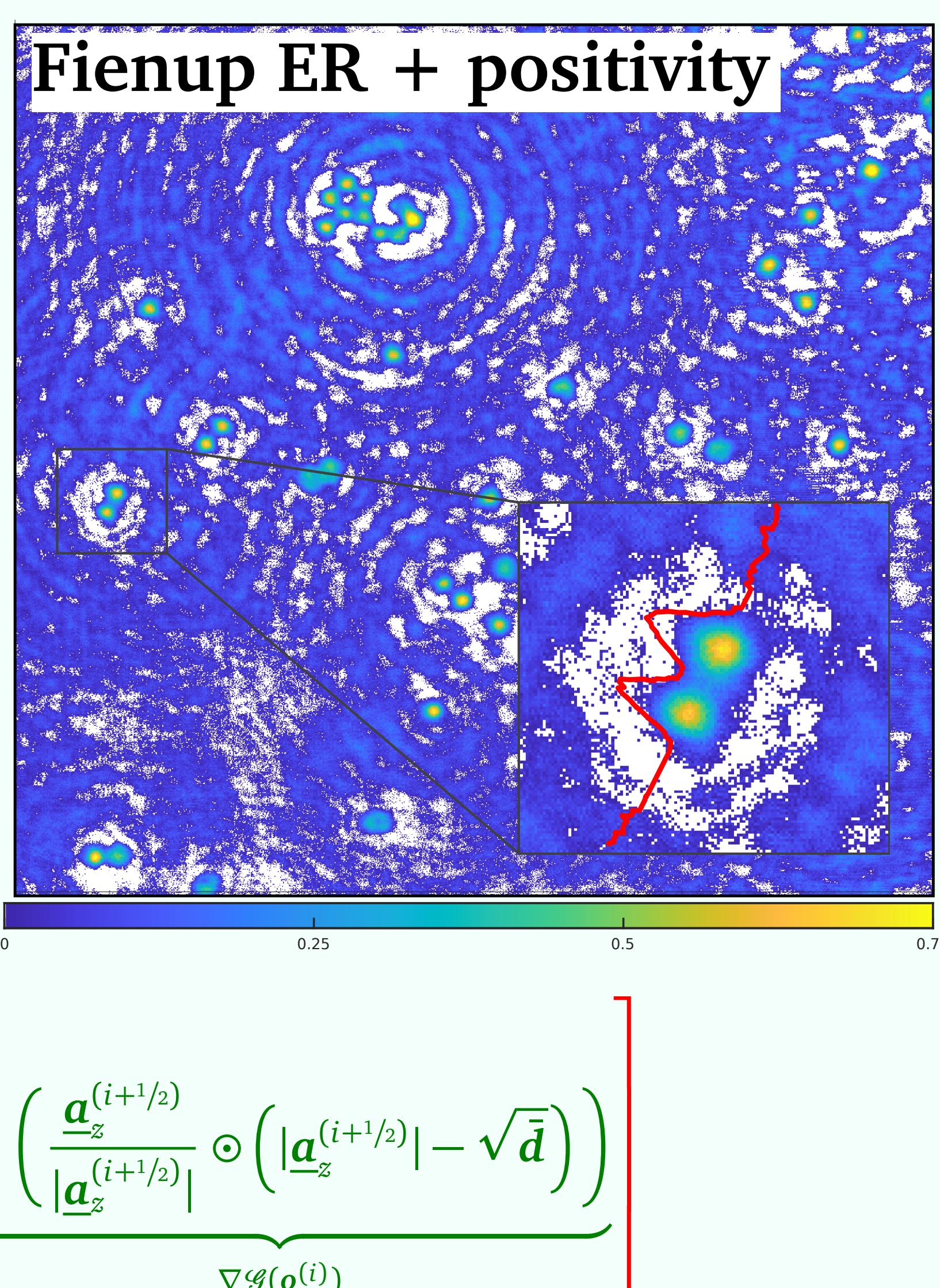
We present our tutorial [1] for building **inverse problems** based image reconstruction strategies in **digital in-line holography (DH)**, starting from the identification of the connections with the well known **alternated projections** strategies. **In-line DH is of great interest for optical microscopy dedicated to biology, as it constitutes a robust, efficient and label-free tool for imaging transparent living samples.**

## Digital Holography (DH) and the phase retrieval problem



## Alternated projections

- Iterative procedure involving a projection step on a feasible domain  $\underline{o} \in \mathbb{O}$  (e.g. positive values of the phase-shift / support constraint) to reduce/suppress twin-image artifacts [2].
- a popular instance is the **Fienup's Error Reduction algorithm (Fienup ER)**:



- gradient descent step of the data-fidelity criterion  $\mathcal{G}$  at current point  $\underline{o}^{(i)}$ .
- projection step  $\mathcal{P}_{\mathbb{O}}$  on the domain  $\mathbb{O}$ , called **proximal mapping** of the indicator function  $\mathcal{l}_{\mathbb{O}}$  at point  $\underline{o}^{(i+1/2)}$ .

This resolution method is an instance of the **ISTA** algorithm (Iterative Shrinkage Thresholding Algorithm), a proximal gradient method.

- Fienup ER is an **Inverse Problems** approach that uses a particular optimization strategy for minimizing the following criterion:

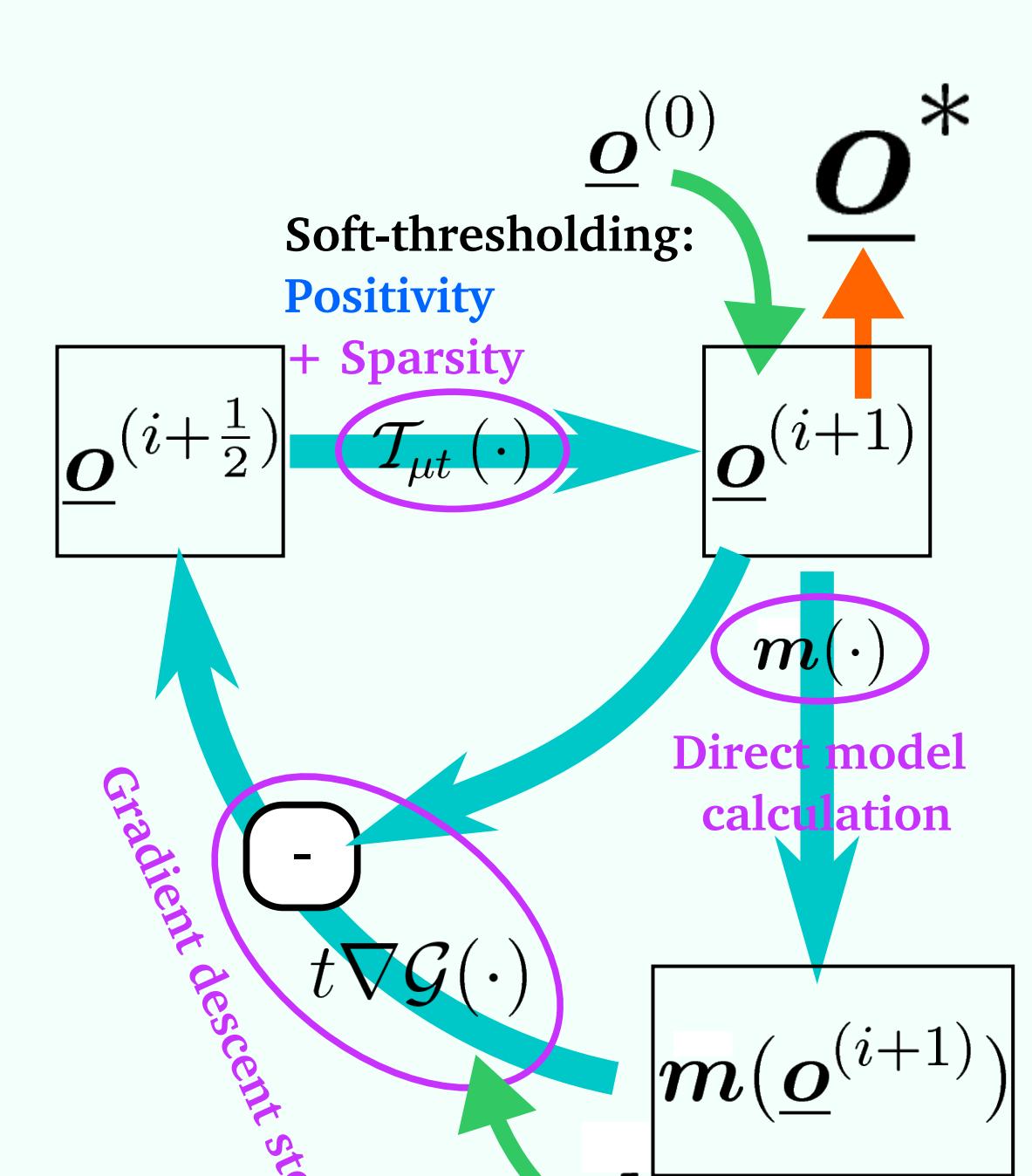
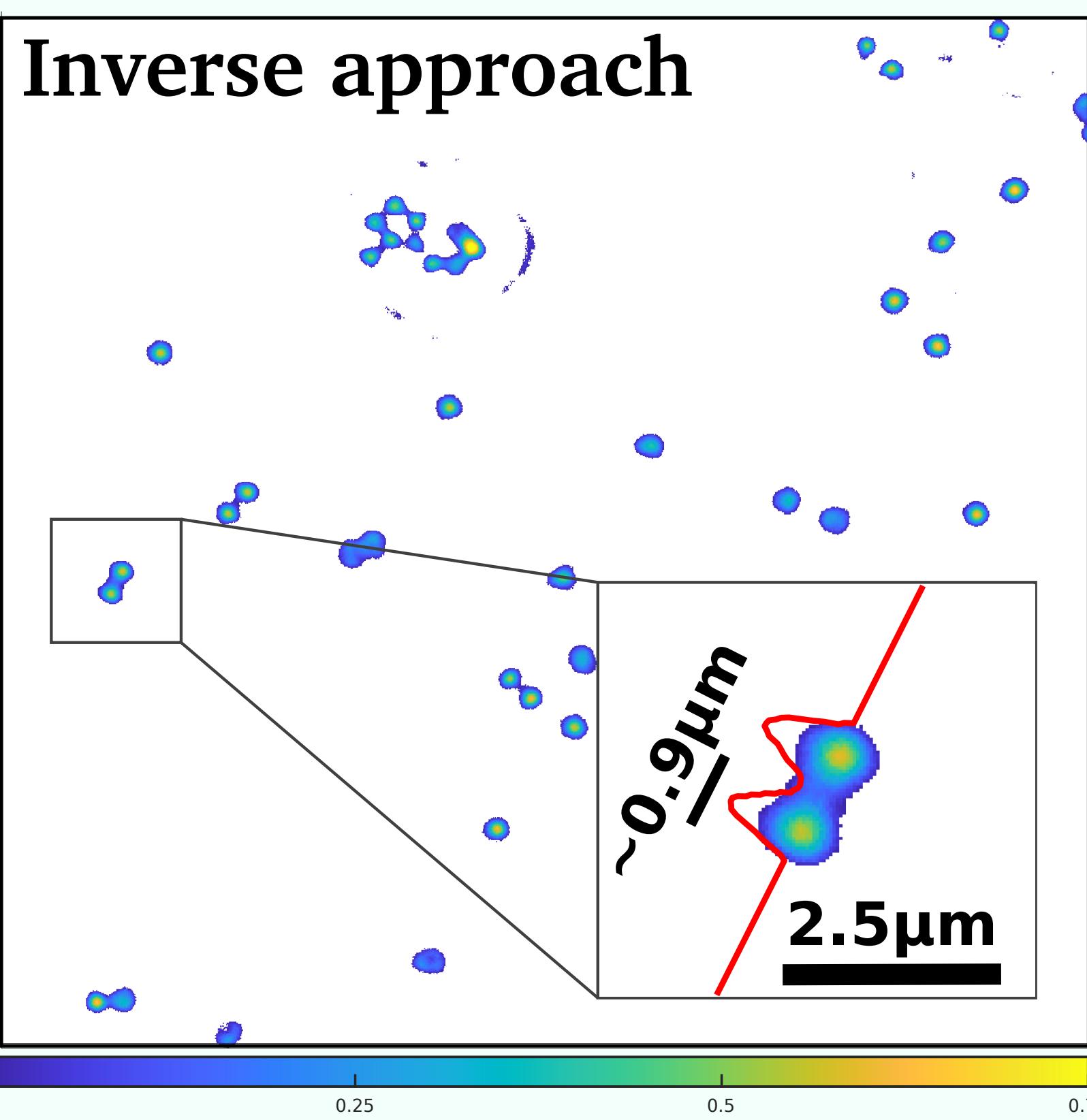
$$\underline{o}^* = \arg \min_{\underline{o}} \underbrace{\|1 + \underline{H}_z \underline{o}\|_2^2}_{\text{data-fidelity } \mathcal{G}(\underline{o}) \text{ (smooth)}} + \underbrace{\mathcal{H}(\underline{o})}_{\text{(non smooth)}}$$

## Towards inverse approaches

Alternating projections methods such Fienup ER can then benefit from the **Inverse Problems** framework to gain flexibility and reconstruction quality.

- More appropriate **direct image formation model**  $\underline{m}(\underline{o})$ .
  - auto-calibration of model parameters made possible.
- Taking into account **data uncertainties** (statistical considerations about the **noise**  $\eta$  in the criterion).
- Inclusion of **more sophisticated non smooth and smooth regularizers**: sparsity constraints ( $\ell_1$ , TV / non smooth), image gradient smoothing (Tikhonov, edge-preserving / smooth).

$$\underline{o}^* = \arg \min_{\underline{o} \geq 0} \|c^* \underline{m}(\underline{o}) - \underline{d}\|_W^2 + \mu_{\text{TV}} \sum_q \underbrace{\sqrt{\|\nabla_q \underline{o}\|_2^2 + \epsilon^2}}_{\mathcal{G}(\underline{o}) \text{ (smooth)}} + \underbrace{\mu_{\ell_1} \|\underline{o}\|_1}_{\mathcal{H}(\underline{o}) \text{ (non smooth)}}$$



## Conclusion & perspectives

- Inverse problems** constitute a rigorous framework for solving image reconstruction and/or parameter estimation tasks in **Digital Holography (DH)**.
- Inverse problems** make possible the extension of classical **2D DH** to deal with **multi-dimensional** and **multi-variate data**: fully-3D (diffractive tomography), multi-spectral, polarimetric data, etc.

A Matlab™ code for **2D DH reconstruction** is available for download at this link: [https://github.com/fabienmomey/Inline\\_Hologram\\_Reconstruction\\_by-Regularized\\_Inversion.git](https://github.com/fabienmomey/Inline_Hologram_Reconstruction_by-Regularized_Inversion.git)

## References

- [1] Momey, F., Denis, L., Olivier, T., and Fournier, C., "From Fienup's phase retrieval techniques to regularized inversion for in-line holography: tutorial," *J. Opt. Soc. Am. A* 36, D62-D80 (2019).
- [2] Fienup, J. R., "Phase retrieval algorithms: a comparison," *Appl. Opt.*, 21, 2758–2769 (1982).