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A high performance computing framework for finite element modelling of conductive and radiative heat transfers within heterogeneous ceramics.

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Architected cellular ceramics with prescribed textural features exhibit high temperature behaviors firmly connected to the spatial radiative heat flux distribution. New insights can be provided by directly investigating the local scale i.e. for this class of materials, the pore or the cell scale [1]. To reach this objective, we plan to use an efficient parallelized framework for numerically simulating the steady-state radiative transfer equation, previously co-developed at LTeN, Nantes and IRIT, Toulouse [2]. In this work, we firstly solve a nonlinear integro-PDEs coupled model for conduction and radiation heat transfers at the continuum-scale. We carry out the analysis and the numerical approximation of the model within a variational framework and we perform the full discretization of the problem based on the discrete ordinates method for angular discretization and the vectorial finite element method for spatial discretization as well as the Newton's method for linearization. Moreover, the efficiency of the numerical strategy in terms of convergence and calculation cost is based on an optimal algorithm, developed for parallel computing. In order to validate the numerical scheme, we consider a three-dimensional geometry, sandwiched between two parallel plates at different temperatures, and filled with a continuous or a homogenized medium, as done in the pioneering work of Viskanta [3]. The results are then discussed for gradual Stark numbers. Secondly, the same numerical approach is extended to a discrete-scale case, namely a cross-bar geometry [4]. Preliminar results are shown.

Keywords: Cellular ceramics, Conductive–radiative transfer, Continuum-scale approach, Discrete-scale approach, Finite element analysis, Parallel computing.

References:

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[2] Badri, M. A., Jolivet, P., Rousseau, B., Le Corre, S., Digonnet, H., Favennec, Y. Vectorial finite elements for solving the radiative transfer equation. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 212 (2018) : 59-74.

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A high performance computing framework for finite element modelling of conductive and radiative heat transfers within heterogeneous ceramics

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Keywords: cellular ceramics, conductive and radiative transfer, continuum-scale & discrete-scale approaches, finite element analysis, parallel computing

Context: In this work, we propose a finite element scheme to numerically solve the conduction radiation behavior within SiC based cellular ceramics. Two approches are discussed: the continuum-scale approach and the discrete-scale approach. This a first step for the elaboration by additive manufacturing of volumetric solar receivers with the company 3D CERAM (Limoges).

Numerical strategy for solving the coupled problem

$\Rightarrow (\mathcal{RTE})$ equation: { Discrete Ordinate Method (angular discretization); Variational Formulation; Full discretization: SUPG - Vectorial FEM } $\Rightarrow (\mathcal{HCE})$ equation: { Linearization of the problem: Newton-Raphson; Variational Formulation; Full discretization: FEM } $\Rightarrow$ { Algorithm $\rightarrow$ Two-step iterative schem; Domain decomposition and parallel computing }

Coupled conduction–radiation problem: Continuum-scale

- $(\mathcal{RTE})$

$$S \cdot \nabla I(x,S) + \beta I(x,S) = \sigma \int_{4\pi} I(x,S')\Phi(S,S')dS' + k\frac{1}{\pi}\sigma_B n^2 T^4 \quad \forall (x,S) \in \Omega \times \mathcal{S}$$

$$I(x,S) = I(x,S'), \quad S \cdot n < 0, \quad x_{\min} < x < x_{\max}$$

$$I(x,S) = \epsilon\frac{1}{\pi}\sigma_B n^2 T^4, \quad x = \{x_{\min}, x_{\max}\}$$

- $(\mathcal{HCE})$:

$$\nabla \cdot Q_c = -\nabla \cdot Q_r \Rightarrow -\nabla \cdot \lambda \nabla T = k \int_{4\pi} I(x,S)ds - k4\pi\{\frac{1}{\pi}\sigma_B n^2 T^4\} \qquad \forall x \in \Omega$$

$$T = T^{\pm}, \quad x = \{x_{\min}, x_{\max}\}$$

$$\nabla T \cdot n = 0, \quad x_{\min} < x < x_{\max}$$

Coupled conduction–radiation problem: Discrete-scale

- $(\mathcal{RTE})$:

$$\{I_m\}_{m=1}^{N_d} : \Omega_{\mathrm{v}} \longrightarrow \mathbb{R} \quad \mathcal{E}_{\mathrm{RTE}} = \{\nabla \cdot \nabla I_m = 0\}_{m=1}^{N_d} \quad \forall x \in \Omega_{\mathrm{v}} \subset \mathbb{R}^3,$$

$$\{I_{m,\mathrm{in}}\}_{m=1}^{N_d} = \frac{1}{\pi}\sigma\, \varepsilon\, n_{\mathrm{v}}^2 \left(T_{\text{s-v}}^{\partial}\right)^4 \quad \text{on } \Gamma, \mathbf{s}_m \cdot \mathbf{n}_{\mathrm{v}} < 0.$$

$$\{I_{m,\mathrm{R}}\}_{m=1}^{N_d} = \{\rho \sum_{j=1}^{N_d} \delta_{m,j}(\mathbf{n}_{\mathrm{v}}) I_j \mathbb{I}_{[\Gamma \cap \mathbf{s}_j \cdot \mathbf{n}_{\mathrm{v}} > 0]}\}_{m=1}^{N_d} \quad \text{on } \Gamma,$$

- $(\mathcal{HCE})$:

$$T : \Omega_{\mathrm{s}} \mapsto \mathbb{R} \quad \mathcal{E}_{\mathrm{HCE}} = -\nabla \cdot \lambda \nabla T = 0 \quad \forall \mathbf{x} \in \Omega_{\mathrm{s}} \subset \mathbb{R}^3,$$

$$T = T_{\mathrm{s},g_-}^{\partial} \quad \text{on } \partial\Omega_{\mathrm{s},D,-} \subset \Gamma; \partial\Omega_{\mathrm{s},D,-} = \{\mathbf{x} \in \partial\Omega_{\mathrm{s}},\ \mathbf{x} < \delta + \min_{\Omega^s} \mathbf{x}'\},$$

$$T = T_{\mathrm{s},g_+}^{\partial} \quad \text{on } \partial\Omega_{\mathrm{s},D,+} \subset \Gamma; \partial\Omega_{\mathrm{s},D,+} = \{\mathbf{x} \in \partial\Omega_{\mathrm{s}},\ \mathbf{x} < -\delta + \max_{\Omega^s} \mathbf{x}'\}.$$

$$-\lambda \nabla T \cdot \mathbf{n}_{\mathrm{s}} = \varepsilon\, \sigma\, n_{\mathrm{v}}^2 \left(T_{\text{s-v}}^{\partial}\right)^4 - \varepsilon \sum_{m=1}^{N_d} I_m w_m |\mathbf{s}_m \cdot \mathbf{n}_{\mathrm{v}}| \mathbb{I}_{[\Gamma \cap \mathbf{s}_m \cdot \mathbf{n}_{\mathrm{v}} > 0]} \quad \text{on } \partial\Omega_{\mathrm{s}} \backslash \partial\Omega_{\mathrm{s},D,\pm}.$$

Results

Plank/Stark parameter, optical thikness parameter and Albedo coefficient: $N = \frac{\lambda\beta}{4\sigma_B T_{max}^3}, \quad \tau_0 = \beta L, \quad \omega = \frac{\sigma}{\beta}, \quad \Theta_0 = \frac{T_{min}}{T_{max}}$

For a family of open cell foam with a porosity $p \in [0.5, 0.9]$ and $d_{\mathrm{nom}} \in [0.5, 2.5]$ mm, we have $N \in [0.2, 2]$.

λ_i	9.46	9.46 10^{-1}	9.46 10^{-2}	9.46 10^{-3}
N_i	10	1	0.1	0.01

Table 1: Values of different conduction-to-radiation parameters N and its corresponding thermal conductivity.$\mathbf{T_{max} = 2222K}$, $\mathbf{T_{min} = 1111K}$, $L = 3.812\ 10^{-4}m$ and $k = 2624.6719m^{-1}$ {Viskanta et al. [2]}

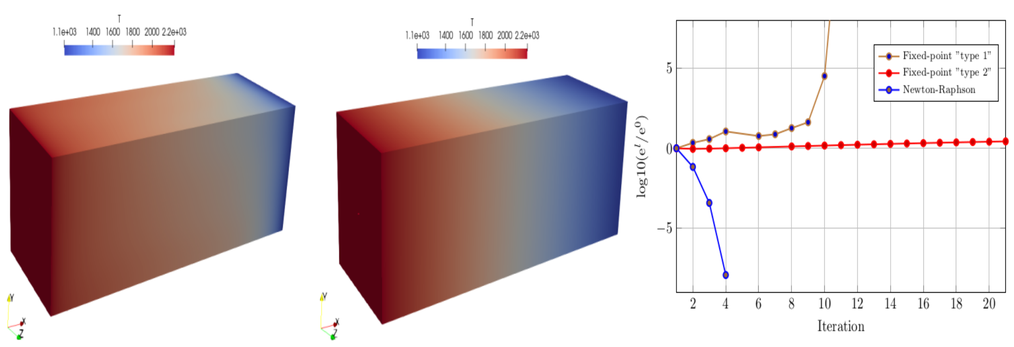


Figure 1: Temperature on the computational domain, for $N = 0.01$ "Radiation predominates" on the left and for $N = 10$ "Conduction predominates" on the middle. Convergence history

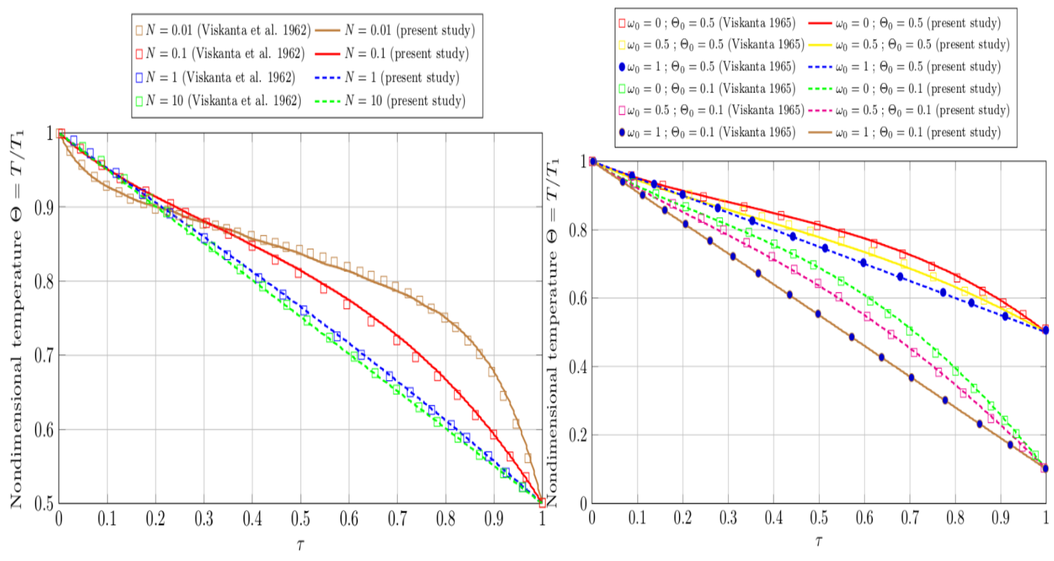


Figure 2: VALIDATION: Comparison of our computational results (3D) with the results from {Viskanta et al. absorbing & absorbing-scattering materials [2]} (1D), for different conduction-to-radiation parameters N, (N=0.01, 0.1, 1, 10) on the left and effect of scattering albedo on the right.

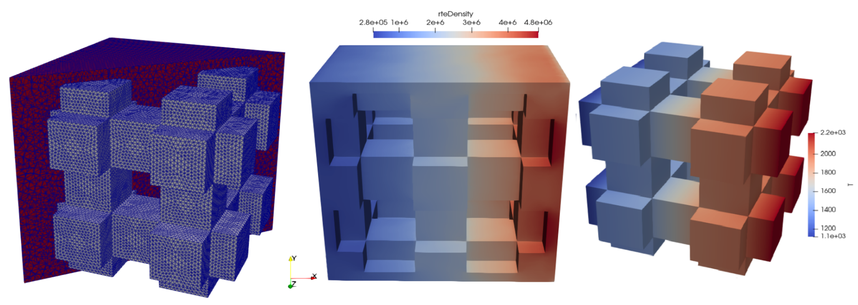
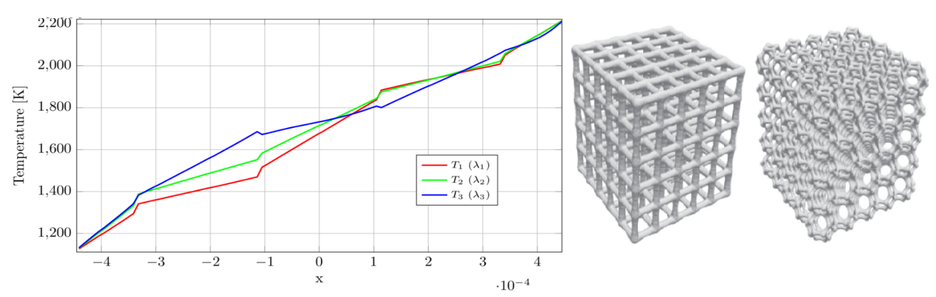


Figure 3: Mesh, radiative density "void part" and temperature "solid part" - with λ_1.



Conclusions and forthcoming research

▷ Validation of the 3D numerical conductive-radiative model;

▷ Efficiency of two-step iterative scheme and Newton–Raphson method;

▷ Numerical experiments in both the continuum-scale and discrete-scale are given;

$\Rightarrow$ A coupled problem of radiation, conduction and flow (Darcy-Forchheimer / Navier-Stokes) in porous media.

References

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