

Unlocking the power of unstructured Isogeometric Analysis: a unified framework and some applications to wave propagation and elasticity.

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ABSTRACT

Isogeometric Analysis (IGA) [1] is enjoying a prosperous career in numerical analysis, having been applied with success to many engineering applications and for the solution of many types of partial differential equations. Based on the use of B-splines and NURBS functions as basis for the Finite Elements Method (FEM), its numerical advantages include an almost perfect representation of the problem geometry, less numerical noise and dispersion, longer simulation timesteps and better stability, among others. In two or more dimensions, multivariate bases are usually obtained via a simple tensor product. This approach, convenient in engineering, is impractical for applications where no Computer Aided Design (CAD) model is available and the problem topology is complex and/or unknown, limiting the penetration of the technology in fields outside engineering, such as geophysics.

Recently, some mathematical results on unstructured spline functions [3, 4, 5] have led to an alternative approach where splines are based on general point clouds. Using these functions as basis for isogeometric analysis allows to simulate a domain with arbitrary topology, including general boundary conditions. Moreover, suitable point repetition allows to reproduce the usual FEM and Discontinuous Galerkin (DG) bases, allowing the natural blending of these schemes in the same simulation, as well as a generalized multi-patch IGA-DG hybrid scheme. Some preliminary results of this method and its application to 2D wave propagation have been first presented in [6].

In our work, we explore the foundations of this method, and especially its relationship with traditional FEM/DG mesh-based approaches and the usual (tensor-product) IGA. The naturalness of our unified point of view allows us to draw some direct, simple connections between these schemes, helping shed some light on a few numerical properties that have made the success of the IGA method. We briefly discuss the numerical advantages and disadvantages of our approach in two applications: 2D and 3D wave propagation and 2D hyperelasticity. For wave propagation, we focus on the Courant–Friedrichs–Lewy timestep constraint and the shape of the mass matrix, in view of highly-parallel applications. For elasticity, we explore the advantages coming from the lack of a fixed mesh in systems subject to large amounts of deformation. Finally, we discuss some possible ways forward and avenues of improvement that could turn our approach into a true general-purpose unstructured multi patch DG-IGA scheme.

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