Addendum to "Isomorphism theorems, extended Markov processes and random interlacements"

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Electronic Journal of Probability (2022) Vol 27, 1-27.

This addendum clarifies Definition 5.1 in section 5.3. Definition 5.1, which defines random interlacements for continuous transient Borel processes in weak duality, should be replaced by a new version presented below. We keep the notation of the original paper and in particular of section 5.3.

To introduce the new version of Definition 5.1, we first set, for any given compact subset B of E: $\lambda_B = \sup\{s \in (b,d) : Z_s \in B\}$, with $\sup \emptyset = -\infty$, and note from Proposition 13.11 in Getoor and Sharpe [17], that $\hat{Q}_\nu[\ .\ ; 0 < \lambda_B < 1]$ is a finite measure.

The capacitary measure $\hat{e}_B$ of B with respect to $\hat{X}$, can also be expressed as follows:

$$\hat{e}_B(f) = \lim_{t\to 0} \frac{1}{t} \hat{Q}_\nu[Z_0 \in E, f(Z_{\lambda_B}); 0 < \lambda_B \leq t]$$

for every nonnegative measurable function f. Since one also has ((5.18) with (5.6)) $\hat{e}_B(f) = \hat{Q}_\nu[f(Z_{\lambda_B}); 0 < \lambda_B \leq 1]$, one obtains:

$$\lim_{t\to 0} \frac{1}{t} \hat{Q}_\nu[0 \leq b < \lambda_B \leq t] = 0. \tag{1}$$

Since λ_B is a stationary time, one has thanks to (2.1) and (2.3) in [14] for every $t > 0$, and every functional F:

$$\frac{1}{t} \hat{Q}_\nu[F(Z_{\lambda_B+s}, s \geq 0); 0 < \lambda_B \leq t] = \hat{Q}_\nu[F(Z_{\lambda_B+s}, s \geq 0); 0 < \lambda_B \leq 1].$$

Setting: $\hat{I\!P}_\nu = \int_E \nu(dy) \hat{I\!P}_y$, this leads together with (1) to

$$\hat{Q}_\nu[F(Z_{\lambda_B+s}, s > 0) f(Z_{\lambda_B}), 0 < \lambda_B \leq 1] = \lim_{t\to 0} \frac{1}{t} \hat{I\!P}_\nu[F(\hat{X}_{\hat{L}_B+s}, s > 0) f(\hat{X}_{\hat{L}_B});\ 0 < \hat{L}_B \leq t]$$

Since $\hat{L}_B$ is a splitting time, one has:

$$\hat{I\!P}_\nu[F(\hat{X}_{\hat{L}_B+s}, s > 0) f(\hat{X}_{\hat{L}_B});\ 0 < \hat{L}_B \leq t] = \hat{I\!P}_\nu[\Gamma(\hat{X}_{\hat{L}_B}, F) f(\hat{X}_{\hat{L}_B});\ 0 < \hat{L}_B \leq t]$$

where $(\Gamma(x,B), x \in E, B \in \mathcal{F})$ is a Markov kernel distinct from $(\hat{I\!P}_x(B), x \in E, B \in \mathcal{F})$ (see Theorem 2.12 in Getoor *Splitting time and Shift functionals Z.W. 47, 69-81(1979)* for a complete description of Γ). Hence

$$\hat{Q}_\nu[F(Z_{\lambda_B+s}, s>0) f(Z_{\lambda_B}), 0 < \lambda_B \leq 1] = \hat{e}_B(\Gamma(.,F)f),$$

which leads, $\hat{e}_B(dx)$ a.e x, to

$$\hat{Q}_\nu[F(Z_{\lambda_B+s}, s>0), 0 < \lambda_B \leq 1 | Z_{\lambda_B} = x] = \hat{I\!P}_\nu[F(\hat{X}_{\hat{L}_B+s}, s>0) | \hat{X}_{\hat{L}_B} = x].$$

We then define for any compact subset B of E and $\hat{e}_B(dx)$ a.e x, the probability measure $\hat{I\!P}_x^B$ on the set of E-valued paths indexed by $I\!R_+$ by

$$\hat{I\!P}_x^B[F(Z_s, s \geq 0)] = \int_E \nu(dy) \hat{I\!P}_y[F(\hat{X}_{\hat{L}_B+s}, s \geq 0) \mid \hat{X}_{\hat{L}_B} = x].$$

Remark that for every $\varepsilon > 0$, one has: $\hat{I\!P}_x^B[\{Z_s, s \geq \varepsilon\} \cap B \neq \emptyset] = 0$.
One finally sets the following definition for random interlacements.

Definition 5.1 *For $u > 0$ the random interlacements at level u associated to $\{\nu, ((P_t)_{t\geq 0}, (\hat{P}_t)_{t\geq 0})\}$ is a PPP with intensity measure $u\mu_\nu$ where μ_ν is the measure on $(\mathcal{W}, \mathcal{A})$ such that $\mu_\nu(\omega \equiv \Delta) = 0$, characterized by the following properties:*

- *for any compact subset B of E, define $H_B = \inf\{t \in (b(\omega), d(\omega)) : \omega(t) \in B\}$ with $\inf \emptyset = +\infty$, then*

$$\mu_\nu[\omega_{H_B} \in dx, H_B < \infty] = \hat{e}_B(dx) \tag{5.13}$$

  where $\hat{e}_B$ is the capacitary measure of B associated to $\hat{X}$;

- *for every couple of $\mathcal{A}$ measurable functionals (F_1, F_2)*

$$\begin{aligned} \mu_\nu \quad & [F_1(\omega(H_B + t), t \geq 0);\ F_2(\omega((H_B - t),) t \geq 0);\ H_B < \infty] \\ = \quad & \int_E \hat{e}_B(dx) I\!P_x[F_1(X_t, t \geq 0)] \hat{I\!P}_x^B[F_2(Z_t, t \geq 0)]]. \end{aligned} \tag{5.14}$$

With this modified Definition 5.1, the rest of the paper is unchanged except for the proof of (5.15) in Theorem 5.1. Indeed the proof of (5.15) stops now page 21 line 20, since we know thanks to (5.6) that $\hat{e}_B(dx)$ a.e. x

$$\begin{aligned} \hat{Q}_\nu \quad & [\quad F_2(Z_{\lambda_B+s}, s \geq 0);\ 0 < \lambda_B < 1 \mid Z_{\lambda_B} = x] \\ = \quad & Q_\nu[F_2(Z_{H_B-s}, s \geq 0); -1 < H_B < 0 \mid Z_{H_B} = x] \\ = \quad & Q_\nu[F_2(Z_{H_B-s}, s \geq 0); 0 < H_B < 1 \mid Z_{H_B} = x] \end{aligned}$$

using again (2.1) and (2.3) in [14].

The seven lines page 21 from line 21 till line 27, should hence be removed.

Besides the formulation of the open question presented in Remark 5.4 has to be reformulated accordingly.

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