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Joint Demosaicing and Fusion of Multiresolution Compressed Acquisitions: Image Formation and Reconstruction Methods

Daniele Picone , *Member, IEEE*, Mauro Dalla Mura , *Senior Member, IEEE*, and Laurent Condat , *Senior Member, IEEE*

Abstract—Novel optical imaging devices allow for hybrid acquisition modalities such as compressed acquisitions with locally different spatial and spectral resolutions captured by the same focal plane array. In this work, we propose to model a multiresolution compressed acquisition (MRCA) in a generic framework, which natively includes acquisitions by conventional systems such as those based on spectral/color filter arrays, compressed coded apertures, and multiresolution sensing. We propose a model-based image reconstruction algorithm performing a joint demosaicing and fusion (JoDeFu) of any acquisition modeled in the MRCA framework. The JoDeFu reconstruction algorithm solves an inverse problem with a proximal splitting technique and is able to reconstruct an uncompressed image datacube at the highest available spatial and spectral resolution. An implementation of the code is available at <https://github.com/danaroth83/jodefufu>.

Index Terms—Color filter array, compressed acquisitions, pan-sharpening, data fusion, demosaicing, multiresolution sensors, nonconventional optical devices.

I. INTRODUCTION

CONVENTIONAL cameras acquire images that are immediately exploitable by the end user with little or no processing of the raw acquisition. When the acquisition relies on a spectral or spatial scanning of the scene, these cameras provide an image as a *datacube* with spatial and spectral dimensions [1], [2]. For example, this is the case of red-green-blue (RGB), multispectral (MS) and hyperspectral (HS) imaging systems, where the datacube has either three, up to a few tens, or more channels, respectively.

A different acquisition approach, following the *computational imaging* paradigm [3]–[6], is based on *compressed acquisitions*. In some cases, this allows to perform acquisitions that are instantaneous (i.e., snapshot) or with a lower number of acquired samples with respect to a conventional full scanning of the datacube. Compressed acquisitions [7], [8] are not necessarily captured in the end user desired domain and need a computational phase to retrieve a datacube that is

intelligible to the final user. In this work, we refer to such acquisition techniques as **image formation** methods, and to the required processing algorithms that recover the desired datacube as **image reconstruction** methods.

Multiple examples of image formation methods are available in the literature, but the interest of this work is mostly focused to two classic acquisition scenarios, the multiresolution sensing and the mosaicing, which are described below:

- The **multiresolution sensing**, shown in Fig. 1a, is an acquisition setup which addresses the technical constraint of single sensors which are not capable of simultaneously achieving the desired spatial and spectral resolution. In this scenario, different technologies for the sensors are employed to provide complementary information of the same scene to be fused in the processing stage. Most commonly, the product is available as a bundle of two images: the high resolution image (HRI), with high spatial and low spectral resolution, and the low resolution image (LRI), with low spatial and high spectral resolution. In the data fusion phase, known as **sharpening**, the target is to produce a synthetic image with the highest available resolutions both in the spectral and spatial domain. This is a more generic formulation of the *pansharpening* problem [9], [10], where the target is to fuse a monochromatic acquisition, known as panchromatic (PAN), and an MS.
- The **mosaicing**, shown in Fig. 1b, is an acquisition technique where the captured samples are the output of a set of image sensors distributed over a focal plane array (FPA) and overlaid with an array of filters, known as *color filter array (CFA)* or *multispectral filter array (MSFA)* [11]. As an effect of filtering, each captured image pixel is associated to a given color/channel component and the full raw acquisition is composed by a mosaic of such components. For example, the reader may be familiar with CFA designs such as the Bayer pattern [12], where the filters are arranged in periodic 2×2 RGB squares with 2 repeated green filters placed on the opposite vertices. The associated image reconstruction method, known as *demosaicing*, consists in recovering the full spectral component of the image at each available position in the FPA.

In some more sophisticated mosaicing-based compressed acquisition systems, a given pixel can also capture a generic linear combination of samples associated to different channels.

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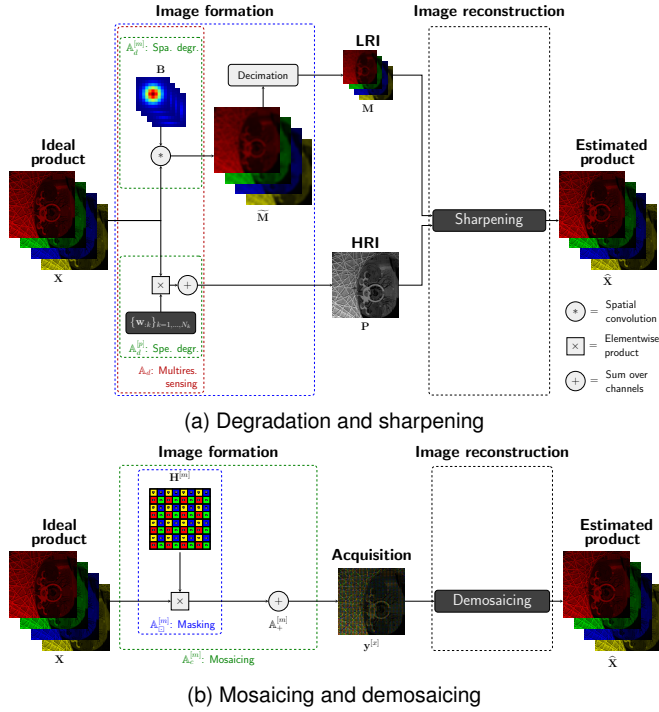


Fig. 1: Classic formulation of the sharpening and demosaicing problem. The red, green, blue, and yellow slices represent each of the available channels of the image to reconstruct.

For example, this is the case of the *compressive coded aperture spectral imaging (CASSI)* [13], for which each available channel is firstly masked with a digital micromirror device and shifted horizontally over the focal plane before recombination.

More recently, both the scientific community and device manufacturers are showing interest for the design of hybrid systems, where compressed acquisitions might have different spatial/spectral resolutions. Specifically, for image formation methods, a series of *RGBW patterns* were proposed where a set of wideband pixels are interleaved to a more classic RGB pattern. This is the case, i.e., of the *Onyx* device by Teledyne e2v [14] and some patents deposited by Kodak [15]. A similar effect could be obtained with the *COLOR SHADES* [16], a technology which allows for a fully customizable spectral response for each filter on the FPA. We can interpret these acquisition as hybrid, since the wideband pixels can be seen as a binning in the spectral domain, and consequently have a different spectral resolution with respect to the color ones.

Modern commercial CFA patterns, such as the *Quad Bayer*, are also starting to implement mechanism of spatial binning across adjacent pixels, in order to improve the SNR of the detected photons in conditions of low illumination [17]. More recently, a novel technology was proposed to focus the incident light rays over photodiodes through customly manufactured microlenses [18]. Consequently, the resulting samples are effectively at a lower spatial resolution with respect to non-binned or less focused alternatives.

Espitia et al. [19] suggested to capture both the LRI and the HRI as separate CASSI acquisitions, introducing a joint Bayesian framework for the reconstruction of the full resolu-

tion image. Fu et al. [20] proposed instead a reconstruction algorithm where a HS image is fused with a MS mosaiced image obtained with a CFA acquisition system. Takeyama and Ono [21] address the *compressed pansharpening*, where the quality of a noisy HRI is restored with the help of an associated compressed acquisition of a LRI.

Armed with this knowledge, the main aim of this work is to develop an unified framework for both image formation and reconstruction which includes all the previously cited examples. For this reason, in the context of image formation methods, we propose the **multiresolution compressed acquisition (MRCA)** framework, a formalization of the multiresolution acquisition model with compressed acquisitions, which includes the previously described devices.

Additionally, such hybrid devices are a reasonable proof that, in the near future, multiresolution sensors could potentially be accommodated over the same focal plane. This is a trend that we aim to intercept with this work, as this model can formalize the design of an optical device based on the assumption that sensors with different characteristics can be accommodated on the same FPA and that the resulting compressed acquisition contains partial information both from the LRI and the HRI, whose physical implementation was presented in our previous work [22]. An example the acquisitions of such device is shown in Fig. 2, where it is shown how the MRCA models the acquisition of a monochromatic raw image which includes samples from both a PAN and a MS, according to arrangement shown in Fig. 3b.

Since we have a unified acquisition model, we also propose the **joint demosaicing and fusion (JoDeFu)**, a generic image reconstruction algorithm that address both the demosaicing and multiresolution fusion, in an extended definition from our previous work [23]. The proposed algorithm both recovers the missing information of the compressed acquisition and fuses the multiresolution samples to reach the maximum available spatial and spectral resolution. The algorithm, which makes use of a Bayesian framework, is not exclusively a demosaicing-style image reconstruction, as we aim to reconstruct a fused product, nor it is a simple fusion, as the acquisition is not given by well-distinguished multimodal sources, but rather by a lossy compressed combination of the two.

The novel contributions of this work include:

- the definition of the MRCA, a flexible model for multiresolution sensors sharing a common focal plane, which includes a series of well-known image formation methods, such as the CFA [24] acquisitions and the multiresolution sensing;
- the derivation of some properties of the direct model operator associated to the MRCA, which enable its use within proximal algorithms;
- the definition of the JoDeFu, an image reconstruction framework capable of simultaneously addressing the problem of demosaicing and the fusion of partial multiresolution acquisition;
- a comparison of the performances of the JoDeFu with respect to classic image reconstruction methods for compressed acquisitions; we also analyze the reconstructed

TABLE I: Notation for the multidimensional arrays defined in this paper. Each variable can be expressed either in their classical order or by its lexicographic order, by reshaping the spatial dimensions of the former into a single dimension.

Variable	Classic representation					Lexicographic order representation				
	Symbol	Dimensions (Row $\times$ col. $\times$ band)	k -th band	(i, j) -th pixel	(i, j, k) -th element	Symbol	Dimensions (Pixel $\times$ band)	k -th band	i -th pixel	(i, k) -th element
Reference	$\mathcal{U}^{[x]}$	$N_i \times N_j \times N_k$	$\mathbf{U}_{::k}^{[x]}$	$\mathbf{u}_{ij:}^{[x]}$	$u_{ijk}^{[x]}$	$\mathbf{X}$	$N_i N_j \times N_k$	$\mathbf{x}_{:k}$	$\mathbf{x}_i$	x_{ik}
HRI	$\mathcal{U}^{[p]}$	$N_i \times N_j \times N_p$	$\mathbf{U}_{::k}^{[p]}$	$\mathbf{u}_{ij:}^{[p]}$	$u_{ijk}^{[p]}$	$\mathbf{P}$	$N_i N_j \times N_k$	$\mathbf{p}_{:k}$	$\mathbf{p}_i$	p_{ik}
LRI	$\mathcal{U}^{[m]}$	$\frac{N_i}{\rho} \times \frac{N_j}{\rho} \times N_k$	$\mathbf{U}_{::k}^{[m]}$	$\mathbf{u}_{ij:}^{[m]}$	$u_{ijk}^{[m]}$	$\mathbf{M}$	$\frac{N_i N_j}{\rho^2} \times N_k$	$\mathbf{m}_{:k}$	$\mathbf{m}_i$	m_{ik}
Upscaled LRI	$\widetilde{\mathcal{U}}^{[m]}$	$N_i \times N_j \times N_k$	$\mathbf{U}_{::k}^{[\widetilde{m}]}$	$\mathbf{u}_{ij:}^{[\widetilde{m}]}$	$u_{ijk}^{[\widetilde{m}]}$	$\widetilde{\mathbf{M}}$	$N_i N_j \times N_k$	$\widetilde{\mathbf{m}}_{:k}$	$\widetilde{\mathbf{m}}_i$	$\widetilde{m}_{ik}$
Acquisition	$\mathbf{Y}$	$N_i \times N_j \times 1$	-	y_{ij}	y_{ijk}	$\mathbf{y}$	$N_i N_j \times 1$	-	y_i	y_i
Estimated product	$\mathcal{U}^{[\widehat{x}]}$	$N_i \times N_j \times N_k$	$\mathbf{U}_{::k}^{[\widehat{x}]}$	$\mathbf{u}_{ij:}^{[\widehat{x}]}$	$u_{ijk}^{[\widehat{x}]}$	$\widehat{\mathbf{X}}$	$N_i N_j \times N_k$	$\widehat{\mathbf{x}}_{:k}$	$\widehat{\mathbf{x}}_i$	$\widehat{x}_{ik}$
Blurring kernels	$\mathcal{U}^{[b]}$	$N_b \times N_b \times N_k$	$\mathbf{U}_{::k}^{[b]}$	$\mathbf{u}_{ij:}^{[b]}$	$u_{ijk}^{[b]}$	$\mathbf{B}$	$N_b^2 \times N_k$	$\mathbf{b}_{:k}$	$\mathbf{b}_i$	b_{ik}
HRI mask	-	-	-	-	-	$\mathbf{H}^{[p]}$	$N_i N_j \times N_p$	$\mathbf{h}_{:k}^{[p]}$	$\mathbf{h}_i^{[p]}$	$h_{ik}^{[p]}$
LRI mask	-	-	-	-	-	$\mathbf{H}^{[m]}$	$N_i N_j \times N_k$	$\mathbf{h}_{:k}^{[m]}$	$\mathbf{h}_i^{[m]}$	$h_{ik}^{[m]}$

products taken with the proposed MRCA, when they embed a different amount of MS bands, and compare the JoDeFu with alternative reconstruction approaches obtained by cascading a set of classical algorithms.

The paper is organized as follows: Section II introduces the notation, Section III describes the MRCA, highlighting how it expands on classic image formation methods; Section IV presents the JoDeFu algorithm, and Section V provides the related experiments.

II. NOTATION

In this paper, we denote:

- *scalars* with lowercase non-bold letters (e.g. u);
- *vectors* with lowercase bold letters (e.g. $\mathbf{u}$);
- *matrices* with uppercase bold letters (e.g. $\mathbf{U}$);
- *tensors* (that is, arrays with more than two dimensions) with bold italic fonts (e.g. $\mathcal{U}$).

A detailed description of the variables used in this paper is given in TABLE I. In this work, the images are represented both in its classic form, as a 3-way tensor whose dimensions are its rows, columns and channels (e.g., $\mathcal{U}^{[x]} \in \mathbb{R}^{N_i \times N_j \times N_k}$) or in *lexicographic order*, by column concatenation of the first two dimensions into one. As both representations contain the same samples, we can switch from the first to the second form without any loss of information, and we denote this operation with $\text{matr}(\cdot)$ (e.g., $\mathbf{X} = \text{matr}(\mathcal{U}^{[x]})$, where $\mathbf{X} \in \mathbb{R}^{N_i N_j \times N_k}$).

When we select a generic k -th slice of the image, this is denoted with a subscripted index k , while the non-sliced dimensions are denoted with a colon (e.g., $\mathbf{U}_{::k}^{[x]}$ denotes the k -th band of $\mathcal{U}^{[x]}$). When confusion may arise, the subscripted indices are separated by a comma (e.g., $\mathbf{U}_{:,i,k}^{[x]}$ is equivalent to $\mathbf{U}_{:,k}^{[x]}$). Once again, a more detailed description of such operations is shown in TABLE I.

Finally, $\|\cdot\|_2$ and $\|\cdot\|_F$ denote the ℓ_2 and Frobenius norm, respectively.

III. PROPOSED IMAGE FORMATION MODEL

In this section, we introduce the mathematical model of the MRCA (Section III-C) and its properties (Section III-D) after defining the image formation models of its two main components: the multiresolution sensing (Section III-A) and the mosaicing (Section III-B).

A. Multiresolution sensing

When multiresolution acquisitions are involved, each of the sensors is characterized by a limited spatial and spectral resolution. In its most general sense, the multiresolution sensing setup is expressed as a whole set of multiple acquisitions, which present a certain spatial and/or spectral degradation with respect to the ideal datacube to reconstruct $\mathbf{X}$. For the sake of exposition but without loss of generalization, we limit our analysis to the most common scenario in the literature [9], [10], [25], in which the acquisition is composed by an HRI and an LRI. The HRI $\mathbf{P}$ and the LRI $\widetilde{\mathbf{M}}$ are respectively obtained as a degradation in the spectral and spatial domain, so that:

$$\begin{cases} \mathbf{P} &= \mathbb{A}_d^{[p]}(\mathbf{X}), \\ \widetilde{\mathbf{M}} &= \mathbb{A}_d^{[m]}(\mathbf{X}), \end{cases} \quad (1)$$

Specifically:

- the **spectral degradation** operation $\mathbf{P} = \mathbb{A}_d^{[p]}(\mathbf{X})$ is described by a linear combination in the form:

$$\mathbf{p}_{:j} = \sum_{k=1}^{N_k} w_{jk} \mathbf{x}_{:k}, \quad \forall j \in [1, \dots, N_p] \quad (2)$$

where $\{w_{jk}\}_{j \in [1, \dots, N_p], k \in [1, \dots, N_k]}$ are the *weight coefficients* associated to the spectral responses of the sensors. This is a widespread choice in the literature, as the sensors perform an integration of the incoming radiance that is modulated by the spectral response of the filters [1].

- the **spatial degradation** operation $\widetilde{\mathbf{M}} = \text{matr}(\mathcal{U}^{[\widetilde{m}]}) = \mathbb{A}_d^{[m]}(\mathbf{X})$ is a convolution by a set of filters $\mathcal{U}^{[b]} \in \mathbb{R}^{N_b \times N_b \times N_k}$. Specifically:

$$\mathbf{U}_{::k}^{[\widetilde{m}]} = \mathbf{U}_{::k}^{[x]} * \mathbf{U}_{::k}^{[b]}, \quad \forall k \in [1, \dots, N_k], \quad (3)$$

where $*$ denotes a spatial convolution operator. Once again, this is a widely employed model in the literature, derived from the assumption that the blurring effect of the sensors can be described by a linear translation-invariant operator, and hence uniquely defined by its *modulation transfer function (MTF)* [25].

Finally, a decimation of $\widetilde{\mathbf{M}}$ by a scale factor ρ (i.e., taking every ρ samples both in the vertical and horizontal direction) produces the actual LRI $\mathbf{M}$.

B. Mosaicing

In its classic formulation [11], shown in Fig. 1b, the CFA/MSFA-based mosaicing is uniquely defined by a mask $\mathbf{H} \in \mathbb{R}^{N_i \times N_k}$. *Binary masks* define a special case of $\mathbf{H}$ whose elements can only be either zeros or ones; in other terms, each pixel of the mask identifies a specific channel, which is transferred on the focal plane. Binary masks can be hence represented by a color-coded matrix and some examples are shown in Fig. 3.

The acquisition $\mathbf{y} = \mathbb{A}_c^{[m]}(\mathbf{X})$ due to the mosaicing is modeled as the cascade of the following two operations:

- **Masking** $\mathbf{X}^\square = \mathbb{A}_\square^{[m]}(\mathbf{X})$: where an element-wise multiplication (denoted by $\odot$) is applied independently on each band, yielding:

$$\mathbf{x}_{:,k}^\square = \mathbf{x}_{:,k} \odot \mathbf{h}_{:,k}, \quad \forall k \in [1, \dots, N_k]. \quad (4)$$

If the mask is binary, the variable $\mathbf{x}_{:,k}^\square$ is commonly known as **sparse channel** in the demosaicing literature [26], since the element-wise multiplication sets most of its values to zero.

- **Sum over channels** $\mathbf{y} = \mathbb{A}_+^{[m]}(\mathbf{X}^\square)$: where the final observation is obtained by summing along the spectral dimension, returning:

$$\mathbf{y} = \sum_{k=1}^{N_k} \mathbf{x}_{:,k}^\square = \sum_{k=1}^{N_k} \mathbf{x}_{:,k} \odot \mathbf{h}_{:,k}. \quad (5)$$

In some more advanced acquisition devices, some optical elements allow to shift the captured light rays of a given channel over the FPA. To model such effect, we introduce an additional **shifting operator** $\mathbb{A}_{\rightarrow}^{[m]}$ in between the previously presented ones, so that the full operation of mosaicing becomes:

$$\mathbf{y} = \mathbb{A}_c^{[m]}(\mathbf{X}) = \mathbb{A}_+^{[m]} \left(\mathbb{A}_{\rightarrow}^{[m]} \left(\mathbb{A}_\square^{[m]}(\mathbf{X}) \right) \right). \quad (6)$$

The generic element s_{ijk} of the shifted image $\mathcal{S}$ in the operation $\text{matr}(\mathcal{S}) = \mathbb{A}_{\rightarrow}^{[m]}(\mathbf{X})$ is given by:

$$s_{ijk} = u_{\mathbf{r}(i,j,k)}^{[x]}, \quad (7)$$

where $\mathbf{r}(i, j, k) : \mathbb{N}^3 \rightarrow \mathbb{N}^3$ is a one-to-one vector function which defines the transformation from a source to a target position.

For example, in the case of the single dispersion CASSI [13], the focal plane associated to each channel can be rigidly translated in the horizontal direction through a diffraction prism. In the problem defined by the original authors, this is a shift by one pixel between adjacent channels, which we reformulated within the MRCA framework by defining a shifted input $\mathcal{S} \in \mathbb{R}^{N_i \times (N_j + N_k - 1) \times N_k}$ such that:

$$\mathbf{r}(i, j, k) = (i, j + k - 1, k). \quad (8)$$

C. Multiresolution compressed acquisition (MRCA)

The main target of the proposed MRCA is to allow for multiresolution images to be stored over the same focal plane. The proposed model $\mathbf{y} = \mathbb{A}(\mathbf{X})$ for the acquisition system is shown in Fig. 2, and is expressed as:

$$\mathbb{A}(\mathbf{X}) = \mathbb{A}_+ \left(\mathbb{A}_b(\mathbb{A}_c^{[p]}(\mathbb{A}_d^{[p]}(\mathbf{X}))), \mathbb{A}_c^{[m]}(\mathbb{A}_d^{[m]}(\mathbf{X})) \right). \quad (9)$$

TABLE II: Description of classical image formation methods under the proposed MRCA framework. In this table, $\mathbf{X}$ marks linear operators of the MRCA being substituted by an identity.

Image formation method	Multires.		HRI mosaicing			LRI mosaicing		
	$\mathbb{A}_d^{[p]}$	$\mathbb{A}_d^{[m]}$	$\mathbb{A}_\square^{[p]}$	$\mathbb{A}_{\rightarrow}^{[p]}$	$\mathbb{A}_+^{[p]}$	$\mathbb{A}_\square^{[m]}$	$\mathbb{A}_{\rightarrow}^{[m]}$	$\mathbb{A}_+^{[m]}$
MRCA	✓	✓	✓	✓	✓	✓	✓	✓
Multiresolution sensing	✓	✓	✗	✗	✗	✓ ¹	✗	✗
CFA mosaicing	✓ ²	✗	✗	✗	✓	✓	✗	✓
CASSI acquisition [13]	✓ ²	✗	✗	✗	✓	✓	✓ ³	✓
Lu et al. [28]	✓	✓	✓	✗	✓	✗	✗	✗
Takeyama and Ono [21]	✓	✓	✗	✗	✗	—	✓ ⁴	✗

¹ where $\mathbf{H}^{[m]}$ is a binary mask, with zeros at the interleaved positions of the LRI;

² with $w_{kl} = 0, \forall k \in [1, \dots, N_p], l \in [1, \dots, N_k]$ to suppress HRI samples;

³ with the condition of eq. (8);

⁴ with any kind of linear compression is allowed for the LRI.

The operator $\mathbb{A}$ is given by the following cascade of operations:

- A **multiresolution sensing operator** $\mathbb{A}_d$, composed by the operators $\mathbb{A}_d^{[m]}$ from eq. (3) and $\mathbb{A}_d^{[p]}$ from eq. (2) of Section III-A, which generate the LRI and HRI branch, respectively;
- A **mosaicing operator** $\mathbb{A}_c$: where the operator $\mathbb{A}_c^{[m]}$ of eq.(6) is applied on the LRI branch and $\mathbb{A}_c^{[p]}$, identical to $\mathbb{A}_c^{[m]}$ except for acting over N_p channels instead of N_k , and applied to the HRI branch.
- A **blur operator** $\mathbb{A}_b$, to adjust its scale ratio of the HRI with respect to the reference and modeled as a spatial convolution.
- A **sum operator** $\mathbb{A}_+(\mathbf{y}^{[m]}, \mathbf{y}^{[p]})$, which sums the LRI mosaic $\mathbf{y}^{[m]}$ and HRI mosaic $\mathbf{y}^{[p]}$ over the same focal plane:

$$\mathbb{A}_+(\mathbf{y}^{[m]}, \mathbf{y}^{[p]}) = \mathbf{y}^{[m]} + \mathbf{y}^{[p]}. \quad (10)$$

The reader may have noticed that the upscaled LRI $\widetilde{\mathbf{M}}$ is not decimated in the MRCA pipeline; in this framework the step is unnecessary as the mask $\mathbf{H}^{[m]}$ can automatically suppress the pixel values that would be eliminated during the decimation process.

The MRCA can model (assuming that no blur is introduced by $\mathbb{A}_b$):

- *multiresolution sensing*, if both the operators $\mathbb{A}_c$ and $\mathbb{A}_+$ are substituted with an identity, as eq. (9) reduces to eq. (1);
- *mosaicing*, if both operators in $\mathbb{A}_d^{[m]}$ is an identity and $\mathbf{y}^{[p]}$ is an all-zero matrix, for which eq. (9) reduces instead to eq. (6).

Therefore, the MRCA framework is a general formation method for acquisition systems that involve multiresolution sensing and mosaicing. A more detailed description of the operators for such special cases, which also includes the frameworks proposed by Li et al. [27] by Takeyama and Ono [21], is given in TABLE II.

D. Properties of the direct model operators

We investigate here the two key properties of the operator $\mathbb{A}$, the adjoint operator and the operator norm, that are necessary to define the generic image reconstruction algorithm to be presented in Section IV. Specifically:

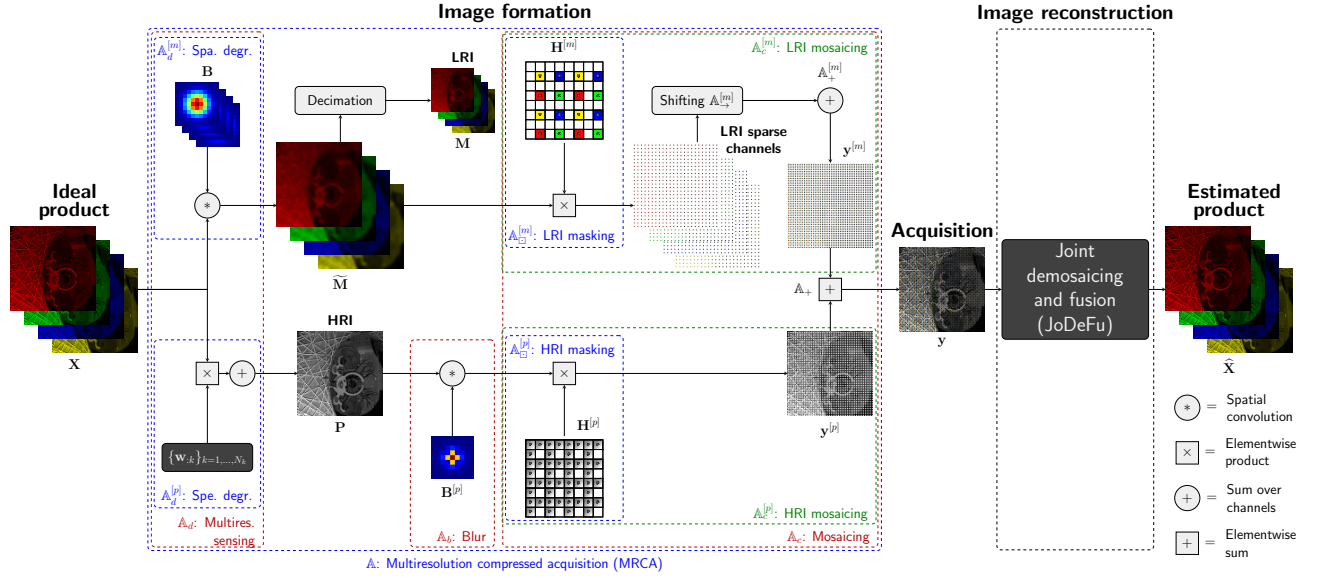


Fig. 2: Representative scheme of the image formation model for the proposed MRCA framework. The multiresolution configuration is represented by a PAN and by a color coded 4-band MS; the white spots in the masks denote zero pixels in all bands. The HRI shifting $\mathbb{A}_{\rightarrow}^{[p]}$ and summing $\mathbb{A}_{+}^{[p]}$ operators are not depicted.

- the **adjoint operator** $\mathbb{A}^*$ is the one that verifies the condition:

$$\langle \mathbb{A}(\mathbf{X}), \mathbf{y} \rangle = \langle \mathbf{X}, \mathbb{A}^*(\mathbf{y}) \rangle, \quad (11)$$

for all $\mathbf{X} \in \mathbb{R}^{N_i N_j \times N_k}$ and $\mathbf{y} \in \mathbb{R}^{N_i N_j}$. Here, $\langle \cdot, \cdot \rangle$ on the left and right side of the equation are the scalar products of the spaces of $\mathbf{y}$ and $\mathbf{X}$, respectively. If $\mathbb{A}$ defines a matrix multiplication applied over vectorized inputs, $\mathbb{A}^*$ is equivalent of applying the Hermitian of the same matrix;

- the **operator norm** $\|\mathbb{A}\|_{op}$ is defined [29] as the smallest scalar γ such that:

$$\|\mathbb{A}(\mathbf{X})\|_2 \leq \gamma \|\mathbf{X}\|_F, \quad \forall \mathbf{X} \in \mathbb{R}^{N_i N_j \times N_k}. \quad (12)$$

If $\mathbb{A}$ defines a matrix multiplication over vectorized inputs, its operator norm is the largest singular value of the matrix itself [29].

These properties are dynamically constructed by combining elementary building blocks, which are associated to the simple operations that were defined in the previous sections, in order to produce the image formation model under test. After defining how to combine them, we will derive the properties for each of these elementary blocks.

The adjoint of a composed operation $\mathbb{A}(\mathbf{X}) = \mathbb{A}_c(\mathbb{A}_d(\mathbf{X}))$ is equal to applying the individual adjoint operators in reverse order:

$$\mathbb{A}^*(\mathbf{y}) = \mathbb{A}_d^*(\mathbb{A}_c^*(\mathbf{y})). \quad (13)$$

Consequently, the only requirement to identify $\mathbb{A}^*$ is to derive the adjoint operator of each component of the MRCA separately. These components can be split into the following categories:

- Spatial convolution** (Operators $\mathbb{A}_d^{[m]}$, and $\mathbb{A}_b$): the adjoint of a convolution by a given kernel is a correlation by the same kernel;

- Sum over channels** (Operators $\mathbb{A}_{+}^{[m]}$, $\mathbb{A}_{+}^{[p]}$, and $\mathbb{A}_{+}$): its adjoint is equivalent to replicating a monochromatic image across all bands;
- Shifting** (Operators $\mathbb{A}_{\rightarrow}^{[m]}$, and $\mathbb{A}_{\rightarrow}^{[p]}$): The adjoint of shifting a sample to a new position is a shift back to its original one.
- Element-wise product** (Operators $\mathbb{A}_d^{[p]}$, $\mathbb{A}_{\square}^{[p]}$, and $\mathbb{A}_{\square}^{[m]}$): The adjoint operator is itself, as this operation is self-adjoint.

We follow a similar approach for the operator norm. Specifically, we apply the Cauchy inequality to a composed operator $\mathbb{A}(\mathbf{x}) = \mathbb{A}_c(\mathbb{A}_d(\mathbf{x}))$ to identify an upper limit for $\|\mathbb{A}\|_{op}$:

$$\|\mathbb{A}\|_{op} \leq \|\mathbb{A}_c\|_{op} \|\mathbb{A}_d\|_{op}. \quad (14)$$

This inequality can be substituted with a strict equality as the reconstruction algorithms we employ only require upper bounds for the operator norm. Nonetheless, we can once again separate the problem into evaluating the operator norms individually:

- Spatial convolution of X by B:** for each band, the convolution by the k -th band $\mathbf{b}_{:k}$ can be rewritten as a multiplication by a circulant matrix. Its singular values are hence defined as the sum of the coefficients $\mathbf{b}_{:k}$ weighted by the complex roots of unity [30]. A conservative estimate for this operator norm (e.g., for $\mathbb{A}_d^{[m]}$) is then given by:

$$\|\mathbb{A}_d^{[m]}\|_{op} = \max_{k \in [1, \dots, N_k]} \sqrt{\sum_{i=1}^{N_b^2} b_{ik}^2}; \quad (15)$$

- Sum over N_k channels:** the operator norm upper bound is $\sqrt{N_k}$, as a result of the triangular inequality applied over every pixel;

- **Shifting:** As shifting is assumed to be a one-to-one operation, the intensity values of each sample are unchanged, hence the operator norm is unitary;
- **Element-wise product by $\mathbf{H}$:** As pixels are scaled by the intensity value of $\mathbf{H}$, the operator norm is equal to the largest value of $\mathbf{H}$ (i.e., it is equal to one if $\mathbf{H}$ is a non-degenerate binary masks).

IV. PROPOSED IMAGE RECONSTRUCTION ALGORITHM

A. Problem statement

This section presents the proposed algorithm to produce an estimation $\hat{\mathbf{X}}$ of the target image. Given the observation $\mathbf{y}$ of the optical device, we aim to make $\hat{\mathbf{X}}$ as close as possible to the ideal (and typically unknown) reference $\mathbf{X}$ which generates it. We operate under the assumption that the observations are affected by noise modeled as an additive independent and identically distributed (i.i.d.) Gaussian distribution with zero mean. While this hypothesis is common for previous Bayesian formulations of similar problems in the literature [10], [31], its validity is currently a point of contention in the scientific community, but is at least reasonable under sufficient high illumination [32].

The proposed **JoDeFu** reconstruction algorithm is based on the following Bayesian formulation of the inverse problem:

$$\hat{\mathbf{X}} = \arg \min_{\mathbf{X}} \frac{1}{2} \|\mathbb{A}(\mathbf{X}) - \mathbf{y}\|_2^2 + f(\mathbf{X}), \quad (16)$$

where the first term of the right side is the maximum a posteriori estimation, also known as data fidelity term, and $f(\mathbf{X}) : \mathbb{R}^{N_i N_j \times N_k} \rightarrow \mathbb{R}^+$ is a regularization function, which we can customize according to our a priori knowledge on the result to reconstruct and is used to counteract the ill-conditioned nature of the formulation [33]. We focus our attention on the expression of the data fidelity term in Section IV-B, on the regularizer in Section IV-C, and on the algorithm for solving this problem in Section IV-D.

B. Data fidelity term

The JoDeFu algorithm can be applied to the observation of any device that can be described within the MRCA image formation model. That is, other than the general MRCA itself, it may also approach the problem of sharpening (if $\mathbb{A}$ models a multiresolution sensing), that of demosaicing (if $\mathbb{A}$ models a mosaicing), and so on. While we specialize here on the MRCA framework, the proposed algorithm admits a solution for any image formation model, as long as we can define the properties of the operator $\mathbb{A}$ defined in Section III-D.

Special care has to be taken in the sharpening scenario (and in general, for every setup in which $\mathbb{A}_+$ is an identity operator). As the observation are two separate acquisitions, the reconstruction problem is equivalent to:

$$\hat{\mathbf{X}} = \arg \min_{\mathbf{X}} \frac{1}{2} \left\| \mathbb{A}_d^{[p]}(\mathbf{X}) - \mathbf{P} \right\|_F + \frac{1}{2} \left\| \mathbb{A}_d^{[m]}(\mathbf{X}) - \widetilde{\mathbf{M}} \right\|_F + f(\mathbf{X}), \quad (17)$$

which is very similar to the Bayesian formulation of the sharpening problem proposed in [10], but assumes the same weight for the data fidelity term associated to the LRI and to the HRI.

Previous works [34] have shown that the algorithms that were proposed to solve image reconstruction problems based on multiresolution acquisition (e.g., pansharpening), are not well suited for the case of reconstruction of missing acquisitions (e.g., inpainting), which is also the case of the demosaicing as binary masks technically perform a subsampling. This motivates the need of an ad-hoc algorithm where these problems can be solved jointly.

To deal with such scenarios, the proposed blur filter $\mathbb{A}_b$ is used to adjust the results even when the HRI is at the same scale of the reference. Its inclusion allows to recast our demosaicing problem from pure inpainting to an hybrid of inpainting/magnification. The formulation implies that some information from the suppressed pixels is contained in adjacent pixels, but this comes at the cost of reducing the spatial resolution of the final product.

C. Regularization

We want to setup here a *proximal algorithm*, as it works in very general conditions, allowing for some nonsmooth real-valued constraint on the cost function, and is relatively fast with respect to other alternatives [35]. To this end, we propose a regularization function in the form:

$$f(\mathbf{X}) = \lambda g(\mathbb{L}(\mathbf{X})), \quad (18)$$

where we denote:

- a **linear operator** $\mathbb{L}(\cdot) : \mathbb{R}^{N_i N_j \times N_k} \rightarrow \mathbf{E}_l$, which describes $\mathbf{X}$ within a sparsity-inducing transformed normed space $\mathbf{E}_l$;
- a **metric function** $g(\cdot) : \mathbf{E}_l \rightarrow \mathbb{R}^+$, for which it is possible to define a proximal operator $\text{prox}_{\gamma g}(\cdot)$ scaled by a generic positive scalar γ ;
- a **regularization parameter** $\lambda \in \mathbb{R}^+$, used to weight the contribute of the regularization with respect to the data term in the cost function; we sometimes refer to this term in its *normalized form* $\bar{\lambda} = \lambda / \rho_y$, where ρ_y is the dynamic range of the observation $\mathbf{y}$ (e.g., $\rho_y = 255$ for 8 bits images).

For the linear transformation $\mathcal{W} = \mathbb{L}(\mathbf{X})$, the CASSI authors proposed to use a *symlet-8 discrete wavelet transform* and a *discrete cosine transform (DCT)* transform in the spatial and spectral domain, respectively [13]. In this work, we propose instead an approach based on the *total variation (TV)*, a regularizing transformation that acts as a discrete representation the *Rudin-Osher-Fatemi model* [36]. In its modern interpretation, the TV is often seen as a sparsity-inducing operator working in the domain of image gradients. This favors piecewise constant images with sparse edges, which are typically a better representation of natural images [34].

Following the generic mathematical description of the third author [37], the TV-based expression of $\mathcal{W}$ is a 4-way tensor $\mathcal{W} \in \mathbb{R}^{N_i \times N_j \times N_k \times N_m}$, whose forth dimension is made up of the gradients of $\mathcal{U}^{[x]}$. For the classic TV in particular, where

$N_m = 2$, the gradients are taken along the horizontal and vertical spatial dimensions, and the elements w_{ijkm} of $\mathcal{W}$ (assuming that the elements out of range in $\mathcal{U}^{[x]}$ are zero) are defined as follows:

$$w_{ijkm} = \begin{cases} u_{ijk}^{[x]} - u_{i-1,j,k}^{[x]} & \text{for } m = 1 \\ u_{ijk}^{[x]} - u_{i,j-1,k}^{[x]} & \text{for } m = 2 \end{cases}. \quad (19)$$

Similar four way tensors can be also defined for alternative TV-based operators, such as the *upwind total variation (UTV)* [38], and the *Shannon total variation (STV)* [39].

To define the metric function $g(\cdot)$, we took inspiration from the framework of the *collaborative total variation (CTV)* [40], [41], where $g(\cdot)$ is defined as a set of norms applied sequentially over different dimensions.

The most relevant that are also employed in this work, are defined below:

- $g(\mathcal{W}) = \|\mathcal{W}\|_{p_1 p_2 p_3}$: this stands for the ℓ_{p_1} , ℓ_{p_2} , and ℓ_{p_3} -norm applied, in this order, respectively to dimension of the gradient, that of the channels, and that of the pixels. Among those, the most widespread are the $\|\mathcal{W}\|_{221}$, which is used in the *vector total variation* [42] and whose mathematical expression is:

$$\|\mathcal{W}\|_{221} = \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} \sqrt{\sum_{k=1}^{N_k} \sum_{m=1}^{N_m} w_{ijkm}^2} \quad (20)$$

and the $\|\mathcal{W}\|_{111}$ norm, known as the *least absolute shrinkage and selection operator (LASSO)* [43] and employed for the classic inversion of the CASSI acquisitions [13].

- $g(\mathcal{W}) = \|\mathcal{W}\|_{S_p \ell_q}$: this stands for the Shatten p -norm firstly applied on both the gradient and the bands' dimensions, and then the ℓ_q -norm applied over the pixels; particularly good performances can be obtained with $\|\mathcal{W}\|_{S_1 \ell_1}$, defined as:

$$\|\mathcal{W}\|_{S_1 \ell_1} = \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} \sqrt{\sum_{m=1}^{N_r} \xi_m^2(\mathbf{W}_{ij::})}, \quad (21)$$

where $\xi_m(\mathbf{W}_{ij::})$ is the m -th singular value and N_r is the total amount of singular values of $\mathbf{W}_{ij::}$.

D. Implementation details

The proposed JoDeFu image reconstruction framework can be summarized as follows:

$$\hat{\mathbf{X}} = \arg \min_{\mathbf{X}} \frac{1}{2} \|\mathbb{A}(\mathbf{X}) - \mathbf{y}\|_2^2 + \lambda g(\mathbb{L}(\mathbf{X})), \quad (22)$$

which is the minimization of a cost function composed by a differentiable data fidelity term and a regularization term whose metric function $g(\cdot)$ is a lower semi-continuous convex function. As long as the adjoint operators for $\mathbb{A}$ and $\mathbb{L}$ are known, and it is possible to define a proximal operator for $g(\cdot)$, a variety of algorithms are available that iteratively converge to the desired solution $\hat{\mathbf{X}}$. Those are known as *proximal algorithms* in the literature [35]. Among those, the *Chambolle-Pock solver* [44] is the most widespread, but we prefer here to employ instead the *Loris-Verhoeven algorithm* [45],

which simplifies the choice of the convergence parameters (as reported in [46]).

The full procedure, described by the Algorithm 1, requires the definition of:

- the adjoint operator $\mathbb{A}^*$ and the operator norm $\|\mathbb{A}\|_{op}$ of $\mathbb{A}$, which were described in Section III-D;
- the adjoint operator $\mathbb{L}^*$ and the operator norm $\|\mathbb{L}\|_{op}$ of $\mathbb{L}$: for orthogonal operators (such as the DCT and some wavelets), their operator norm is unitary. For the classic TV the generic v_{ijk} element of $\mathcal{V} \in \mathbb{R}^{N_i \times N_j \times N_k}$, of $\text{matr}(\mathcal{V}) = \mathbb{L}^*(\mathcal{W})$ is:

$$v_{ijk} = (w_{i,j,k,1} - w_{i-1,j,k,1}) + (w_{i,j,k,1} - w_{i,j-1,k,2}), \quad (23)$$

where we assume once again that the elements out of range of $\mathcal{W}$ are null. Its operator norm is $\|\mathbb{L}\|_{op} = \sqrt{8}$ [37]. For the other TV-like operators, we redirect the reader towards the related articles [38], [39].

- the scaled *proximal operator* $\text{prox}_{\lambda g^*}(\cdot)$ of the Fenchel conjugate of $g(\cdot)$: a summarizing table of its expression is provided in [41]. We just remind here that, for the $\|\mathcal{W}\|_{221}$ norm, this is equal to:

$$\text{prox}_{\lambda g^*}(\mathbf{y}) = \frac{\mathcal{W}}{\max \left(\sqrt{\frac{1}{\lambda} \sum_{k=1}^{N_k} \sum_{m=1}^{N_m} \mathbf{W}_{::km}}, 1 \right)}, \quad (24)$$

where $\max(\mathbf{X}, 1)$ is an operator substituting with 1 all elements of $\mathbf{X}$ that are superior to 1, and the fraction stands for an element-wise division broadcasted over 4 dimensions.

The JoDeFu framework allows for multiple ways to construct a custom cost function, which include the choice for the regularization parameter λ , for the metric function $g(\cdot)$, and the linear transformation operator $\mathbb{L}$. We propose here two possible solutions, whose specifics are shown in table III: the **JoDeFu v1**, provides reasonable performances while keeping the computation time relatively short, while the **JoDeFu v2** variant produces optimal performances if there is no constraint on time.

V. EXPERIMENTS

In this section, the proposed image formation/reconstruction framework is tested under different viewpoints:

- In Section V-B, we analyze the acquisition obtained with a variety of image formation methods. For each acquisition under test, we compare the reconstructed products obtained both with a representative state-of-the-art classic reconstruction algorithm and with the proposed JoDeFu algorithm. This experiment aims to show the effectiveness of the MRCA in modeling a wide variety of capturing techniques, by verifying that the quality of the reconstructed image meets the standards set up by the previous literature when this model is employed within the proposed reconstruction algorithm;
- in Section V-C, the image formation is fixed to the proposed MRCA and we inspect a variety of solutions for the image reconstruction, by comparing the results when

Algorithm 1: JoDeFu algorithm, based on the Loris-Verhoeven solver [45] with over-relaxation [46].

Result: Estimated product $\hat{\mathbf{X}}$

Input:

- Acquisition: $\mathbf{y} \in \mathbb{R}^{N_i N_j}$ (with the LRI samples histogram matched to the HRI)
- Direct model operator $\mathbb{A}(\cdot)$, according to eq. (9), with $\mathbb{A}^*$ and $\|\mathbb{A}\|_{op}$ defined in Section III-D;
- Domain transformation operator $\mathbb{L}(\cdot)$, e.g. a TV according to eq. (19), an UTV, or a STV, with $\mathbb{L}^*$ and $\|\mathbb{L}\|_{op}$ defined in Section IV-D;
- Proximal operator $\text{prox}_{\lambda g^*}(\cdot)$, e.g. from eq. (24);
- Regularization parameter: λ (default: $10^{-3}\rho_y$, where ρ_y is the dynamic range of $\mathbf{y}$);
- Over-relaxation parameter: ρ_o (default: 1.9);
- Maximum number of iterations: $q^{[max]}$ (default: 250);

Initialization:

- First convergence parameter: $\tau = 0.99/\|\mathbb{A}\|_{op}^2$
- Second convergence parameter: $\sigma = 1/(\tau\|\mathbb{L}\|_{op}^2)$
- Primal variable: $\mathbf{X}^{(0)} = \mathbb{A}^*(\mathbf{y})$
- Dual variable: $\mathbf{W}^{(0)} = \mathbb{L}(\mathbf{X}^{(0)})$
- Iteration: $q = 0$

while $q < q^{[max]}$ **do**

$$\begin{aligned} \mathbf{V}^{(q)} &= \mathbb{A}^*(\mathbb{A}(\mathbf{X}^{(q)}) - \mathbf{y}) \\ \mathbf{X}^{(q+\frac{1}{2})} &= \mathbf{X}^{(q)} - \tau \left(\mathbf{V}^{(q)} + \mathbb{L}^*(\mathbf{W}^{(q)}) \right) \\ \mathbf{W}^{(q+\frac{1}{2})} &= \text{prox}_{\lambda g^*} \left(\mathbf{W}^{(q)} + \sigma \mathbb{L} \left(\mathbf{X}^{(q+\frac{1}{2})} \right) \right) \\ \mathbf{X}^{(q+1)} &= \mathbf{X}^{(q)} - \rho_o \tau \left(\mathbf{V}^{(q)} + \mathbb{L}^*(\mathbf{W}^{(q+\frac{1}{2})}) \right) \\ \mathbf{W}^{(q+1)} &= \mathbf{W}^{(q)} + \rho_o \left(\mathbf{W}^{(q+\frac{1}{2})} - \mathbf{W}^{(q)} \right) \\ q &\leftarrow q + 1 \end{aligned}$$

return $\hat{\mathbf{X}} = \mathbf{X}^{(q^{[max]})}$

TABLE III: Suggested setups for MRCA image reconstruction algorithms. The first half refers to the JoDeFu algorithm, while the rows labeled with classic v1 and v2 refer to the alternative algorithms described in Section V-C.

JoDeFu setups			
	$g(\mathbf{W})$	$\mathbb{L}(\cdot)$	$\mathbb{A}_b(\cdot)$
JoDeFu v1	$\ \mathbf{W}\ _{221}$ [40]	TV [36]	Identity
JoDeFu v2	$\ \mathbf{W}\ _{S_2\ell_1}$ [40]	UTV [38] or STV [39]	$\rho_b = 1-1.5$ px
Cascaded classic algorithms setups			
	PAN interpolation	Demosaicing	Sharpening
Classic v1	TPS-RBF [47]	ARI [48]	MTF-GLP-HPM [49]
Classic v2	TPS-RBF [47]	ID [26]	GSA [50]

a different amount of LRI channels are encoded in the acquisition. This experiments aims to show the capability of the JoDeFu in recovering the relevant information embedded in the compressed acquisition under different conditions.

- in Section V-D, we analyze the effect of the parameters associated to the JoDeFu algorithm. The experiment aims to test the robustness of the proposed reconstruction algorithm with respect to deviations from an ideal parametric setup.

To introduce the experiments, a description of the employed

TABLE IV: Characteristics of the GT of the datasets employed in the tests of Section V. The ground sample distance (GSD) refers to the spatial resolution of the GT.

Label	Scene	Sensor	GSD	Sizes [px]
Beijing	Bird's nest, China	WV2	1.6 m	512×512
Janeiro	Bay area, Brazil	WV3	1.2 m	512×512
Washington	Capitol building, U.S.	WV3	1.6 m	512×512

datasets, of the experimental setup, and of the validation method is given in Section V-A. The experiments provided in this section are fully reproducible with the MATLAB implementation of the algorithm¹ and additional results are available in the *supplementary materials*.

A. Experimental setup

Our validation setup consists of four steps:

- **Reference choice:** where we select a high resolution image as reference $\mathbf{X} \in \mathbb{R}^{N_i N_j \times N_k}$, that is referred as ground truth (GT);
- **Simulation:** where the acquisition $\mathbf{y} = \mathbb{A}(\mathbf{X})$ is evaluated from the direct model $\mathbb{A}$ under test (e.g., with the architecture in Fig. 2 for the MRCA);
- **Testing:** where the reconstruction algorithm under test is applied to the observation, in order to achieve an estimation $\hat{\mathbf{X}}$ of $\mathbf{X}$;
- **Comparison:** where the estimated product $\hat{\mathbf{X}}$ and the reference $\mathbf{X}$ are compared by evaluating a series of quality indices.

Each reference dataset is composed of a HRI/LRI image bundle acquired almost simultaneously, originally featuring a scale ratio of 1 : 4, although the tests are performed at reduced resolution with a scale ratio of $\rho = 2$. The HRI is monochromatic (i.e. a PAN) and the LRI has up to 8 channels.

The bundles were acquired by the WorldView-2 (WV2) and WorldView-3 (WV3) satellites, and are available for download on the MAXAR Technologies website [51]. Their characteristics are shown in TABLE IV. Additional experiments over different datasets are available in the supplementary materials.

For the simulation step, when applicable, the HRI is given as a spatial degradation of a HRI at higher resolution, instead of spectral degradation of the GT, in order to follow the Wald's protocol for reduced resolution validation [52].

In the spectral degradation model $\mathbb{A}_d^{[p]}$, the weighting coefficients are always set as equal to $w_{1,k} = 1/N_k$, for all $k \in [1, \dots, N_k]$, so that the HRI is modeled as the average of the channels of the GT. In the spatial degradation model $\mathbb{A}_c^{[p]}$, the blurring kernels $\mathbf{B}$ are Gaussian functions whose cutoff frequency matches the one of the MTF of the sensors. The $\mathbb{A}_b(\cdot)$ is implemented by a infinite impulse response (IIR) Butterworth filter whose bilateral cutoff frequency is $1/\rho_b$, with ρ_b denoting the diameter of the blurring filter and expressed in pixels (px). All JoDeFu algorithms are run for 250 iterations. In the JoDeFu preprocessing stage, the overall mean and standard deviation of the LRI samples are equalized to that of HRI samples.

¹<https://github.com/danaroth83/jodefuf>

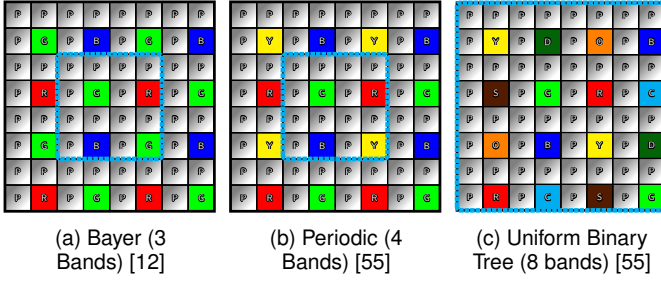


Fig. 3: Masks used for the experiments of Section V-C. The masks are obtained by mosaicing a set of HRI sensors (in gray) to classical literature designs for LRI masks, color coded with their characteristic channel. The dashed outline denotes the periodicity of each mask.

For the comparison step, we employ as quality indices the *peak signal to noise ratio (PSNR)*, the *spectral angle mapper (SAM)* [53], and the *structural similarity (SSIM)* [54], which is given as average over all bands.

B. Image formation

We consider here a selection of image formation methods and compare the obtained quality of the estimated product, produced both with a carefully selected classic image reconstruction algorithm and with the JoDeFu. For completeness, this approach is also applied to an observation obtained with the complete declination of the MRCA method, using the mask of Fig. 3b.

Since formation methods can be interpreted as compressed acquisitions, the final observation generally contains less samples than the datacube to reconstruct. Therefore, their *compression ratio* ρ_c can be defined as the ratio between the amount of samples of the observation $\mathbf{y}$ and the reconstructed product $\hat{\mathbf{X}}$. For example, if the dynamic range of all samples is the same, the compression ratio of the full MRCA model is equal to:

$$\rho_c = \frac{N_i N_j}{N_i N_j N_k} = \frac{1}{N_k}, \quad (25)$$

as the multiresolution setup compresses the acquisition over a single FPA matching the spatial dimensions of the reconstructed product.

We investigate the follow image formation methods:

- **Multiresolution sensing:** with the same scale ratio of the MRCA, this is compared with the *MTF-matched generalized Laplacian pyramid with high pass modulation injection (MTF-GLP-HPM)* algorithm [56], the best performing one for this dataset among the classic ones that were tested;
- **Mosaicing:** applying a 4-band mask with period 2×2 , which also reaches the same compression ratio of the MRCA, which is compared with the *intensity difference (ID)* demosaicing algorithm;
- **CASSI:** using the model proposed by the author employing one acquisition in their single dispersion variant and comparing with the proposed algorithm for the reconstruction [13].

The validation procedure is applied to the “Washington” dataset and compared to the baseline JoDeFu, with the results shown in TABLE V and a visual comparisons in Fig. 4.

The JoDeFu algorithm achieves results which are at least comparable to the state-of-the-art. The benefit of the TV-style regularizer is immediately evident for the CASSI reconstruction (Fig. 4h). For the pansharpening problem, the final product obtained with the JoDeFu (Fig. 4f) shows subtler improvements, as the final product achieves slightly more accurate color quality. For the demosaicing problem, however, there are still margins of improvement (Fig. 4g). The proposed algorithm is not fully capable of eliminating some texture effects; this is a known weakness of TV-based regularizer, which are not well suited for the reconstruction of thread-like structures [34].

C. Image reconstruction

In this section, we shift the focus on image reconstruction methods, and compare the quality of the estimated products achieved with different methods, starting from an observation acquired with the full version of the proposed MRCA architecture. Our main target is to show the robustness of the proposed algorithm to recover the desired information when a sufficiently large amount of channels are embedded in the observation.

The analysis is carried out by simulating the observation starting from the “Janeiro” dataset bundle. For the LRI, we select either 3, 4, or 8 channels, by either only choosing the RGB in the first case, adding a near infrared response (NIR) in the second one, or selecting all the visible (VIS) and NIR channels in the last case. The MRCA model employs the periodic masks shown in Fig. 3.

Other than with the JoDeFu, this specific problem can be also approached with a custom image formation method, composed of the following three-step procedure:

- **HRI interpolation:** recover the sparse channel associated to the HRI ($\mathbf{p}_{:,1}^{\square} = \mathbf{y} \odot \mathbf{h}_{:,1}^{[p]}$) and estimate the missing HRI samples with a multivariate interpolation.
- **Demosaicing:** obtain the LRI mosaic by decimating the observation $\mathbf{y}$. Furthermore, apply any classic demosaicing algorithm to estimate all the channels of the LRI;
- **Sharpening:** perform a fusion on the reconstructed HRI and LRI from the previous two steps.

In our tests, we employ a *thin plate spline radial basis function (TPS-RBF)* [47] for the HRI interpolation. For a comprehensive comparison, we isolated two viable configurations for the demosaicing and pansharpening algorithms, whose specifics are given in TABLE III:

- The **classic v1** setup is optimized for the RGB setup and employs the *adaptive residual interpolation (ARI)* demosaicing method [48], which is only applicable to Bayer masks.
- The **classic v2** method can be applied to all cases, and employs the *Gram-Schmidt adaptive (GSA)* fusion method [50], as it provides more robust results for larger amount of bands.

TABLE V: Results of classic and proposed reconstruction algorithms for different methods of image formation with the 4-bands “Washington” dataset. Best results for each image formation method are shown in bold.

Image formation	ρ_c	Image reconstruction	$\bar{\lambda}$	SSIM	PSNR	SAM
Reference (GT)	1.000	-	-	1	∞	0
Multiresolution sensing	0.500	Pansharpening [56]	-	0.9772	31.14	3.751
		JoDeFu v1	1×10^{-3}	0.9868	33.22	2.797
Mosaicing	0.250	Demosaicing [57]	-	0.9613	30.38	4.361
		JoDeFu v1	2×10^{-3}	0.9312	28.36	4.371
CASSI acquisition [13]	0.251	CASSI reconstruction [13]	3×10^{-3}	0.7645	24.48	9.750
		JoDeFu v1	2×10^{-3}	0.8600	26.89	5.954
MRCA	0.250	Classic v2 (TABLE III)	-	0.9156	28.13	6.246
		JoDeFu v1	1×10^{-3}	0.9446	29.34	4.553

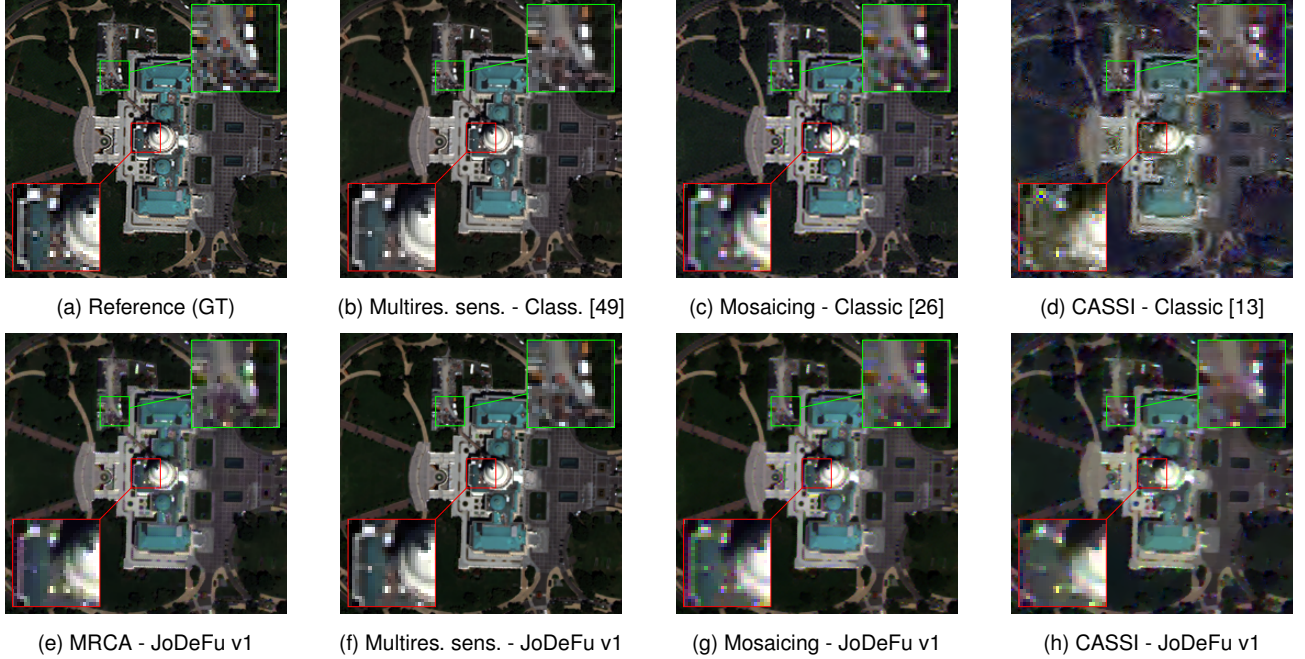


Fig. 4: Visual comparison for different image formation methods, comparing the reconstructed product obtained with the proposed JoDeFu algorithm and with classic solutions available in the literature. The images show a 256×256 cropped area of the 4-band “Washington” dataset. Zoomed areas are provided for a detailed comparison.

The analysis of TABLE VI and its associated visual comparison of Fig. 5 shows that the JoDeFu yields sharper results than the classic v2, regardless of band setup. However, it does not provide a consistent reconstruction of large homogeneous zones, such as the swimming pool in Fig. 5c. This is probably due to the spatial uniformity of the regularization parameter λ . As we did not provide any mechanism to change the value of λ locally, its choice is not necessarily optimized for differently scaled objects on the scene.

D. Setting the parameters

We test here various possible parameters for the optimization of the JoDeFu algorithm applied to the MRCA acquisition obtained with the mask of Fig. 3b. For this test, we firstly define as *baseline* the JoDeFu v1 with $\bar{\lambda} = 1 \times 10^{-3}$ (whose specifics are given in TABLE III) and evaluate its estimated product. Then, for each parameter under test, we estimate the reconstruction results obtained by solely varying a single

parameter from the baseline setup. There is no guarantee on the combined effect of varying multiple parameter, but we empirically experienced that optimizing each parameter separately still returns performances within a reasonable ballpark of the overall best optimization. The tests are applied to the 4-band “Beijing” dataset, and a summary of the measured quality indices is given in Fig. 6, with an associated visual comparison in Fig. 7.

A more in-depth discussion for each of the parameters under test is given in the following list:

- **Regularization Parameter λ :** as a rule of thumb, $\bar{\lambda} = 10^{-3}$ is a good compromise in most scenarios and can be used as starting test to further refine the parameters if higher quality is required. In Fig. 7b and 7g, some reconstructed products are shown for implausible low and high values of λ , respectively. If λ is too low, we impose no structure of the final image, and most texture effects from the mosaicing are not flattened. If λ is too high, the

TABLE VI: Results of MRCA image reconstruction employing different amount of mosaiced bands for the “Janeiro” dataset. The image construction is obtained with the MRCA model using the masks shown in Fig. 3. Best results are marked in bold fonts.

	3 bands (RGB)			4 bands (RGB + NIR)			8 bands (All VIS/NIR)		
	SSIM	PSNR	SAM	SSIM	PSNR	SAM	SSIM	PSNR	SAM
Reference (GT)	1	∞	0	1	∞	0	1	∞	0
Classic v1	0.9634	32.30	2.502	-	-	-	-	-	-
Classic v2	0.8680	28.77	5.253	0.9049	28.11	6.463	0.8917	28.50	9.166
JoDeFu v1 ($\bar{\lambda} = 2 \times 10^{-3}$)	0.8803	28.78	4.988	0.9159	28.38	5.218	0.9080	29.08	7.220
JoDeFu v2 with UTV	0.9050	29.57	3.901	0.9264	28.58	4.891	0.9247	29.63	6.693
JoDeFu v2 options	$\bar{\lambda} = 2 \times 10^{-3}, \rho_b = 1$			$\bar{\lambda} = 1 \times 10^{-3}, \rho_b = 1.4$			$\bar{\lambda} = 1 \times 10^{-3}, \rho_b = 1.4$		

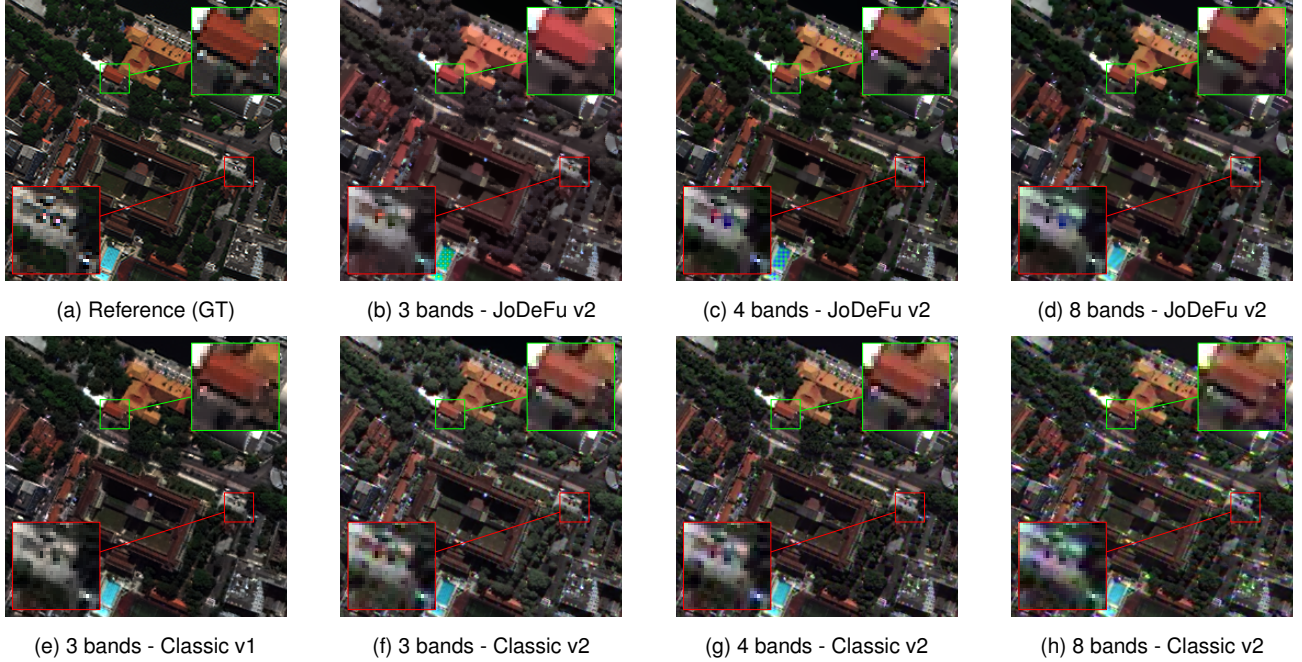


Fig. 5: Visual comparison of different MRCA image reconstruction algorithms, employing a different amount of embedded bands from the “Janeiro” dataset. In all cases, we visualize the RGB bands of 256×256 px cropped area with zoomed details.

smoothing effect applies to relevant image features;

- **Metric function norm $g(\cdot)$:** the quantitative verification shows that the ℓ_{221} norm is the best compromise between quality of the reconstructed product and computational speed. Among the remaining choices, better performances are only achieved with the $S_1\ell_1$, due to the noise whitening effect that this constraint imposes across different bands. The visual analysis shows some spectral spot-shaped spectral distortions in Fig. 7h;
- **Linear operator $\mathbb{L}(\cdot)$:** for this configuration we tested the classic TV, the UTV [38] and the STV [39] with an upscaling factor of 2 and 3, but no noticeable differences were found;
- **Diameter ρ_b of the blurring operator $\mathbb{A}_b(\cdot)$:** in our tests, the optimal value of the blur diameter was found to be in the range $1.3 - 1.5$ px for a scale ratio $\rho = 2$. This optimal value has to be chosen as a trade-off between a more accurate recovery of the HRI samples and avoiding out-of-focus effects in the final product.

VI. CONCLUSION

In this paper, we proposed the MRCA, a novel multiresolution compressed acquisition system with enough flexibility to model classic image formation methods, including CFA/MSFA-based mosaicing, HRI/LRI image bundles, non-conventional acquisition system, such as the CASSI, and various hybrid methods. The proposed design aims to intercept the future trends for unconventional optical devices, and allow for an immediately available procedure to model the acquisition.

We also proposed the JoDeFu, a Bayesian solver for image reconstruction applicable to any available variant of MRCA-based acquisitions, and specialized on data with a strong LRI component. The proposed algorithm jointly addresses the problem of image fusion and reconstruction of compressed data, exploits the CTV regularization to recover the desired product, and can be declined into two different variants with respect of the requirements in term of computation time. The proposed method does not necessarily match the state-of-

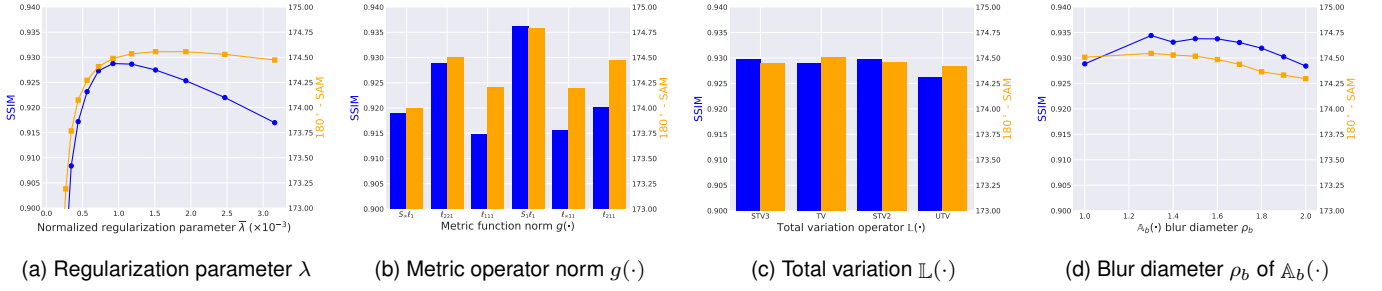


Fig. 6: Quality indices results obtained with different setups of the JoDeFu image reconstruction algorithm applied to the 4-band “Beijing” dataset. For each figure, we vary the indicated parameter with respect to the baseline JoDeFu v1 setup. Better reconstruction performances are associated to higher values of the “SSIM” (in blue) and of the “180° – SAM” (in orange).

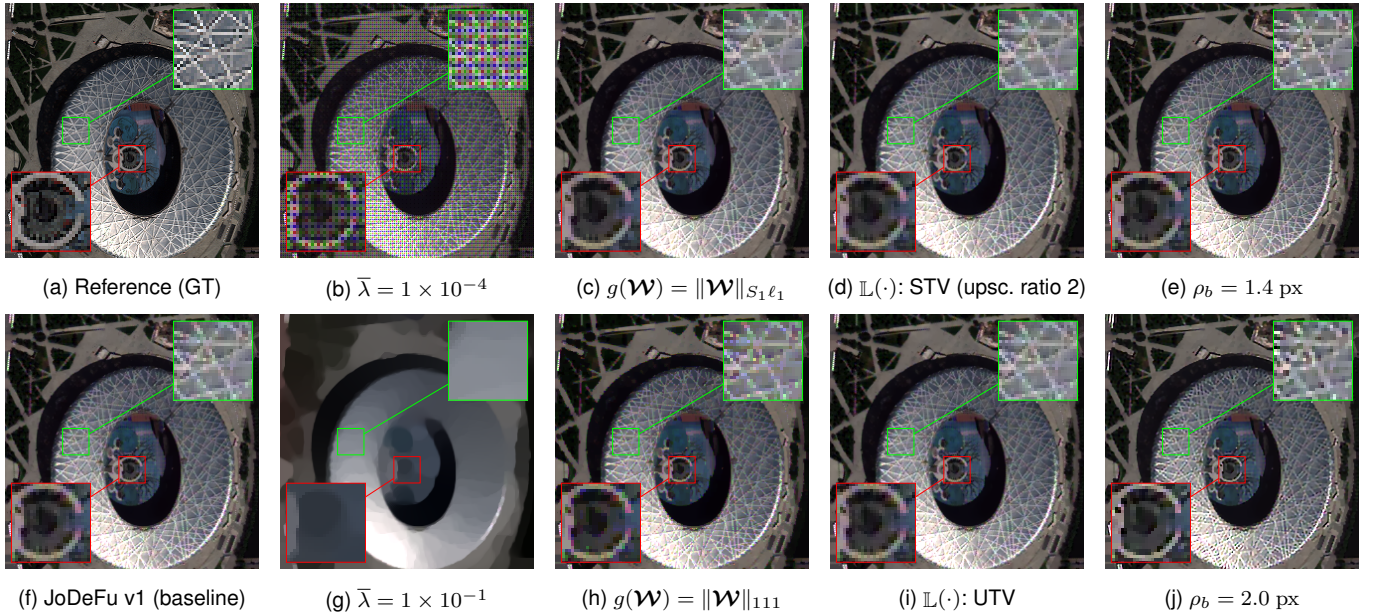


Fig. 7: Visual comparison of the effects of changing parameters in the baseline setup of the JoDeFu image reconstruction algorithm applied to the MRCA acquisition of the 4-band “Beijing” dataset (256 × 256 px cropped area).

the-art for every available image formation setup, but some alternatives were analyzed for particularly simple ones, which employ a cascade of classic techniques.

A possible extension of this work may involve supervised learning [58], [59] to fine tune the parameters of the JoDeFu algorithm. Additionally, improved robustness to a higher number of channels can be obtained by capturing samples directly in a sparse domain, and the reconstructed products can achieve higher quality with suitable mask designs based on compressed sensing, expanding the results of our previous works [60], [61].

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