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► **To cite this version:**

G Vincent, Marta Costa-Bouzo, Christophe Guerin, Jean-Louis Coulomb, Olivier Chadebec. COMPARISON OF SPARSE LINEAR SYSTEMS SOLVERS ON 3-D SUPERCONDUCTOR PROBLEMS. EMF 2003, 6th International Symposium on Electric and Magnetic Fields, Oct 2003, Aachen, Germany. hal-03856962

HAL Id: hal-03856962

<https://hal.science/hal-03856962>

Submitted on 17 Nov 2022

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COMPARISON OF SPARSE LINEAR SYSTEMS SOLVERS ON 3-D SUPERCONDUCTOR PROBLEMS

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ABSTRACT

This paper proposes to compare several solvers on two transient magnetic problems using superconductor material.

Because of the highly non-linearity of the conductivity law, the two superconductor problems generate particularly ill-conditioned matrices.

We choose to test three solvers. They are the well-known "Incomplete Gauss Bi-Conjugate Gradient" (IGBCG), "Bi-Conjugate Gradient Stabilized" (Bi-CGStab) from H.A. Van Der Vorst and "Generalized Minimum RESidual" (GMRES) from Y. Saad. Our aim is to compare their respective robustness.

After having presented solvers and problems, we compare the obtained results. We conclude that Bi-CGStab and GMRES are more robust than IGBCG on both superconductor problems.

I. INTRODUCTION

The tests we present are transient magnetic problems using superconductor material. Because of the highly non-linearity of the conductivity law, they generate ill-conditioned matrices. We model them thanks to the FLUX software (3D application), developed by CEDRAT and LEG ("Laboratoire d'Électrotechnique de Grenoble"). It uses the T electric vector potential formulation. In this case, the extract matrix of the linear system is non symmetric. Most of the time, the solver used for this type of matrix is the well-known "Incomplete Gauss Bi-Conjugate Gradient" (IGBCG) [1]. However this solver often does not converge.

This paper proposes to compare results obtained on the two superconductor problems with three solvers. These are IGBCG, "Bi-Conjugate Gradient Stabilized" (Bi-CGStab) from H.A. Van Der Vorst [3] and "Generalized Minimum RESidual" (GMRES) from Y. Saad [4].

We present first the solvers tested, then the two superconductor problems and finally the results obtained with all the solvers on the two last examples.

II. DESCRIPTION OF THE SOLVERS TESTED

We are interested in the resolution of the systems of the form:

$$A.x = b \quad (1)$$

Where x and b are vectors of length N and A is a matrix of rank N . In the 3D application of FLUX, the use of the electric vector potential formulation generates non-symmetric matrices. Then, we will be interested more precisely in the non-symmetric matrix solvers.

In this part, we describe firstly the preconditioning techniques and secondly the solvers tested.

a. Preconditioning

To accelerate convergence, an iterative method requires a coupling with a preconditioning technique.

These techniques consist in transforming the original system (1) into one which has the same solution, but which is likely to be easier to solve with an iterative solver. For example, instead to solve (1), we can solve the following system:

$$M^{-1}.A.x = M^{-1}.b$$

Where M is a matrix of rank N .

If M is well-chosen, the matrix of the new system has now a better conditioning number than the original system.

The conditioning number $K(A)$ of the matrix A is defined so that:

$$K(A) = \|A\| \|A^{-1}\|$$

This is a typical index to know the convergence speed of the iterative solvers. For example, the speed of convergence of the well-known Conjugate Gradient algorithm is proportional to $\sqrt{K(A)}$. In general, the more $K(A)$ is small, the more the iterative solver will converge quickly.

The first preconditioning technique we describe is the Incomplete Factorisation of Gauss of type LDU [2][6]. It uses the same principle than the full LDU factorisation. But to reduce computation time, it only computes the matrix elements corresponding to the non-zero structure of the matrix A . For example, if A is tri-diagonal then M is tri-diagonal as well. That preconditioning computes a matrix M so that $M=LDU$, where L , D and U are respectively the approximations of the unit lower triangular, diagonal and upper triangular matrices computed by the full LDU factorisation algorithm. When conditioning number is bad and in order to increase

convergence speed of the iterative solver, we multiply the element of D by a number a little higher than 1 [2][6].

The second preconditioning technique we describe is the ILUT algorithm [5]. Its principle is also based on the full LU factorisation, but does not only takes into account the non-zero structure of A. In fact, it uses two parameters: the level of fill, LFIL and the tolerance TOL. For each line of A, it calculates the LFIL larger elements with the last algorithm and drops the smallest thanks to TOL.

That preconditioning is interesting for matrices with bad conditioning because we can tune the quality of the preconditioning. In the other hand, it needs more memory requirement and more computation time than the Incomplete Factorisation of Gauss of type LDU.

b. Solvers

In this part, we describe the solvers we test. These are Bi-CG, Bi-CGStab and GMRES.

Bi-CG [1] is an extension of the conjugate gradient to the non symmetric matrices. In fact, it applies the conjugate gradient method to solve in the same time the two systems $A.x=b$ and $A'.x'=b$ (x and x' are the solution of the two different systems). Each iteration of this algorithm requires a product matrix by vector with the transpose of A.

The main idea of Bi-CGStab [3] is the same than Bi-CG. It is based on the fact that the two descent directions are computed using the same recurrence. Thanks to a polynomial interpretation, the algorithm does not require matrix by vector product with the transpose of A.

The principle of GMRES [4] is different than the two last ones. Indeed, the other solvers find, at each iteration one direction of descent and compute the best approximation of the solution in this direction. The GMRES algorithm finds first all descent directions and then computes the approximate in the space created by those directions. In practice, because of the necessary memory requirement, we do not compute the N directions but only a number m given by the user. This algorithm is called the restart GMRES or GMRES(m). In this paper, we use this last one.

For our tests, we choose to compare three “combinations” of solvers. These are Bi-CG with the Incomplete Factorization of Gauss of type LDU, Bi-CGStab and GMRES(m). These two last solvers are combined with the ILUT preconditioning.

A version of these solvers coming from the Sparskit library of Y. Saad [5] has been implemented in the FLUX software (3D application).

III. DESCRIPTION OF THE SUPERCONDUCTOR PROBLEMS

The solvers are tested on two transient magnetic problems using superconductor material. In a first paragraph, we describe the law used for describe the relation between E and J in the superconductor material. Afterwards, we describe geometry and physical properties of the two problems.

a. The power law

In our problems, the relation between the electric field E and the current density J is given by the power-law [7]:

$$|E| = E_c(|J|/J_c)^n$$

The curves obtained for different values of n are shown on the figure 1.

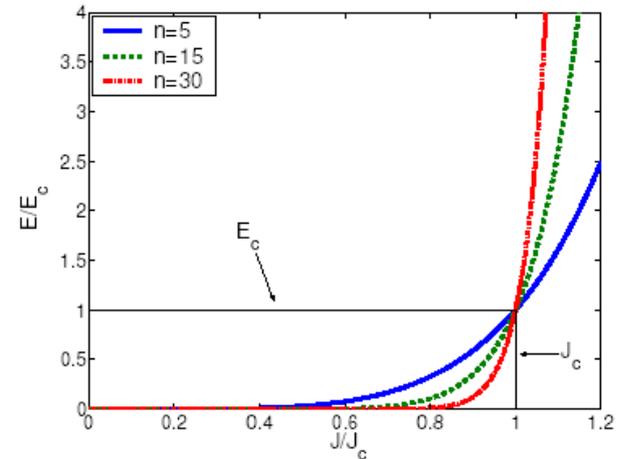


FIGURE 1: CURVE E(J) OF THE POWER LAW

For high n, the curve has a sharp knee. The limit case, when n tends to infinity is a curve with a right angle.

However, the matrix to solve contains elements with $\frac{\partial E}{\partial J}$ in factor. Then, when we are just near the curve knee, the value of this factor will be high compared to other matrix elements. That difference damages the conditioning of the matrix.

b. First Problem

Geometry and materials properties are shown on figure 2. For reasons of symmetry, only one eighth of the geometry is represented ($\omega=2\pi f$, where f is the frequency and is equal to 50 Hertz).

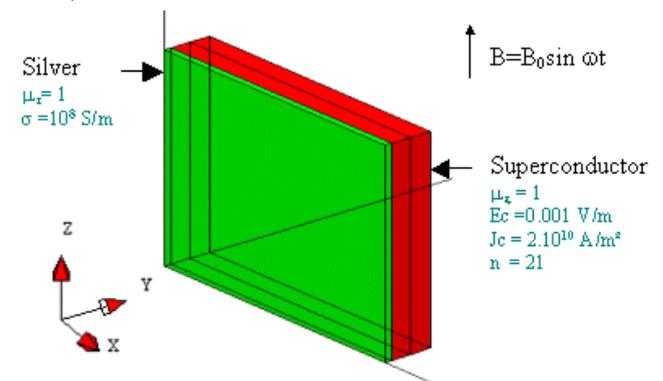


FIGURE 2: GEOMETRY AND PHYSICAL PROPERTIES OF THE FIRST PROBLEM

The example contains a silver sheet. On both sides of this silver sheet there is a superconductor material with high non-linear conductivity properties. The relation between the electric field E and the current density J is given by the power-law.

The source is a sinusoidal time varying uniform magnetic field and we use the nodal electric vector potential formulation [8].

c. Second problem

The second problem represents superconductor filaments in a silver matrix. The geometry is shown on the figure 3. The silver has a conductivity equal to $4 \cdot 10^8$ S/m. The superconductor filaments have a relative permeability equal to 1 and a relation between J and E given by the power law, with the parameters: $E_c = 10^{-3}$ V/m, $J_c = 1,61 \cdot 10^8$ A/m² and $n = 16,8$.

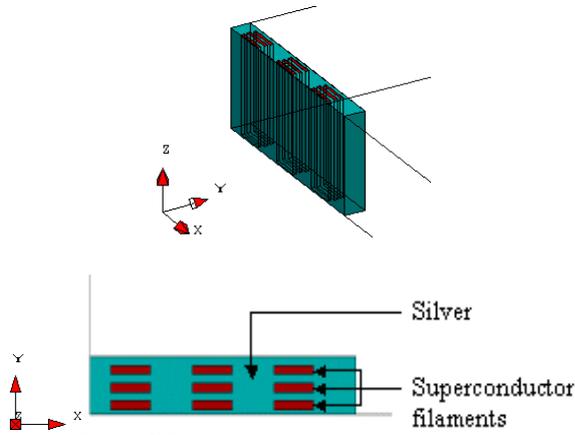


FIGURE 3: DESCRIPTION OF THE SECOND PROBLEM (GENERAL AND TOP SIGHT)

A current source of value $5,92 \cdot \sin(2\pi ft)$ A (f is the frequency and is equal to 50 Hz) is imposed thanks to solid conductors on the top side of each filament.

For this problem, we do not use the nodal T vector potential because it gives errors in re-entrant corners [10]. We prefer to use the edge elements that give correct results in this case.

IV. OBTAINED RESULTS

This part shows the results obtained with the solvers we have just described. These solvers are Bi-CG with the LDU preconditioning, Bi-CGstab with the ILUT preconditioning and GMRES(m) with ILUT preconditioning. In this paper, these solvers are respectively referred to as IGBCG, BCGS and GMRES.

For each problem, we compare robustness, speed and memory requirement for each solver.

In the 3D application of FLUX, the transient model is taken into account by a step by step solving process that is the Euler scheme. Because of the non-linearity of the conductivity

properties, the system to solve will be also non-linear. In this case, the 3D application of FLUX uses the Newton-Raphson process, which is an iterative method solving a linear system at each iterations. For the first problem, we will be particularly interested in the second (case 1) and the seventh (case 2) iteration of the Newton-Raphson process of the first time step. For the second problem, we will be interested in the first (case 3) and the second (case 4) iteration of the Newton-Raphson process of the first time step.

For the first problem, the matrix has 37703 lines and 2579157 non-zeros values. For the second problem it has 29739 lines and 1441607 non-zeros values.

	Memory requirement (Number of real*10 ⁶)			Time (sec)		
	IGBCG	BCGS	GMRES	IGBCG	BCGS	GMRES
1st problem (case 1)	8,148	10,161	23,591	1430	173	184
1st problem (case 2)	8,148	10,161	23,591	DIV	129	320
2nd problem (case 3)	5,144	22,194	10,653	8379	857	780
2nd problem (case 4)	5,144	22,194	10,653	DIV	773	720

TABLE I: Obtained results on the two problems tested. DIV means the solver diverges.

The results obtained with BCGS and GMRES on the second superconductor problem are shown on figures 4 and 5. We can see the shaded map of the current density on the three filaments on the right of the figure 3.

We see they are almost the same in both cases and are conform to what we expected.

The same remark can be made for the first superconductor problem.

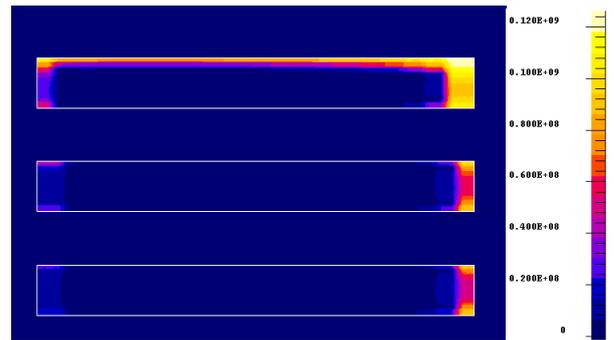


FIGURE 4: CURRENT DENSITY ON THE RIGHT PART OF THE TOP FACE OBTAINED WITH BCGS (A/m²)

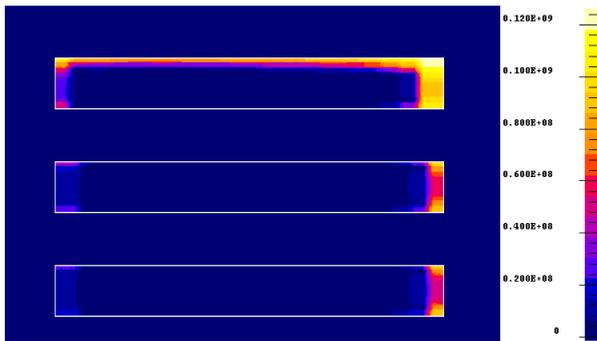


FIGURE 5: CURRENT DENSITY ON THE RIGHT PART OF THE TOP FACE OBTAINED WITH GMRES (A/m²)

BCGS and GMRES converge then on the two problems while the IGBCG does not converge. Moreover, obtained results seem to be correct. When all the solvers converge, BCGS and GMRES require also less time. We can conclude that the two solvers we have introduced are more robust and faster than IGBCG on our ill-conditioned matrices.

On the other hand, this progress is done by using more memory requirement than IGBCG. It is then possible we cannot solve problems with a lot of degrees of freedom.

V. CONCLUSION

In this paper, we present two combinations formed by the GMRES and Bi-CGStab solvers with ILUT preconditioning. We find that they are more robust on our 3-D superconductor problems than the well-known Bi-CG solver with Gauss preconditioning of type LDU.

In the other hand, GMRES and Bi-CGStab require more memory space. We will test them on matrices of higher rank N so as to see their limits.

The second example uses another origin of ill-conditioned matrices, that is the edge elements for approximating T electric vector potential. We can think our two combinations of solvers are also more robust with several other categories of electromagnetic problems generating ill-conditioned matrices. To be sure, we will test these solvers on a largest list of benchmark tests containing examples from very different physical applications (thermal magnetic coupling, dielectric,...). These tests on a large list can also allow us to compare more precisely the memory requirement and the computation time for both Bi-CGStab and GMRES solvers.

VI. ACKNOWLEDGMENTS

We sincerely thank Y. Saad for authorizing us to use routines of its "Sparskit" library (Version 2) [10].

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