



Heat transfer enhancement by the suppression of the stratified stagnant core in a rectangular differentially heated cavity

P Huerta, R Gers, O Skurtys, Florian Moreau, Didier Saury

► To cite this version:

P Huerta, R Gers, O Skurtys, Florian Moreau, Didier Saury. Heat transfer enhancement by the suppression of the stratified stagnant core in a rectangular differentially heated cavity. *International Journal of Thermal Sciences*, 2023, 186, pp.108137. 10.1016/j.ijthermalsci.2022.108137 . hal-03934938

HAL Id: hal-03934938

<https://cnrs.hal.science/hal-03934938>

Submitted on 11 Jan 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Heat transfer enhancement by the suppression of the stratified stagnant core in a rectangular differentially heated cavity

P. Huerta^a, R. Gers^b, O. Skurtys^{a,*}, F. Moreau^c, D. Saury^c

^a*Department of Mechanical Engineering, Universidad Técnica Federico Santa María, Avenida Vicuña Mackenna 3939, Santiago, Chile*

^b*Department of Mechanical Engineering, Universidad Técnica Federico Santa María, Avenida España 1680, Valparaíso, Chile*

^c*Institut Pprime UPR CNRS 3346, CNRS – ENSMA – Université de Poitiers, Département FTC, Téléport 2, 1 Avenue Clément Ader, BP40109, F-86961 Futuroscope Cedex, France*

Abstract

This work investigates numerically a natural convection flow in a rectangular differentially heated cavity. The aspect ratio of the cavity (height over width) is 4, the working fluid is air and the Rayleigh number based on the height of the cavity is 9×10^7 . A large rectangular obstacle is placed in the center of the cavity. The influence of the size of that obstacle and the effect of a new geometry (obtained by extrusion of two rectangular forms from the obstacle) on the flow dynamics and heat transfers are investigated. The global and local heat transfers modifications, compared to the case without obstacle, are highlighted for several configurations. In such a flow regime, an optimal geometry and a solid-to-fluid conductivity ratio are determined. This geometry leads, for an adiabatic obstacle, to a global heat transfer increase of 4.39% with a local heat transfer increase up to 80%.

Keywords: Differentially-Heated Cavity, Natural Convection, Heat Transfer Modification

1. Introduction

In the past decades, rectangular differentially-heated cavities (DHC) have been widely studied experimentally and numerically for laminar, transient and turbulent flows [1–6]. Natural convection in a rectangular DHC is known to have a lot of industrial and engineering applications as solar collectors [7], cooling of electronic systems [8], air-conditioning [9], double pane windows, and others. However, the size, performance and effectiveness of those engineering systems are often limited by the ability to transfer heat to or from the system. Thus, working toward an energy-efficient society, it is fundamental to understand and control the attendant phenomena to increase or reduce the associated heat transfers [10, 11]. To manipulate heat transfer in a rectangular DHC, both passive and active perturbation strategies of the flow were investigated. The studies showed that active actuators, located at the isothermal wall to modify the thermal or velocity fields, can be either mechanical [12] or thermal [13–15]. In every cases, the main goal is to excite instability waves inside the boundary layer (e.g. Tollmien-Schlichting waves) or inside the core region (e.g. gravity waves), in order to observe a significant increase of temperature gradients at the wall [16]. A numerical investigation (2D-DNS) for a cavity of aspect ratio four was carried out by Chorin et al. [14] using a thermal actuator located at the hot wall. They found that a thermal perturbation has a significant impact on heat transfer up to 16% downstream the disturbed area and up

**Corresponding author.*
Email addresses: pablo.huerta@usm.cl (P. Huerta), romain.gers@usm.cl (R. Gers), olivier.skurtys@usm.cl (O. Skurtys), florian.moreau@ensma.fr (F. Moreau), didier.saury@ensma.fr (D. Saury)
Prepared for submission to International Journal of Thermal Sciences
December 28, 2022

to 2% on the cold wall. Recently, Thiers et al. [15] have shown optimal positions of two local thermal disturbances, one per isothermal wall: the optimum height of the actuator at the hot wall is 70% of the hot plate height and at 30% of the cold plate height. At those positions, the increase of heat transfer reached 5.5% by synchronised square waves.

On the other hand, in general, passive actuators are mechanical. They can be located at the wall or away from it. If we consider techniques applied to the wall, parameters that can influence heat transfer at the isothermal walls are the roughness [17, 18] or a modification of the surface by adiabatic or conducting fins [19–21]. For a two-dimensional differentially heated rough square cavity, effects of frequency and dimensionless amplitude of sinusoidal roughness elements on the hot, and both the hot and cold walls simultaneously, have been investigated numerically by Yousaf and Usman [18] using a single relaxation time Bhatnagar–Gross and Krook (BGK) model in a Lattice Boltzmann method (LBM). The range of Rayleigh numbers explored was $Ra \in [10^3; 10^6]$ and a Newtonian fluid of Prandtl number equals to 1.0 was considered. For all studied cases, a reduction in the overall Nusselt number was noted, up to 17%. The unsteady natural convection flow adjacent to the finned sidewall of a differentially heated cavity has been studied by Xu et al. [22]. In particular, a comparison between cases with a conducting fin and with an adiabatic fin was carried out. The results show that the fin may significantly influence natural convection in the cavity and even trigger the transition to unsteady natural convection in the cavity. Indeed, the authors found that the conducting fin improves the transient convective flows in the cavity and enhances heat transfer across the cavity (by up to 52% in comparison with the case without a fin). Recently, Chorin et al. [23] studied experimentally, a natural convection flow in a rectangular DHC disturbed by introducing a localized obstacle which acts as a small spatial extent passive system. The obstacle is placed inside the hot boundary layer. The influence of the length and the vertical location for an insulating and a conducting obstacle have been analysed. A relative heat transfer increase up to 83% is observed downstream the insulating obstacle for the largest length and highest vertical location.

Another way of altering the natural convection flow and heat transfer is to locate a solid obstacle somewhere in the DHC. Obviously, the size, number, position and physical properties of solid obstacles have deep influences on the enclosed fluid flow and heat transfer structures. A literature review reveals the existence of a large number of numerical studies only devoted to natural convection phenomenon occurring in square DHC containing solid bodies with different thermal behaviours: including heat generation [24, 25], adiabatic [26–28] and conducting properties [29, 30]. In particular, for $Ra \in [10^3; 10^6]$ and $Pr = 0.071, 0.71$ and 7.1 , Bhave et al. [27] reported the existence of an optimum size of the adiabatic solid which allows a maximum of heat transfer, for each Ra . Authors also added correlations, predicting optimum solid sizes, and the corresponding maximum heat transfer as a function of $Ra Pr$. Karki et al. [31] also confirmed the existence of an optimum size of the adiabatic solid. These authors underlined that heat transfer across the cavity decreases by splitting the solid into two or four fragments, but outside of the zone where conduction dominates. In practice,

such solids are not exactly adiabatic bodies: they only have a low thermal conductivity. Thus, the Rayleigh number is not enough. The solid-to-fluid thermal conductivity ratio and the number of solids have to be considered in a square differentially heated cavities with conducting solids. In an earlier paper, House et al. [32] numerically showed that heat transfer across a differentially heated cavity may be enhanced (reduced) by the addition of a single solid with a solid-to-fluid thermal conductivity ratio smaller (greater) than unity. They also concluded that heat transfer may reach a minimum as the solid size was increased for large solid-to-fluid conductivity ratios. The results published by Zhao et al. [33] confirm the heat transfer enhancing with a weakly conductive solid inside a differentially heated cavity for a moderate solid-to-fluid volume ratio.

In conclusion, this review of the literature shows that few publications deal with the influence on heat transfer of a solid located in the core region of a rectangular cavity. However, in a rectangular DHC, there is also a stratified stagnant core of fluid that does not participate in the convection heat transfer between the isothermal walls. If this stagnant region is completely replaced by a centrally-placed adiabatic obstacle, a corresponding steady-state heat transfer enhancement could also be observed. Thus, the main objective of this work is to study numerically the impact of a solid obstacle on the heat transfer, in DHC of vertical aspect ratio equal to 4, when the flow is two-dimensional and laminar ($Ra_H = 9 \times 10^7$). In particular, the influence of the geometry of the solid obstacle is investigated in detail. The present article is organized as follows: in Sec. 2 the problem studied, its modelling as well as a summary of the numerical methods used are presented. In Sec. 3 the numerical results obtained are exhibited for some relevant situations. Flow fields, thermal fields and the heat transfer behaviour are presented and analyzed.

2. Problem and mathematical model

2.1. Problem definition

Consider a two-dimensional vertical differentially heated cavity of height H and width W , filled with air, assumed to be a Newtonian fluid, of kinematic viscosity ν , thermal diffusivity α , thermal conductivity k_f , thermal expansion β and density ρ . The Prandtl number $Pr = \frac{\nu}{\alpha}$ is supposed to be constant and equal to 0.71. The cavity aspect ratio $A_z = \frac{H}{W}$ is set to 4. The two opposite vertical walls of the cavity (x -direction) are maintained isothermal: T_{hot} at $x = 0$ and T_{cold} at $x = \frac{W}{H} = \frac{1}{4}$ with $T_{hot} > T_{cold}$. The top and bottom walls of the cavity are adiabatic. The gravitational acceleration $\vec{g}$ acts in negative z -direction. Three cases are considered: case 1 is the rectangular DHC without solid, only filled with the working fluid (see Fig. 1a); case 2 is the rectangular DHC with an adiabatic or conductive rectangular solid placed at the center of the cavity (see Fig. 1b); finally, case 3 is the rectangular DHC with an adiabatic or conductive extruded solid placed at the center of the cavity (see Fig. 1c).

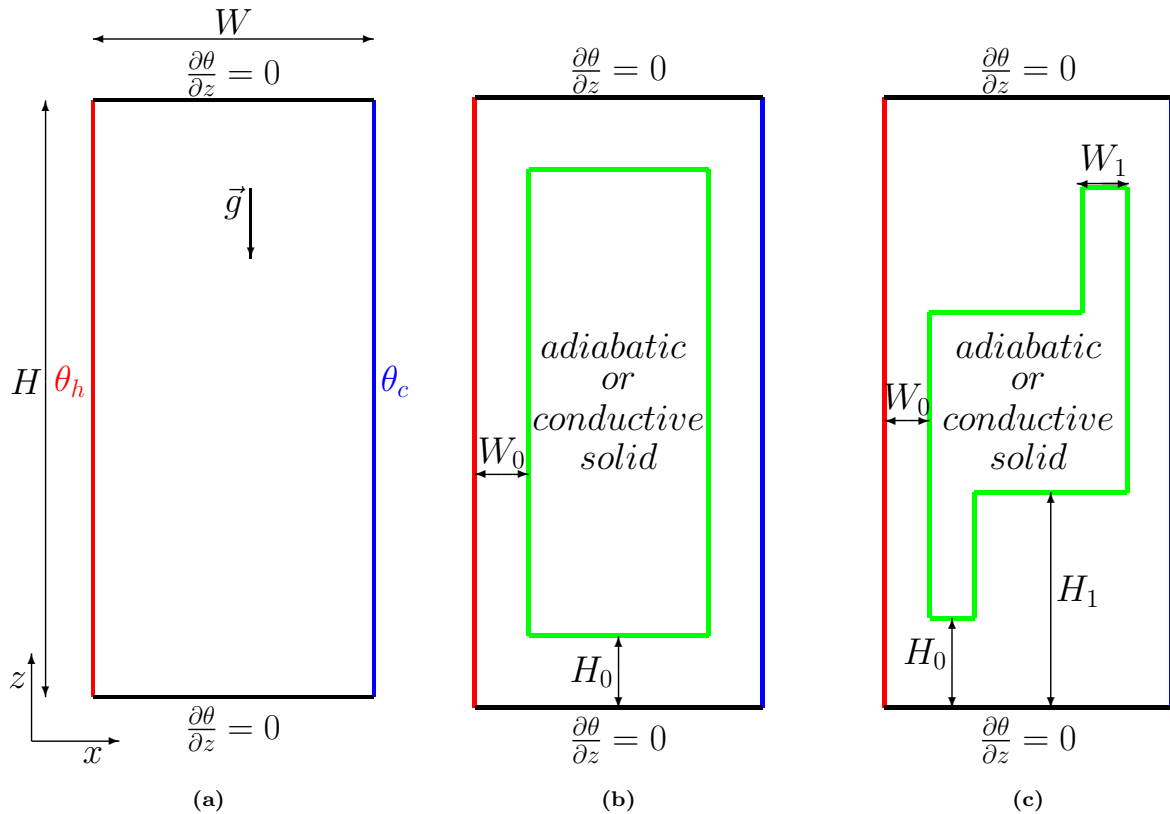


Fig. 1. Schematic of three configurations studied: a) Base case; b) Rectangular case; c) Extruded case.

2.2. Mathematical model

Heat and fluid flows can be described by the unsteady two-dimensional Navier-Stokes equations under the Oberbeck-Boussinesq hypothesis. The Oberbeck-Boussinesq approximation ignores all density variations of the fluid in governing equations, except when associated with the gravitational term. Using the temperature difference $\Delta T = T_{hot} - T_{cold}$ and the mean temperature $T_{mean} = \frac{T_{hot} + T_{cold}}{2}$, the reduced temperature θ can then be defined as $\theta = \frac{T - T_{mean}}{\Delta T}$. Using the cavity height H as the reference length and the Rayleigh number based on this cavity height $Ra = g\beta\Delta TH^3/(\nu\alpha)$, the reference velocity can be defined as $u_{ref} = \alpha Ra^{1/2}/H$, the convection time as $t_{ref} = H^2 Ra^{-1/2}/\alpha$ and the reference pressure as $P_{ref} = \rho u_{ref}^2$. The dimensionless form of the two-dimensional Navier-Stokes equations under the Oberbeck-Boussinesq hypothesis yields:

$$\begin{cases} \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \\ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \frac{Pr}{Ra^{0.5}} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) \\ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{Pr}{Ra^{0.5}} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) + Pr\theta \\ \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + w \frac{\partial \theta}{\partial z} = \frac{1}{Ra^{0.5}} \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \end{cases} \quad (1)$$

where u is the non-dimensional x-direction velocity component, w is the non-dimensional z-direction velocity component, p the non-dimensional pressure.

Within the solid obstacle, the heat equation is expressed as:

$$\frac{\partial \theta}{\partial t} = a_s \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \quad (2)$$

where a_s is the thermal diffusivity of the solid obstacle.

2.3. Boundary conditions

Non-slip boundary conditions are imposed on the solid walls of the cavity and the solid obstacle, i.e.

$$u = w = 0. \quad (3)$$

Horizontal walls of the enclosure are assumed to be perfectly adiabatic. Thus, boundary conditions for temperature are expressed as:

$$\left. \frac{\partial \theta}{\partial z} \right|_{(x,z=0)} = \left. \frac{\partial \theta}{\partial z} \right|_{(x,z=1)} = 0. \quad (4)$$

The thermal boundary conditions of the vertical cavity-walls are given by:

$$\theta(x = 0, z) = 0.5, \quad \theta(x = W/H = 0.25, z) = -0.5. \quad (5)$$

The conservation of the heat flux at the solid obstacle–fluid interfaces could be expressed as following,

$$\left. \frac{\partial \theta}{\partial n} \right|_{fluid} = k_r \left. \frac{\partial \theta}{\partial n} \right|_{solid}, \quad (6)$$

where n is a vector normal to solid surface pointing outwards and k_r is the solid-to-fluid-ratio of thermal conductivity: $k_r = k_s/k_f$. Four specific values of k_r are considered in the last section 3.3: $k_r = 0$ for the perfectly adiabatic solid ($k_s = 0$), $k_r = 10^{-3}$ (low k_s), $k_r = 1$ (air-like k_s), and $k_r = 10^2$ (high k_s).

2.4. Convective transport evaluation and flow visualization

The fluid flows in the DHC are visualized using the Line Integral Convolution (LIC). It was first proposed by Cabral and Leedom [34]. For each grid point in a vector field, a one-dimensional low-pass filter kernel function is used to convolve the noise texture so that it is bidirectionally symmetrical, thereby providing an output texture value. Thus, LIC could characterize flow fields more compactly and comprehensively than geometric visualization methods such as pathlines [35].

To estimate the impact of the obstacles on heat transfer at isothermal walls, the averaged Nusselt number at the hot wall can also be calculated as follows:

$$\langle \text{Nu} \rangle_{x=0} = \int_0^1 - \left. \frac{\partial \theta}{\partial x} \right|_{x=0} dz, \quad (7)$$

The same expression without the minus sign and evaluated at $x = W/H$ is applied at the cold wall. Due to the energy conservation across the enclosure, the average Nusselt numbers at the hot and cold walls must be equal, that is, $\langle \text{Nu} \rangle_{x=0} = \langle \text{Nu} \rangle_{x=W/H}$. In order to compare heat transfer with or without an obstacle, the following parameter is defined:

$$\langle G_{\text{Nu}} \rangle = \frac{\langle \text{Nu} \rangle_{x=0} - \langle \text{Nu} \rangle_{\text{bf}}}{\langle \text{Nu} \rangle_{\text{bf}}} \times 100 \quad (8)$$

where $\langle \text{Nu} \rangle_{\text{bf}}$ denotes the corresponding value of the Nusselt number for the base flow (bf), that is to say without solid. Thus G_{Nu} represents the relative gain ($G_{\text{Nu}} > 0$) or loss ($G_{\text{Nu}} < 0$) of heat transfer compared to the case without modifications ($G_{\text{Nu}} = 0$) when

160 a solid obstacle is located at the cavity center (see Fig. 1). In the same way, the height-
 161 dependant relative gain $G_{\text{Nu}}(z)$ is defined:

$$G_{\text{Nu}}(z) = \frac{\text{Nu}(z) - \text{Nu}_{\text{bf}}(z)}{\text{Nu}_{\text{bf}}(z)} \times 100 \quad (9)$$

162 2.5. Numerical Methods

163 The system of equations (1) is solved by the computational fluid dynamics open-source
 164 program Nek5000 developed and maintained by Fischer et al. [36] at the Argonne National
 165 Laboratory. Nek5000 is a Navier-Stokes solver, which uses a spectral element method pro-
 166 posed by Patera [37], to accurately resolve the velocity field $\vec{u}(x, z, t)$ and temperature field
 167 $\theta(x, z, t)$. This method combines the benefits of high-order spectral methods with finite ele-
 168 ment methods. The computational domain is made of $N_x \times N_z$ rectangular spectral elements,
 169 in the x and z -directions, respectively. The spatial resolution is selected using as a guideline,
 170 the mesh reported by [15]. In Figure 2, four examples of meshes are presented. For the
 171 adiabatic cases, only the flow domain needs a mesh (see Fig. 2a-b) whereas for conductive
 172 obstacles a mesh inside the solid is needed to calculate the temperature field (see Fig. 2c-d).
 173 For the thinnest case ($W_1 \rightarrow 0$, see Fig. 2d) only one point is considered; that is to say for
 174 example, from $z \in [0.7; 0.95]$ (or $z \in [0.05; 0.3]$) the obstacle is only a boundary.

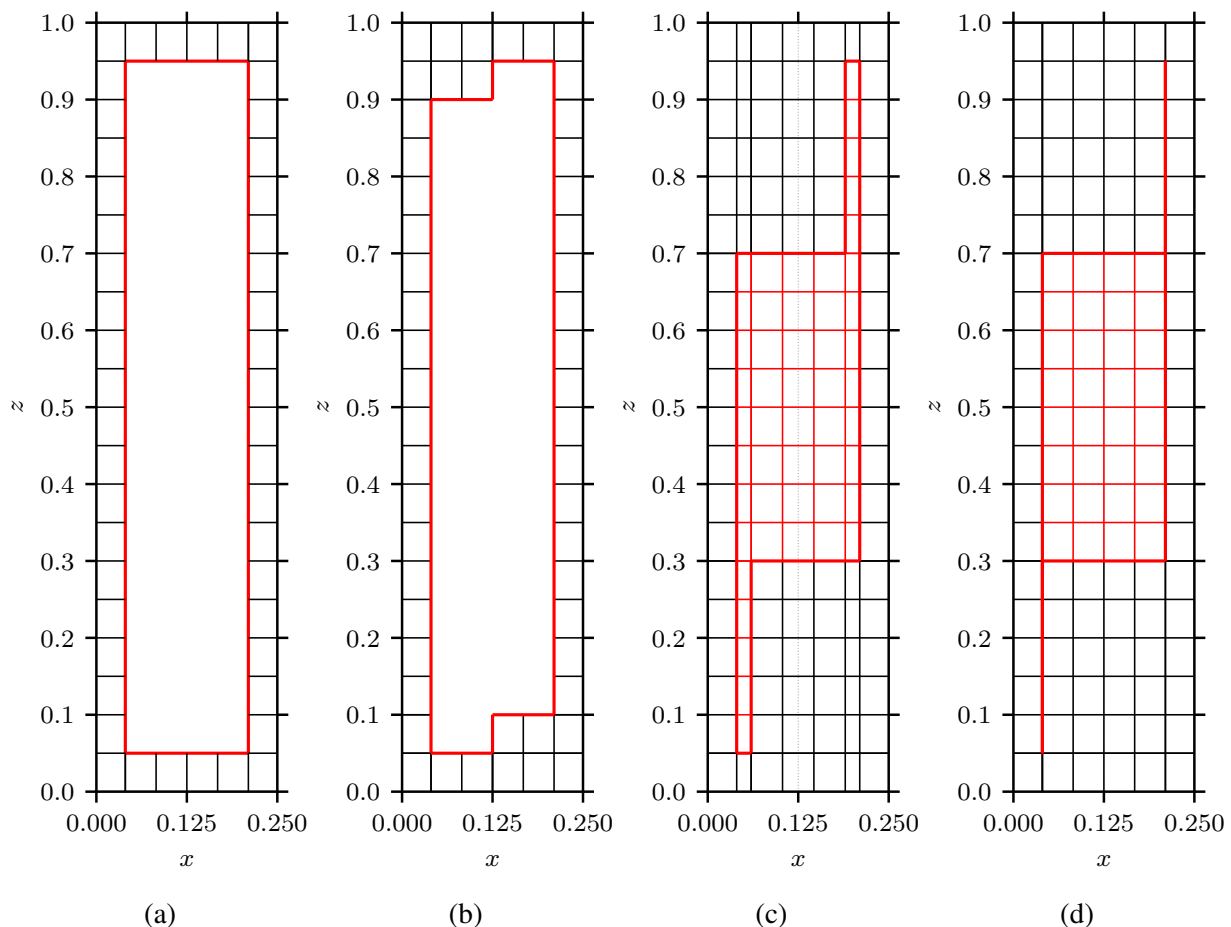


Fig. 2. For $H_0 = 0.05$ and $W_0 = 0.04$, four examples of meshes with $N_x \times N_z = 7 \times 21$: a) an adiabatic rectangular obstacle; b) an adiabatic extruded obstacle ($W_1 = 0.08$, $H_1 = 0.1$); c) a conductive extruded obstacle ($W_1 = 0.02$, $H_1 = 0.3$); d) thinnest conductive extruded obstacle ($W_1 \rightarrow 0$, $H_1 = 0.3$).

175 On each spectral element, the Navier-Stokes equations (see Eq. 1) are rewritten in the
 176 weak formulation and discretized by a Galerkin method, where test and trial functions are

sought in different polynomial spaces. Indeed, a $\mathbb{P}_N - \mathbb{P}_{N-2}$ formulation is used: the velocity and temperature fields are discretized using N th degree Lagrange interpolants, defined on the Gauss-Lobatto-Legendre (GLL) quadrature points, as basis and trial functions, while the pressure field is discretized using Lagrange interpolants of degree $N - 2$ defined on the Gauss-Legendre quadrature points. The time-derivative terms are discretized by the third-order backward differentiation formula (BDF3). The nonlinear convective terms are computed explicitly using a third-order extrapolation scheme (EXT3), while the linear terms are treated implicitly. This high-order splitting method (BDF3-EXT3) leads to a Poisson equation for pressure and Helmholtz equations for temperature and velocity components that are solved using a generalized minimal residual method (GMRES) where the tolerance is set to 10^{-8} . More details on the numerical scheme and appropriate grid resolution can be found in Deville et al. [38].

2.6. Mesh convergence and code validation

The numerical method is validated, comparing our results for the base case (see Fig. 1a) with those reported by Gadoin et al. [39] on a wider range of Rayleigh numbers, between $Ra_H = 6.4 \times 10^6$ and 1.92×10^8 . A mesh with 6×20 spectral elements and a grid polynomial order $N = 24$ resulting in 144×480 grid points is chosen. For different Rayleigh numbers, averaged Nusselt numbers at the isothermal wall $\langle Nu \rangle$ are reported in Table 1. It can be seen that the present results match very well those reported by Gadoin *et al.* [39] and that the following correlation $\langle \overline{Nu} \rangle = 0.311 \times Ra_H^{0.2493}$ is obtained. This relationship is also very close to the numerical correlation $\langle Nu \rangle = 0.29 Ra_H^{0.25}$ reported by Grondin and Roux [40] and experimental results $\langle Nu \rangle = 0.288 Ra_H^{0.25}$ reported by Belleoud *et al.* [41]. For all other cases (see Figs. 1b and 1c), a mesh convergence study is performed using a p -refinement technique in which the grid polynomial order N was increased: $N = 22, 24, 26$ and $N = 28$. It appeared that $N = 24$ is sufficient to obtain an accurate Nusselt number at the active wall and a sufficiently high resolution of the velocity profile. An increase of the polynomial order does not affect the simulation results. We have also checked that the cold and hot Nusselt numbers are equal. Close to the isothermal walls, the wall-normal distance is smaller than $\frac{(\Delta x)_{min}}{H} = 3.3 \times 10^{-4}$. A constant time-stepping with a target Courant-Friedrichs-Lewy number of 0.5 is used, which is sufficient to guarantee the stability during the simulation. Each calculation is extended until the flow reaches a steady stage.

Table 1

For different Rayleigh numbers, the average Nusselt number at the isothermal wall, $\langle \overline{\text{Nu}} \rangle$. Comparison with the numerical results reported by [39] and the correlation $\langle \overline{\text{Nu}} \rangle = 0.311 \times \text{Ra}_H^{0.2493}$.

Rayleigh	$0.311 \times \text{Ra}_H^{0.2493}$	Present	Gadoin [39]
6.4×10^6	15.47	15.49	15.49
6.4×10^7	27.46	27.46	27.46
1.28×10^8	32.65	32.69	32.69
1.6×10^8	34.51	34.56	34.56
1.92×10^8	36.12	36.17	36.17

2.7. Base flow

The Rayleigh number, $\text{Ra} = 9 \times 10^7$, is chosen slightly below the first critical Rayleigh number $\text{Ra}_c = 1.052 \times 10^8$ [5]. Thus, the base flow corresponds with a steady state and our results can be compared with those of our previous works [15]. The thickness $\delta_\theta(z)$ of the thermal boundary is defined at a given elevation z , as the minimal horizontal distance, from the wall, for which the core temperature is reached [41, 42]. In the same way, the thickness of the dynamic boundary layer $\delta_w(z)$ is defined, at a given elevation z , as the minimal horizontal distance, from the wall, for which the vertical velocity of the fluid vanishes [4]. It can also be observed that the temperature gradients decrease along the hot wall as z increases, whereas the maximum of vertical velocity w increases. In the middle of cavity, $z = 0.5$, for this Rayleigh number, $\delta_\theta(z) = 0.03$ and $\delta_w(z) = 0.05$.

3. Numerical results and discussion

The primary goal of this work is to find a way to enhance the overall heat transfer in the cavity. A possible way to achieve this goal is to increase the boundary layer velocity at the beginning of the vertical boundary layers by using a solid obstacle. For this purpose, two main configurations are tested. First, a rectangular obstacle is located in the cavity core to suppress the secondary flows. In a second part, the best geometry of the previous configuration is modified by a subsequent extrusion process to generate a new secondary flow by a backward-facing step. In each configuration, the influence of the solid obstacle on the velocity and temperature fields is detailed. Finally, in the third section, effects of heat conduction in the solid obstacle are considered.

3.1. Suppression of base secondary flows induced by an adiabatic rectangular obstacle (case (b) in Fig. 1)

In this section, for forty pairs of (W_0, H_0) , we present in detail the effects of the size of an adiabatic rectangular obstacle (see Fig. 1-b): first on the flow dynamics and in a second part on heat transfer.

234 3.1.1. Flow dynamics

235 After evaluating the base case, in order to explain the influence of the spacing H_0 between
 236 the adiabatic wall and the top or bottom wall of the rectangular obstacle, a steady flow is
 237 obtained and the flow visualizations using the LIC-method are shown in Fig. 3. The spacing
 238 W_0 between the isotherm wall and the vertical wall of the obstacle is constant: $W_0 = 0.04$.
 239 The base case is also presented to show how two recirculation zones are modified by the
 240 obstacle: one at $z \approx 0.9$ (see point A) and one at $z \approx 0.8$ (see point B). The same behaviour
 241 is observed at the bottom of the cavity due to the centrosymmetry property of the flow (see
 242 Fig. 3a). When a rectangular obstacle of height $H_0 = 0.15$ is introduced inside the cavity,
 243 the recirculation zone A and B get smaller (see Fig. 3b). If a larger obstacle is inserted
 244 ($H_0 = 0.05$), the two recirculation zones disappear while a small recirculation zone (noted C)
 245 is formed at the right of the bottom side (or left of the top side) of the obstacle (see Fig. 3c).
 246 This is due to the fact that the velocity inside the vertical channel gets higher than inside
 247 the horizontal one ($H_0 > W_0$); that is to say C corresponds to an hydraulic jump. Finally,
 248 if we increase slightly the rectangle height so that $H_0 = 0.03$, as $H_0 < W_0$, the velocity
 249 inside the horizontal channel is higher than inside the vertical one. As a consequence, a new
 250 recirculation zone is created at the bottom of the vertical left side (or top of the vertical
 251 right side) of the obstacle (see Fig. 3d).

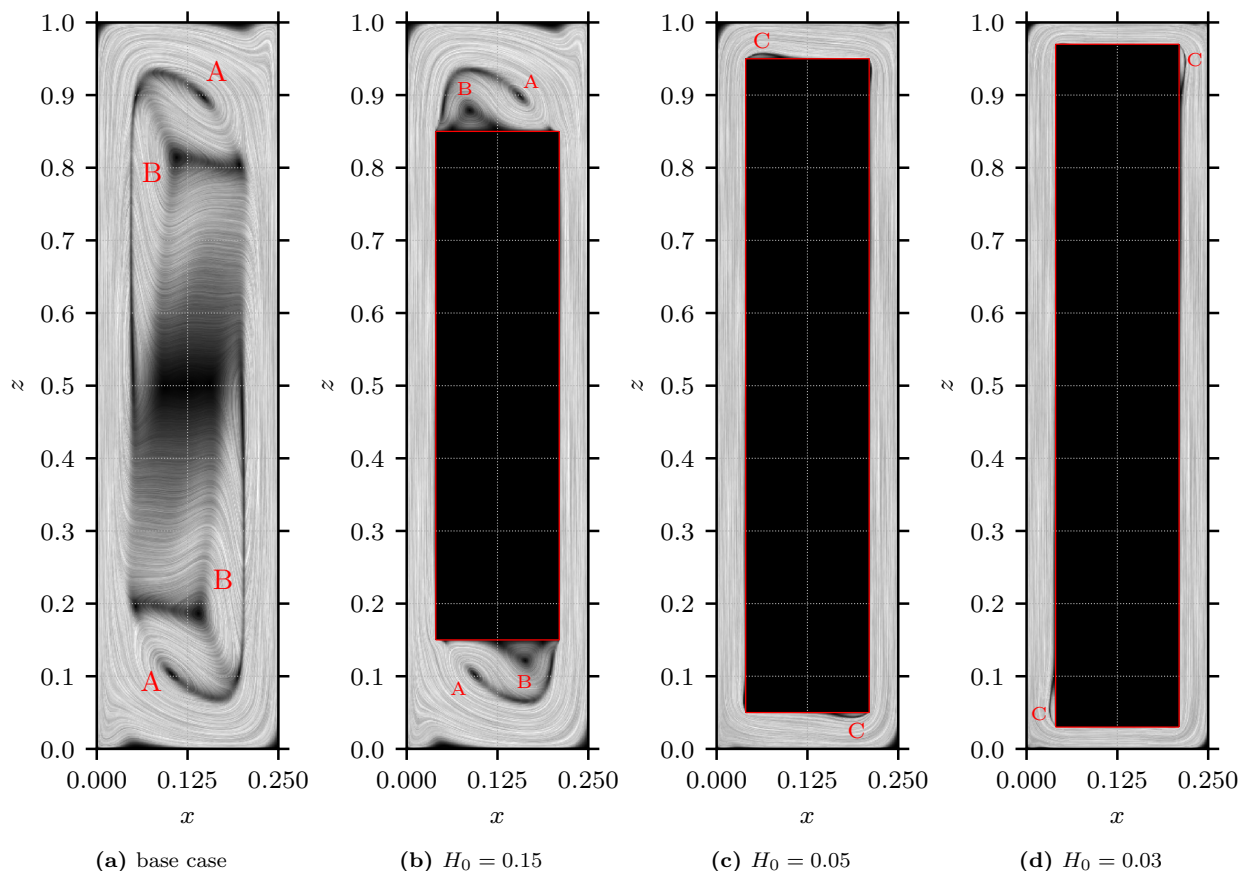


Fig. 3. Line Integral Convolution (LIC) visualisation for three values of H_0 when $W_0 = 0.04$. The base case is also presented for comparison.

252 Figure 4 shows vertical velocity profiles at six heights along the hot wall ($z = 0.05$,
 253 $z = 0.1$, $z = 0.5$, $z = 0.7$, $z = 0.8$ and $z = 0.9$), for the three cases previously presented
 254 ($H_0 = 0.15$, $H_0 = 0.05$, and $H_0 = 0.03$). For $H_0 = 0.15$, as expected, the vertical velocity
 255 profiles are close to those of the base case in the lower part of the cavity (see Fig. 4a).

However, in the upper part of the cavity ($z = 0.9$), the maximum velocity is greater than that of the base case (see Fig. 4b). For smaller values of H_0 , $H_0 = 0.05$ or $H_0 = 0.03$, at the beginning of the boundary layer ($z = 0.05$ and $z = 0.1$) it can be observed that the vertical velocity profiles w are higher than those of the base case. As a consequence, in this area, heat transfer is expected to be higher than in the base case. Using the rectangular obstacle, the flow is forced to pass through a vertical channel. It implies that the air flow benefits from a greater exchange length and consequently G_{Nu} should increase. It can be noted that for $z = 0.05$ and $H_0 = 0.03$, a slight counterflow seen for $x \in [0.03; 0.04]$ corresponds to the C point in Fig. 3d. Finally, it can also be observed for $z = 0.5$ and 0.7 that vertical velocity profiles are slightly lower than those of the base case; this may be explained because the base flow is naturally concentrated in the mid-section of the cavity due to secondary recirculation (see point B, Fig 3a) or because the three cases studied must overcome frictional resistance from the channel walls.

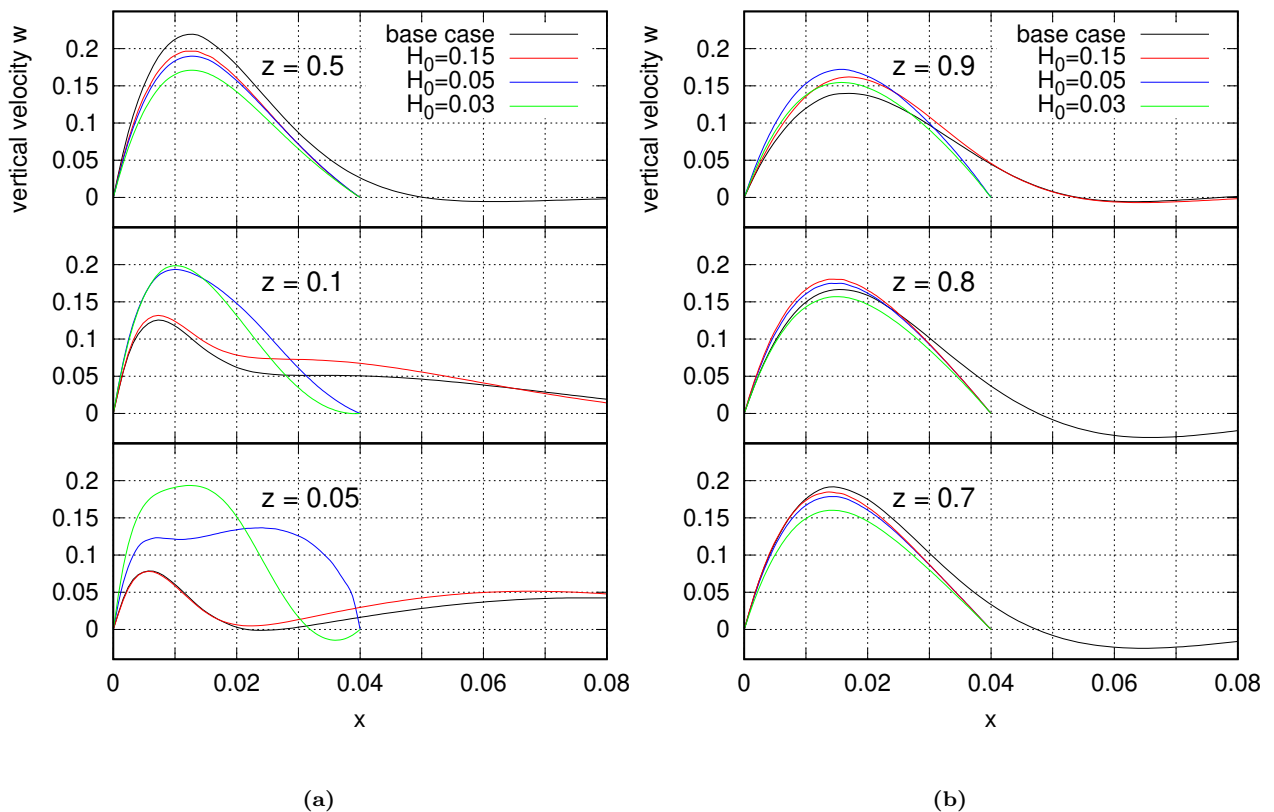


Fig. 4. Vertical velocity profiles w along the hot wall for six different heights: a) upstream section; b) downstream section. Comparison between the base case and three values of H_0 .

268

In Fig. 5, for $H_0 \in [0.03, 0.25]$ and $W_0 \in [0.03, 0.05]$ (i.e. $W_0 \in [\delta_\theta(z = 0.5), \delta_w(z = 0.5)]$ for the base flow case.), the relative mean shear stress at the isothermal wall Γ_T and at the adiabatic wall Γ_Q are presented. They are defined as:

$$\Gamma_T = \left(\frac{\tau}{\tau_{bf}} \right)_{x=\{0; W/H\}}, \quad \Gamma_Q = \left(\frac{\tau}{\tau_{bf}} \right)_{z=\{0; 1\}} \quad (10)$$

272

where $(\tau)_{x=\{0; W/H\}}$ and $(\tau)_{z=\{0; 1\}}$ are, considering a rectangular obstacle at the center, the mean shear stress at isothermal and adiabatic cavity walls respectively. For the base case, the same magnitudes are defined as $(\tau_{bf})_{x=\{0; W/H\}}$ and $(\tau_{bf})_{z=\{0; 1\}}$. The values of H_0 for the thickness of the thermal and dynamic boundary layers for base flow case, $\delta_\theta(z = 0.5)$ and $\delta_w(z = 0.5)$, is also superimposed. Clearly, Γ_T and Γ_Q seems to be dependent on H_0

276

277 than on W_0 since all curves are almost superimposed (see Figs. 5a and 5b). Moreover, it
 278 can be observed in Fig. 5a that $(\tau)_{x=\{0;W/H\}}$ is close to $(\tau_{bf})_{x=\{0;W/H\}}$ as $\Gamma_T \approx 1$. That is
 279 to say, the rectangular obstacle does not modify significantly the mean shear stress at the
 280 isothermal walls. On the contrary, the influence of the obstacle is more important on the
 281 adiabatic walls since $\Gamma_Q \in [0.6; 1.9]$ (see Fig. 5b). One can see that if the vertical aperture
 282 $H_0 < \delta_w(z = 0.5)$ a significant increase of $(\tau)_{z=\{0;1\}}$ occurs (see Fig. 5b) whereas $(\tau)_{x=\{0;W/H\}}$
 283 decreases.

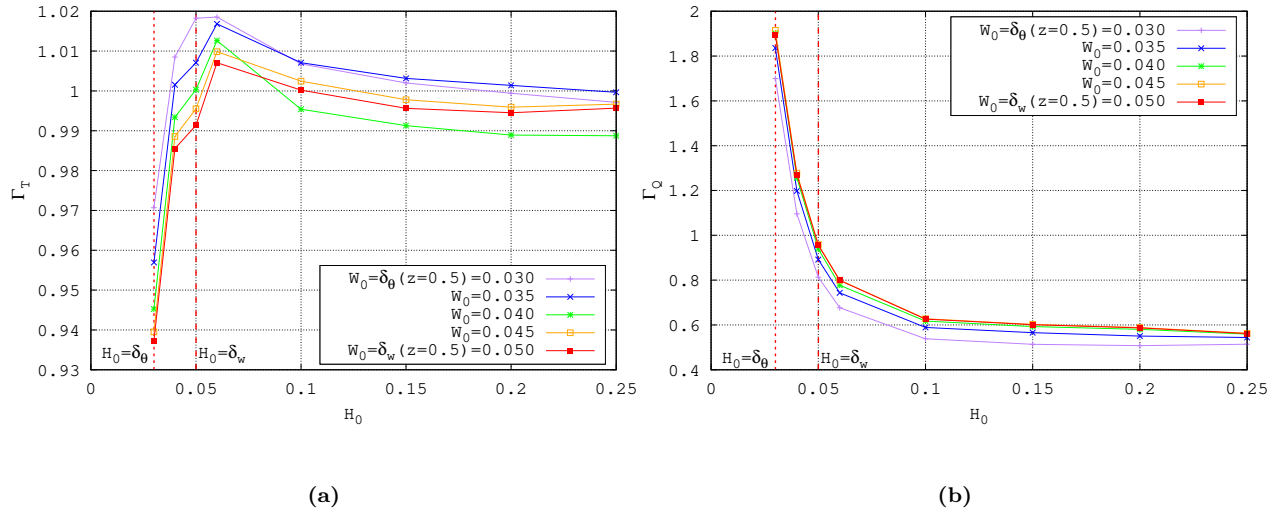


Fig. 5. For various values of H_0 and W_0 , the relative mean shear stress: a) at the isothermal wall $\Gamma_T = \left(\frac{\tau}{\tau_{bf}}\right)_{x=0,0.25}$; b) at the adiabatic wall $\Gamma_Q = \left(\frac{\tau}{\tau_{bf}}\right)_{z=0,1}$. The values of H_0 for the thickness of the thermal and dynamic boundary layers, $\delta_\theta(z = 0.5)$ and $\delta_w(z = 0.5)$, are superimposed.

284 3.1.2. Heat transfer

285 The size effect of adiabatic rectangular obstacle on the heat transfer is now studied in
 286 detail. In Figure 6, the evolution of the relative gain of heat transfer G_{Nu} is shown for various
 287 values of the vertical and horizontal gaps, H_0 and W_0 , respectively. For $H_0 \geq 0.15$, whatever
 288 the value of W_0 , a small value of the relative heat transfer gain $\langle G_{Nu} \rangle$ is obtained, less than
 289 1%. As H_0 decreases to $\delta_w(z = 0.5)$, the heat transfer is higher than 1%; $\langle G_{Nu} \rangle$ increases
 290 continuously and reaches a local maximum for all W_0 . If the separation between the adiabatic
 291 wall and the obstacle is narrowed, $H_0 < \delta_w(z = 0.5)$, $\langle G_{Nu} \rangle$ diminishes continuously probably
 292 because of the friction. It can also be noted that if $W_0 = \delta_\theta(z = 0.5) = 0.03$, the relative
 293 gain is always negative, i.e. the heat transfer is less efficient than the base case whatever the
 294 value of H_0 again probably due to the increase of the pressure lost. Finally, it can be noted
 295 that the heat transfer is maximum, $\langle G_{Nu} \rangle = 2.6\%$, for $H_0 = 0.05$ and $W_0 = 0.04$. This case
 296 is called from this point the best rectangular case.

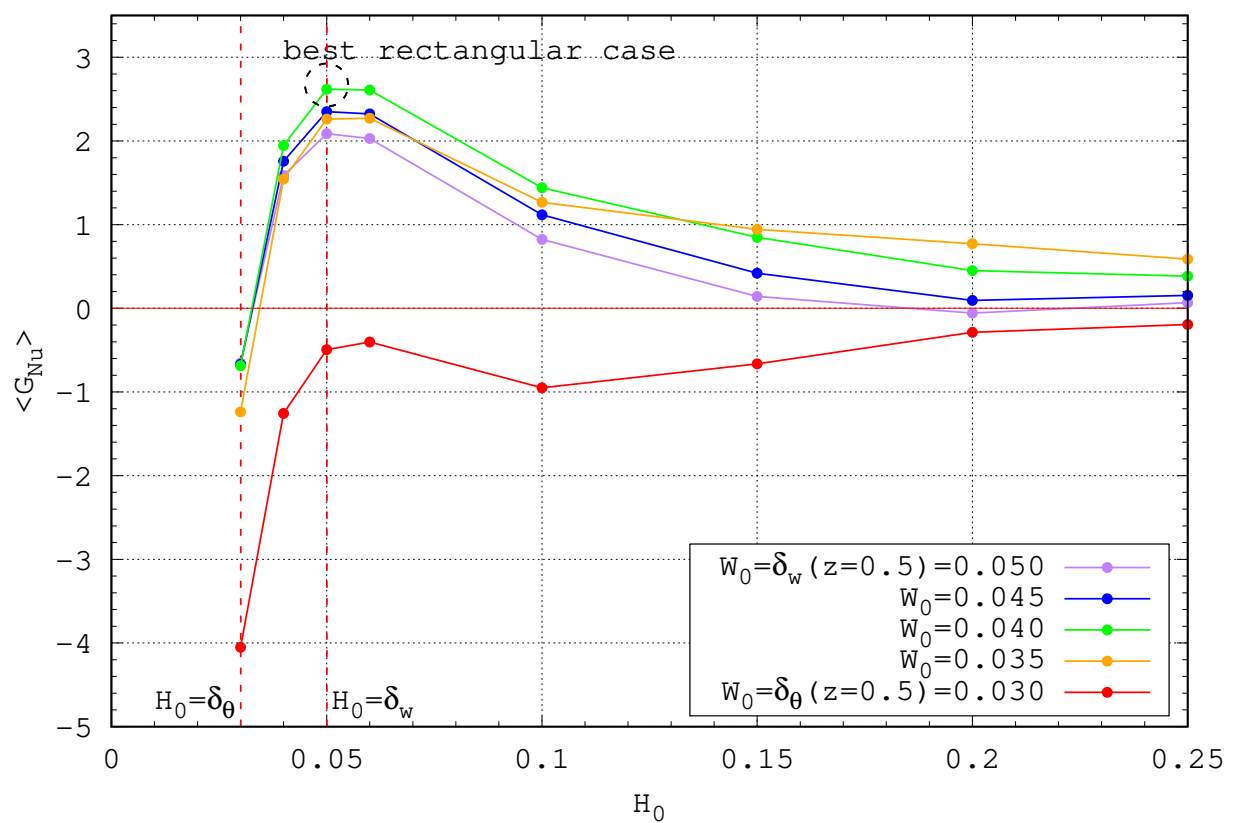


Fig. 6. For various values of H_0 and W_0 , the relative gain of heat transfer $\langle G_{Nu} \rangle$. Values of H_0 for the thickness of the thermal and dynamic boundary layers, $\delta_\theta(z = 0.5)$ and $\delta_w(z = 0.5)$, is superimposed.

297 In Figure 7, for a fixed $W_0 = 0.04$ and three values of H_0 , profiles of Nusselt number
 298 $Nu(W_0 = 0.04, H_0)$ along the hot wall are shown. The base case is also presented for
 299 comparison. It can be observed that when a small rectangular obstacle ($H_0 = 0.25$) is
 300 introduced into the core of the cavity, the heat transfer behaviour is only slightly modified,
 301 whatever the elevation z considered. In this case, the profile of the Nusselt number is close
 302 to that of the base case and the global maximum of Nusselt number is near the bottom of the
 303 cavity ($z = 0.01$). If the distance H_0 is decreased by 2.5 times ($H_0 = 0.10$), the modifications
 304 are more noticeable. Indeed, a global maximum is always present near the adiabatic wall
 305 of the cavity but a local maximum appears at the height $z = 0.1$ which corresponds to the
 306 bottom of the rectangular obstacle. For $z \geq 0.05$ and $z \leq 0.25$, $Nu_{base} < Nu(H_0 = 0.1, W_0 =$
 307 $0.04)$. For $z \geq 0.25$, $Nu(H_0 = 0.1, W_0 = 0.04)$ is slightly lower than the base case. Finally,
 308 if the space between the obstacle and the adiabatic wall of the cavity is 2 times smaller (i.e.
 309 $H_0 = 0.05$), the heat transfer is highly increased for $0.025 < z < 0.25$, compared to the base
 310 case. In particular, a higher global maximum of Nu is observed in comparison with the other
 311 cases at the height $z = H_0 = 0.05$.

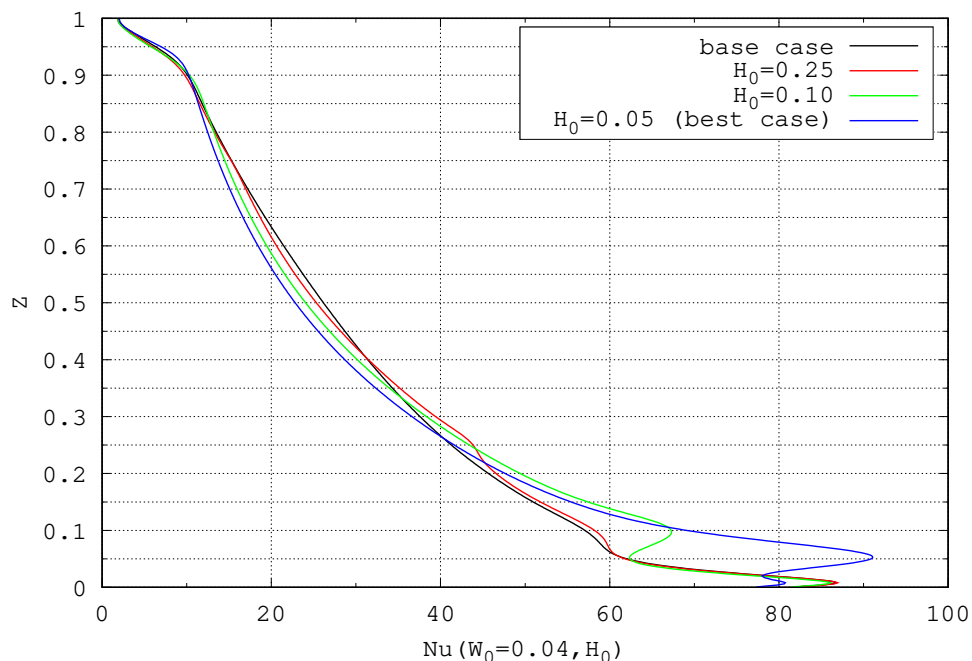


Fig. 7. Profiles of Nusselt number along the hot wall for various values of H_0 when $W_0 = 0.04$. The base case is also presented for comparison.

312 To evaluate more thoroughly the efficiency of heat transfer along the isothermal wall, a
 313 local comparison of the Nusselt numbers is presented in Table 2 for the different separations
 314 H_0 . The isothermal wall is divided in two parts: upstream $z \leq 0.25$ and downstream
 315 $z > 0.25$. It appears clearly that the global enhancement of heat transfer is due to its
 316 increase in the upstream section (see $\langle G_{Nu} \rangle(z \leq 0.25)$ in Tab. 2). So, the main goal of this
 317 work has been achieved.

318 3.2. Generation of new secondary flows induced by a backward-facing step (case (c) in Fig. 1)

319 In this section, the best geometry of the rectangular case is modified by a subsequent ex-
 320 trusion process. The main objective is to generate new secondary recirculations downstream

Table 2

For various H_0 , the relative gain of heat transfer $\langle G_{Nu} \rangle(z)$ in the upstream ($z \leq 0.25$) and downstream ($z > 0.25$) parts when W_0 is fixed to 0.04.

H_0	$\langle G_{Nu} \rangle(z \leq 0.25)$	$\langle G_{Nu} \rangle(z > 0.25)$	$\langle G_{Nu} \rangle(0 \leq z \leq 1)$
0.25	+0.61%	-0.22%	+0.39%
0.20	+1.09%	-0.64%	+0.45%
0.15	+2.05%	-1.20%	+0.85%
0.10	+3.53%	-2.08%	+1.44%
0.05	+6.92%	-4.30%	+2.62%
0.03	+9.09%	-9.77%	-0.68%

the hot and cold boundary layers by a cut rectangular obstacle as those observed in the previous work of [15] who used a local thermal perturbation to obtain them. To obtain the optimal extrusion, two steps are considered. Firstly, the spacing H_0 and W_0 are fixed while the distances H_1 (distance between the cavity adiabatic wall and the horizontal obstacle surface) and W_1 (horizontal obstacle width after extrusion) are varied (see Figure 1c). In a second step, starting from the best geometry which allowed to obtain the highest heat transfer in the first step (called Extrusion 1), the distances obtained previously H_1 and W_1 are fixed whereas H_0 and W_0 are varied again. At the end of step two, the most favourable case to the heat transfer is called: Extrusion 2. In following parts, velocity and temperature fields are successively presented.

3.2.1. Flow dynamics

In Figure 8, steady flow visualizations using the LIC-method are shown for two extruded obstacles and the best rectangle case determined previously. To compare with the best excited case obtained by a localised time varying thermal perturbations at hot and cold walls, the Fig. 8a presenting results from Thiers et al. [15] (see Fig. 8-E) is added. Indeed, for the same Rayleigh number, the authors shown that an efficient way to enhance the heat transfer is to disturb the hot wall at $z = 0.7$ and the cold wall at $z = 0.3$ using synchronised local square waves of amplitude $\varepsilon = 1$ and frequency $f = 0.403$. At this thermal disturbance position, the relative gain of global heat transfer by both disturbances is $\langle G_{Nu} \rangle = 5.5\%$ [15]. In Fig. 8a it can be observed that a large and unique recirculation zone is formed. Moreover, the horizontal boundary layer along the adiabatic wall are thicker than those for the base flow (see Fig. 3a). To take advantage of the modification of the downstream boundary layer and the recirculation zone, the best rectangle is extruded gradually (see Fig. 8b). Figure 8c shows that when a small extrusion is created a small recirculation is formed (see point A). On the contrary in Fig. 8d, the horizontal flow is divided in two parts when it impacts the vertical obstacle tip ($z = 0.95$ or $z = 0.05$): the upper part of the flow is going to the isothermal wall whereas the lower part follows the recirculation zone and is coming back to the isothermal wall (see the red streamline in Fig. 8d). As a consequence, a stagnant zone close to the obstacle (see the zone between the recirculation A and the obstacle). Thus in Fig.

8d, the flow behaviour is closer to the flow when two thermal disturbances are simultaneously introduced at the active walls (see Fig. 8a). Therefore it may be expected a higher heat transfer comparatively with our best rectangular case.

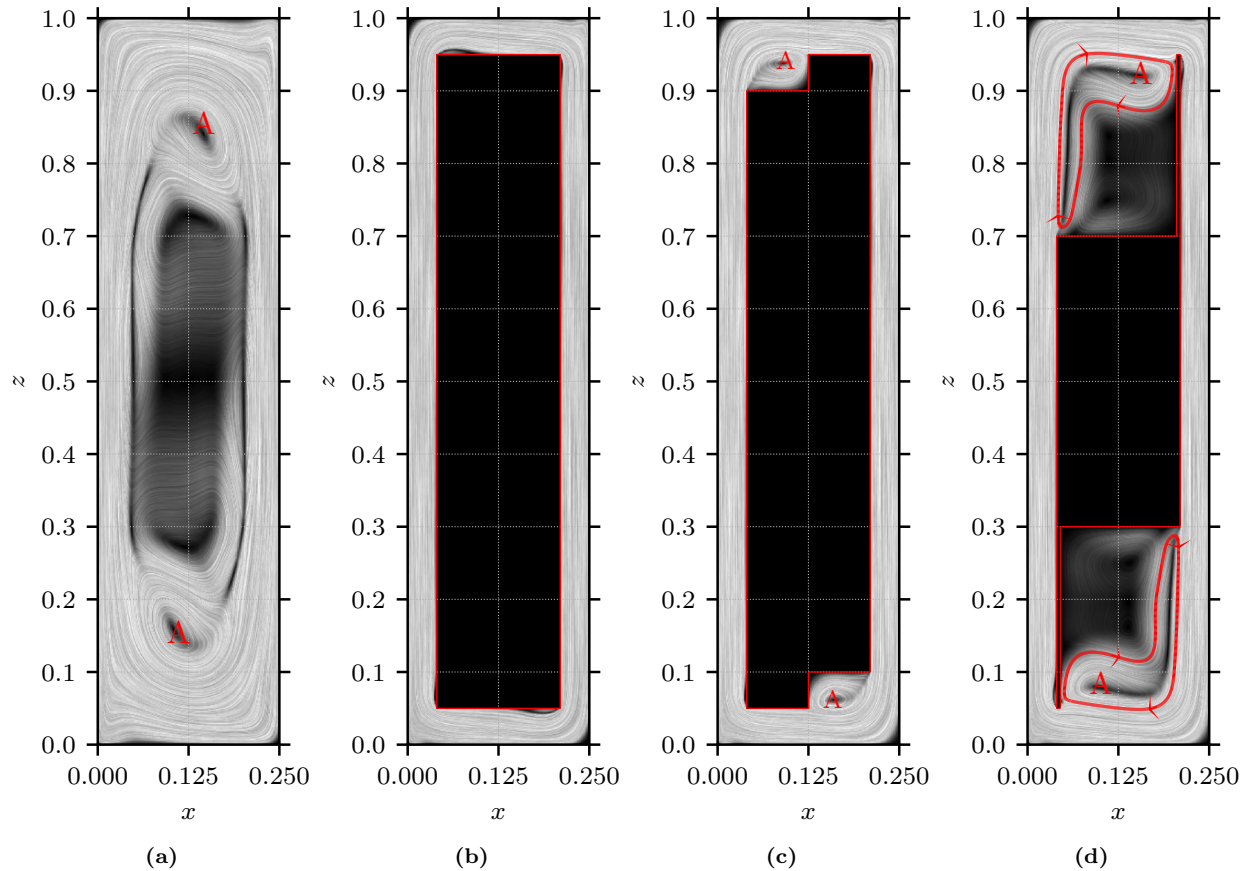


Fig. 8. Line Integral Convolution (LIC) visualisation: a) best excited case by a local thermal perturbation (see previous work [15]) and three obstacles where $H_0 = 0.05$, $W_0 = 0.04$; b) The best rectangular case; c) obstacle 2: $H_1 = 0.1$, $W_1 = 0.085$; d) obstacle 1: $H_1 = 0.3$, $W_1 = 0.005$.

In Figure 9, flows are detailed in a more quantitative way. The best excited case is not more taken into account. Vertical velocity profiles along the hot wall for the base case, the best rectangular case, the case called Extrusion 1 and the case called Extrusion 2 are compared for six different heights $z \in \{0.05; 0.1; 0.5; 0.7; 0.8; 0.9\}$. For each case, the distances H_0 , W_0 , H_1 and W_1 are reported in Tab. 3. In the upstream section ($z = 0.05$ and $z = 0.1$) and the downstream section ($z = 0.9$), it is clear that each obstacle increases the velocity gradient and the maximum velocity compared to the base case (see Fig. 9a). The fluid is forced to flow in the upstream section where the Nusselt number tends to be lower (see Fig. 7). However at $z = 0.5$ the base case has a higher maximum velocity. It can be explained by the fact that in the other cases wall friction has decreased the available energy (see Fig. 9a). In the downstream section ($z = 0.7$, $z = 0.8$ and $z = 0.9$) it can be observed that velocity profiles are higher for the obstacle cases than for base case on the border of the boundary layer ($x > 0.04$). For the Extrusion 2 case, reducing W_0 , the vertical maximum velocity is increased in the upstream section (see green line in Fig. 9a), more than two times at the height $z = 0.05$.

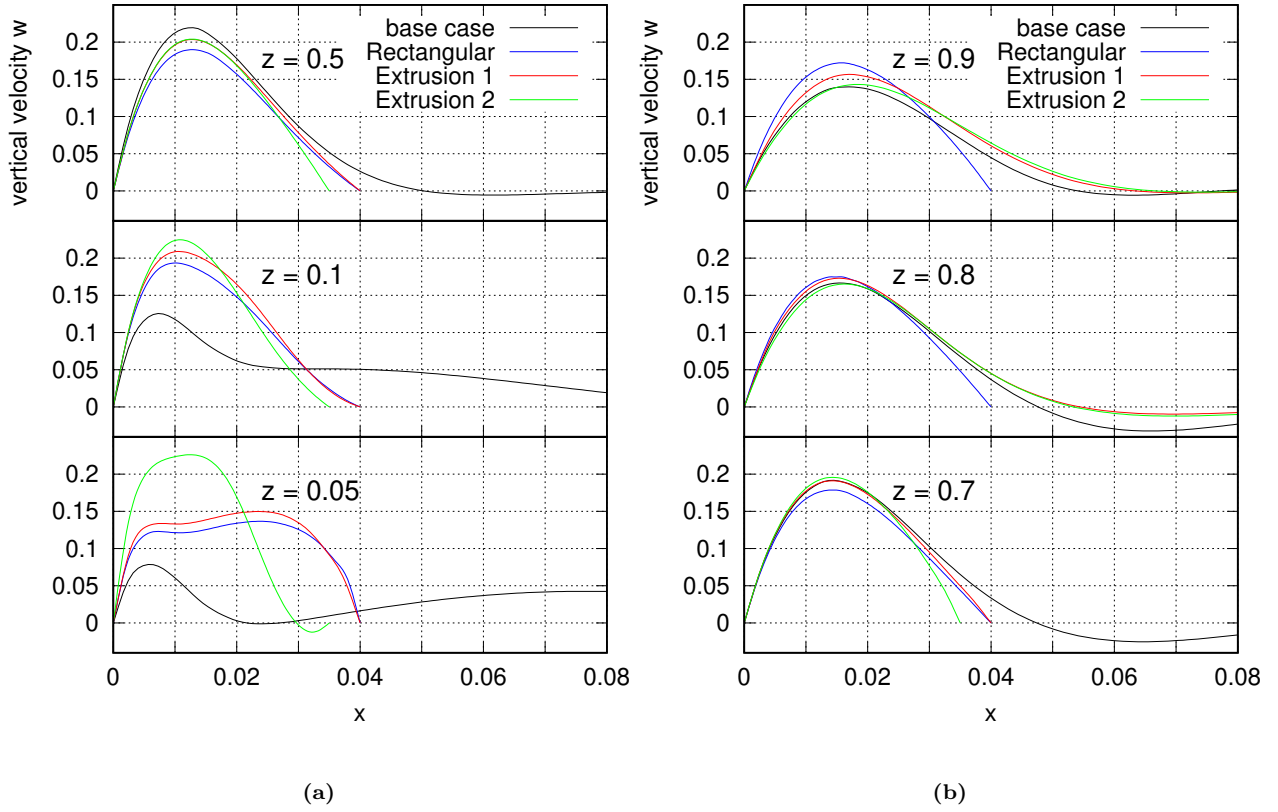


Fig. 9. Vertical velocity profiles along the hot wall for six different heights: a) upstream section; b) downstream section. Comparison between the base case, the best rectangular case ($H_0 = 0.05$, $W_0 = 0.04$), the extruded obstacle 1 ($H_1 = 0.3$, $W_1 = 0.005$) and the extruded obstacle 2 ($H_0 = 0.035$, $W_0 = 0.035$).

Table 3

For the three obstacles, the relative gain of heat transfer $G_{Nu}(z)$ in the upstream ($z \leq 0.25$) and downstream ($z > 0.25$) parts.

case	H_0	H_1	W_0	W_1	$\langle G_{Nu} \rangle(z \leq 0.25)$	$\langle G_{Nu} \rangle(z > 0.25)$	$\langle G_{Nu} \rangle(0 \leq z \leq 1)$
Rectangular	0.05	-	0.04	-	+6.92%	-4.30%	+2.62%
Extrusion 1	0.05	0.3	0.04	0.005	+7.04%	-3.37%	+3.67%
Extrusion 2	0.035	0.3	0.035	0.005	+18.00%	-13.61%	+4.39%

3.2.2. Heat transfer

The relative gain of heat transfer $\langle G_{Nu} \rangle$ is plotted for various values of H_1 and W_1 in Fig. 10a. The distances H_0 and W_0 are maintained constant: $H_0 = 0.05$ and $W_0 = 0.04$. Clearly, all simulated geometries generate a better heat transfer than the rectangular obstacle (which has higher heat transfer than the base case). It can also be observed that there is a local maximum of $\langle G_{Nu} \rangle$ for each W_1 value. This local maximum increases as W_1 diminishes until it reaches a global maximum value for $W_1 \rightarrow 0$ and $H_1 = 0.25$. In this study, for an experimental consideration, the smallest thickness considered is $W_1 = 0.005$, which is 2% of the total obstacle width. Thus, the best extruded case, called Extrusion 1, corresponds to a relative gain $\langle G_{Nu} \rangle = +3.67\%$.

The size of the obstacle which allows the highest heat transfer ($H_1 = 0.3$) and ($W_1 = 0.005$) are now fixed whereas H_0 and W_0 are varied again. In Fig. 10b, the relative gain of heat transfer $\langle G_{Nu} \rangle$ for various values of H_0 and W_0 is shown. The G_{Nu} value of Extrusion 1 is also presented for comparison (red dashed line). From $W_0 = H_0 = 0.05$, both values decrease

until 0.03. It means that a compression of the vertical and horizontal boundary layers occurs. It can be observed that the best tested case, corresponding to $\langle G_{Nu} \rangle = +4.39\%$, is obtained for $W_0 = H_0 = 0.035$. This case is called Extrusion 2.

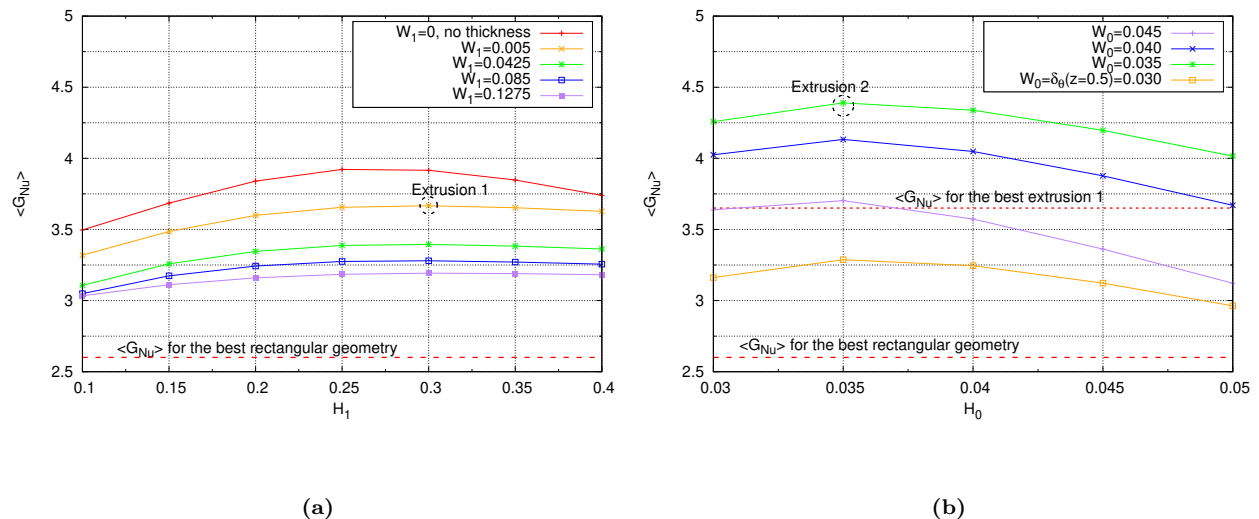


Fig. 10. For two geometries, the relative gain of heat transfer $\langle G_{Nu} \rangle$: a) for various values of H_1 and W_1 when $H_0 = 0.05$ and $W_0 = 0.04$; b) for various values of H_0 and W_0 when $H_1 = 0.3$, $W_1 = 0.005$. The $\langle G_{Nu} \rangle$ value of the best rectangular geometry and best extrusion 1 are also presented for comparison (red dashed lines).

In the Figure 11, the vertical evolution of the relative gain of heat transfer, $G_{Nu}(z)$, along the hot wall are presented for the rectangular case, Extrusion 1 and Extrusion 2. Clearly, for the upstream area $z \in [0; 0.25]$, the three obstacles increase the global heat transfer: +6.92% for the rectangular case, +7.04% for Extrusion 1 and +18% for Extrusion 2 (see Table 3). Moreover, it can be observed that close to $z = 0.05$ the local gain reaches up to 80% for the extrusion 2. However, the local heat transfer is reduced, $G_{Nu}(z) < 0$ for both extrusions when $z \in [0.25; 1]$ and for $z \in [0.25; 0.9]$ in the rectangular case. This is due to the fact that more cold air is in contact with the hot plate, leading to an increase of heat transfer in the upstream part ($z \in [0; 0.25]$); but as a consequence, the ascent of more hot air along the hot plate provokes a decrease of heat transfer in the downstream part $z \in [0.25; 0.9]$. For the rectangular case, when $z \in [0.9; 1]$, the gain is again positive, up to 10%. Finally, at the height $z = 0.7$, it can be observed that the sudden widening diminishes the gain for the extrusion 2 case contrary to the rectangular case (see the change of the slope of the red curve).

3.3. Effects of the thermal conductivity

As it can be noticed experimentally, solids are not exactly adiabatic bodies, effects of fluid and solid thermal conductivity on the heat transfer are studied by considering $k_r = 10^{-3}$, 1.0 and 10^2 (where k_r is the ratio of solid thermal conductivity k_s to the fluid thermal conductivity k_f : $k_r = k_s/k_f$). In Table 4, the relative gain of heat transfer $\langle G_{Nu} \rangle$ is reported for the three solid obstacles. For $k_s = 10^{-3}$, that is to say for a weakly conductive solid, all values of $\langle G_{Nu} \rangle$ are equal or very close to those of the adiabatic case. However, for $k_r = 1$ and $k_r = 10^2$, it is clear that the conductive condition reduces the gain of heat transfer

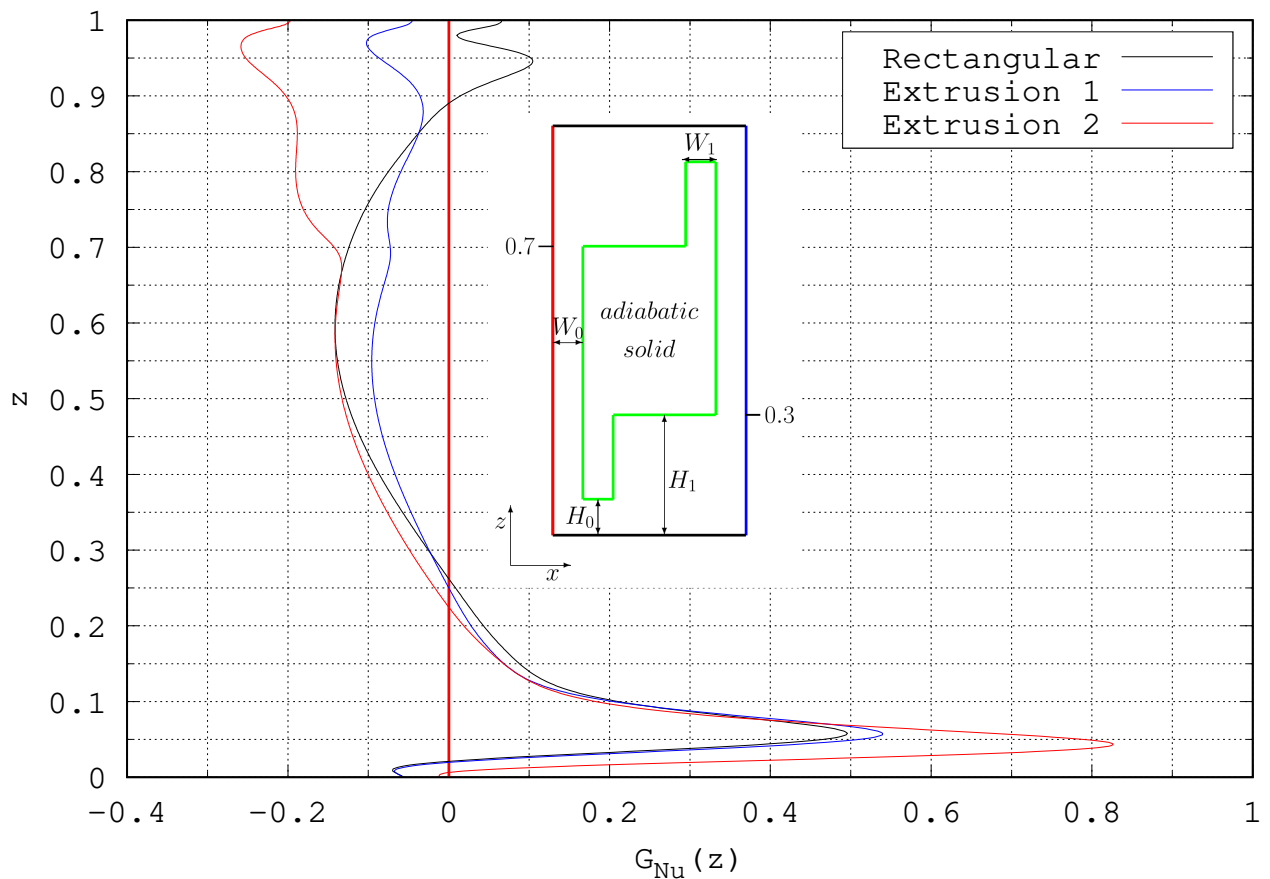


Fig. 11. Vertical evolution of the relative gain of heat transfer, $G_{Nu}(z)$, along the hot wall for three obstacles: rectangular case, extrusion 1 and extrusion 2.

significantly. For $k_s = 1$, which represents a thermal conduction close to that of polystyrene, the gain diminishes around 1% for all solid geometries.

Table 4

Effects of the geometry and thermal conductivity of the solid obstacle k_s on the relative gain of heat transfer.

k_s	k_r	$\langle G_{Nu} \rangle_{\text{rectangular}}$	$\langle G_{Nu} \rangle_{\text{extrusion 1}}$	$\langle G_{Nu} \rangle_{\text{extrusion 2}}$
adiabatic	0	2.62%	3.67%	4.39%
weakly conductive	10^{-3}	2.62%	3.67%	4.38%
air-like	1	2.45%	3.54%	3.69%
highly conductive	10^2	-2.51%	1.15%	0.58%

4. Conclusion

This work report the 2D direct numerical simulations of an air flow in a differentially heated cavity of aspect ratio 4, at a Rayleigh number of 9×10^7 chosen just below than the first critical Rayleigh number ($Ra_c = 1.02 \times 10^8$). In order to improve the heat transfer at the beginning of the boundary layers at the isotherm walls, three kinds of obstacle are tested looking for the optimum size: rectangular, and two cut rectangular forms. With an obstacle, in the middle of the cavity, which compresses the dynamic boundary layer at the isotherm wall, it is possible to enhance the heat transfer globally up to 4.39% compared to the heat transfer in a cavity without obstacle, thanks to an extruded rectangle. This is due to the increase of the velocity downstream the boundary layers at the isotherm walls. Locally, the gain could be higher than 80%. Thus the main conclusion of this work is that the presence

of an adiabatic obstacle increases the heat transfer in the cavity by a passive way. We finally verify that a conductive obstacle does not improve the heat transfer as an adiabatic one does. On the contrary, as the thermal conductivity of the obstacle increases, the global heat transfer decreases. It has to be noted that this study is valid for any Rayleigh number below the first critical Rayleigh number.

The next step of this work is a generalization to a 3D cavity in order to study how the obstacle control the interactions between 3D flow structures and the heat transfer, for laminar and turbulent flows.

Acknowledgements

The authors would like to thank the financial support provided by CONICYT (Fondecyt project 1171281). P. H. thanks UTFSM through its Programa de Incentivos a la investigación científica (PIIC) on its 2021 version.

References

- [1] P. Le Quéré, T. A. de Roquefort, Transition to unsteady natural convection of air in vertical differentially heated cavities: influence of thermal boundary conditions on the horizontal walls, in: Proceeding of International Heat Transfer Conference 8, Begell-house, 1986. doi:10.1615/ihtc8.3100.
- [2] S. Xin, P. Le Quéré, Direct numerical simulations of two-dimensional chaotic natural convection in a differentially heated cavity of aspect ratio 4, *Journal of Fluid Mechanics* 304 (1995) 87–118. doi:10.1017/s0022112095004356.
- [3] F. X. Trias, M. Soria, A. Oliva, C. D. Pérez-Segarra, Direct numerical simulations of two- and three-dimensional turbulent natural convection flows in a differentially heated cavity of aspect ratio 4, *Journal of Fluid Mechanics* 586 (2007) 259–293. doi:10.1017/s0022112007006908.
- [4] D. Saury, N. Rouger, F. Djanna, F. Penot, Natural convection in an air-filled cavity: Experimental results at large rayleigh numbers, *International Communications in Heat and Mass Transfer* 38 (6) (2011) 679–687. doi:10.1016/j.icheatmasstransfer.2011.03.019.
- [5] S. Xin, P. Le Quéré, Stability of two-dimensional (2d) natural convection flows in air-filled differentially heated cavities: 2d/3d disturbances, *Fluid Dynamics Research* 44 (3) (2012) 031419. doi:10.1088/0169-5983/44/3/031419.
- [6] R. Gers, O. Skurtys, N. Thiers, F. Moreau, D. Saury, Three-dimensional effects induced by depth variation in a differentially heated cavity, *Physics of Fluids* 34 (9) (2022) 093602. doi:10.1063/5.0100218.
- [7] S. Amraoui, A. Mezrhab, C. Abid, Combined natural convection and surface radiation in solar collector equipped with partitions, *Applied Solar Energy* 47 (1) (2011) 36–47. doi:10.3103/s0003701x11010051.
- [8] H. J. Kim, B. H. An, J. Park, D.-K. Kim, Experimental study on natural convection heat transfer from horizontal cylinders with longitudinal plate fins, *Journal of Mechanical Science and Technology* 27 (2) (2013) 593–599. doi:10.1007/s12206-012-1236-3.
- [9] L. Moosavi, N. Mahyuddin, N. A. Ghafar, M. A. Ismail, Thermal performance of atria: An overview of natural ventilation effective designs, *Renewable and Sustainable Energy Reviews* 34 (2014) 654–670. doi:10.1016/j.rser.2014.02.035.
- [10] M. G. El-Hak, *Flow Control*, Cambridge University Press, 2011.
URL https://www.ebook.de/de/product/3776239/mohamed_gad_el_hak_flow_control.html
- [11] V. Badescu, *Optimal Control in Thermal Engineering*, Springer International Publishing, 2017. doi:10.1007/978-3-319-52968-4.

- [12] Y. Lin, B. Farouk, Heat transfer in a rectangular chamber with differentially heated horizontal walls: Effects of a vibrating sidewall, *International Journal of Heat and Mass Transfer* 51 (11-12) (2008) 3179–3189. doi:10.1016/j.ijheatmasstransfer.2007.08.005.
- [13] F. Penot, O. Skurtys, D. Saury, Preliminary experiments on the control of natural convection in differentially-heated cavities, *International Journal of Thermal Sciences* 49 (10) (2010) 1911–1919. doi:10.1016/j.ijthermalsci.2010.05.008.
- [14] P. Chorin, F. Moreau, D. Saury, Heat transfer modification induced by a localized thermal disturbance in a differentially-heated cavity, *International Journal of Thermal Sciences* 125 (2018) 101–110. doi:10.1016/j.ijthermalsci.2017.11.018.
- [15] N. Thiers, R. Gers, O. Skurtys, Heat transfer enhancement by localised time varying thermal perturbations at hot and cold walls in a rectangular differentially heated cavity, *International Journal of Thermal Sciences* 151 (2020) 106245. doi:10.1016/j.ijthermalsci.2019.106245.
- [16] B. Podvin, P. L. Quéré, Low-order models for the flow in a differentially heated cavity, *Physics of Fluids* 13 (11) (2001) 3204–3214. doi:10.1063/1.1408919.
- [17] S. Shakerin, M. Bohn, R. Loehrke, Natural convection in an enclosure with discrete roughness elements on a vertical heated wall, *International Journal of Heat and Mass Transfer* 31 (7) (1988) 1423–1430. doi:10.1016/0017-9310(88)90251-7.
- [18] M. Yousaf, S. Usman, Natural convection heat transfer in a square cavity with sinusoidal roughness elements, *International Journal of Heat and Mass Transfer* 90 (2015) 180–190. doi:10.1016/j.ijheatmasstransfer.2015.06.049.
- [19] F. Xu, J. Patterson, C. Lei, Unsteady flow and heat transfer adjacent to the sidewall wall of a differentially heated cavity with a conducting and an adiabatic fin, *International Journal of Heat and Fluid Flow* 32 (3) (2011) 680–687.
- [20] J. Ma, F. Xu, S. C. Saha, Flows and heat transfer of the transition to an unsteady state in a finned cavity for different prandtl numbers, *International Communications in Heat and Mass Transfer* 88 (2017) 220–227. doi:10.1016/j.icheatmasstransfer.2017.09.012.
- [21] A. Abdi, V. Martin, J. N. Chiu, Numerical investigation of melting in a cavity with vertically oriented fins, *Applied Energy* 235 (2019) 1027–1040. doi:10.1016/j.apenergy.2018.11.025.
- [22] F. Xu, J. C. Patterson, C. Lei, Effect of the fin length on natural convection flow transition in a cavity, *International Journal of Thermal Sciences* 70 (2013) 92–101. doi:10.1016/j.ijthermalsci.2013.03.012.
- [23] P. Chorin, F. Moreau, D. Saury, Heat transfer modification of a natural convection flow in a differentially heated cavity by means of a localized obstacle, *International Journal of Thermal Sciences* 151 (2020) 106279. doi:10.1016/j.ijthermalsci.2020.106279.

- [24] M. Y. Ha, M. J. Jung, A numerical study on three-dimensional conjugate heat transfer of natural convection and conduction in a differentially heated cubic enclosure with a heat-generating cubic conducting body, *International Journal of Heat and Mass Transfer* 43 (23) (2000) 4229–4248. doi:10.1016/s0017-9310(00)00063-6.
- [25] M. E. Abdallaoui, M. Hasnaoui, A. Amahmid, Lattice-boltzmann modeling of natural convection between a square outer cylinder and an inner isosceles triangular heating body, *Numerical Heat Transfer, Part A: Applications* 66 (9) (2014) 1076–1096. doi:10.1080/10407782.2014.894398.
- [26] M. Y. Ha, I.-K. Kim, H. S. Yoon, K. S. Yoon, J. R. Lee, S. Balachandar, H. H. Chun, Two-Dimensional and Unsteady Natural Convection in a Horizontal Enclosure with a Square Body, *Numerical Heat Transfer, Part A: Applications* 41 (2) (2002) 183–210. doi:10.1080/104077802317221393.
- [27] P. Bhawe, A. Narasimhan, D. Rees, Natural convection heat transfer enhancement using adiabatic block: Optimal block size and prandtl number effect, *International Journal of Heat and Mass Transfer* 49 (21-22) (2006) 3807–3818. doi:10.1016/j.ijheatmasstransfer.2006.04.017.
- [28] J. R. Lee, Numerical simulation of natural convection in a horizontal enclosure: Part i. on the effect of adiabatic obstacle in middle, *International Journal of Heat and Mass Transfer* 124 (2018) 220–232. doi:10.1016/j.ijheatmasstransfer.2018.03.077.
- [29] J.-T. Hu, X.-H. Ren, Di Liu, F.-Y. Zhao, H.-Q. Wang, Conjugate natural convection inside a vertical enclosure with solid obstacles of unique volume and multiple morphologies, *International Journal of Heat and Mass Transfer* 95 (2016) 1096–1114. doi:10.1016/j.ijheatmasstransfer.2015.12.070.
- [30] A. E. Mansouri, M. Hasnaoui, A. Amahmid, M. Alouah, Numerical analysis of conjugate convection-conduction heat transfer in an air-filled cavity with a rhombus conducting block subjected to subdivision: Cooperating and opposing roles, *International Journal of Heat and Mass Transfer* 150 (2020) 119375. doi:10.1016/j.ijheatmasstransfer.2020.119375.
- [31] P. Karki, A. K. Yadav, D. Arumuga Perumal, Study of Adiabatic Obstacles on Natural Convection in a Square Cavity Using Lattice Boltzmann Method, *Journal of Thermal Science and Engineering Applications* 11 (3), 034502 (02 2019). doi:10.1115/1.4041875.
- [32] J. M. House, C. Beckermann, T. F. Smith, Effect of a centered conducting body on natural convection heat transfer in an enclosure, *Numerical Heat Transfer, Part A: Applications* 18 (2) (1990) 213–225. doi:10.1080/10407789008944791.
- [33] F.-Y. Zhao, D. Liu, G.-F. Tang, Conjugate heat transfer in square enclosures, *Heat and Mass Transfer* 43 (9) (aug 2006). doi:10.1007/s00231-006-0136-4.

- [34] B. Cabral, L. Leedom, Imaging vector fields using line integral convolution, SIGGRAPH, Computer Graphics Proceedings, 1993, p. 263–270.
- [35] B. Tang, H. Shi, Intelligent vector field visualization based on line integral convolution, Cognitive Systems Research 52 (2018) 828–842.
- [36] P. Fischer, J. Lottes, S. Kerkemeier, Nek5000: open source spectral element cfd solver (2016).
URL <http://nek5000.mcs.anl.gov>
- [37] A. T. Patera, A spectral element method for fluid dynamics: Laminar flow in a channel expansion, Journal of Computational Physics 54 (3) (1984) 468–488. doi:10.1016/0021-9991(84)90128-1.
- [38] M. O. Deville, P. F. Fischer, E. H. Mund, High-Order Methods for Incompressible Fluid Flow, Cambridge University Press, 2002. doi:10.1017/cbo9780511546792.
- [39] E. Gadoin, P. L. Quéré, O. Daube, A general methodology for investigating flow instabilities in complex geometries: application to natural convection in enclosures, International Journal for Numerical Methods in Fluids 37 (2) (2001) 175–208. doi:10.1002/fld.173.
- [40] J. C. Grondin, B. Roux, Recherche de corrélations simples exprimant les pertes convectives dans une cavité bidimensionnelle, inclinée, chauffée différentiellement 14 (1) 49–56. doi:10.1051/rphysap:0197900140104900.
- [41] P. Belleoud, D. Saury, P. Joubert, D. Lemonnier, F. Djanna, Experimental investigations in an air-filled differentially-heated cavity at large rayleigh numbers, Journal of Physics: Conference Series 395 (2012) 012119. doi:10.1088/1742-6596/395/1/012119.
- [42] P. Belleoud, D. Saury, D. Lemonnier, Coupled velocity and temperature measurements in an air-filled differentially heated cavity at $Ra = 1.2 \times 10^{11}$, International Journal of Thermal Sciences 123 (2018) 151–161. doi:10.1016/j.ijthermalsci.2017.09.018.