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# Admittance of the Earth Rotational Response to Zonal Tide Potential

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17 **Abstract**

18 This study is devoted to the determination of the admittance parameters describ-  
 19 ing the Earth rotational response to the components of the zonal tide potential. First,  
 20 in order to better grasp the physical content of those admittance coefficients, we revisit  
 21 the theoretical description of the length of day (LOD) changes at sub-decadal time scale,  
 22 where forcing is dominated by zonal tides and hydro-atmospheric mass transports. This  
 23 theoretical reminder specifies the rheological coefficients permitting to apply the hydro-  
 24 atmospheric corrections to isolate the tidal part of the LOD. Then, the admittances are  
 25 determined from the LOD series corrected from hydro-atmospheric contributions at the  
 26 frequencies of the dominant zonal tidal terms between 7 and 365 days. In contrast of the  
 27 former kindred studies, we both address the discrepancy of the results brought by var-  
 28 ious EOP series and the hydro-atmospheric corrections on the LOD. Our study forwards  
 29 the complementary corrections brought by the ocean, the land water and sea level changes.  
 30 Below 32 days, removing the atmospheric-oceanic excitation from LOD allows to much  
 31 better constraint the admittance complex coefficients  $\kappa$  than applying the atmospheric  
 32 correction only: the discrepancy with respect to modelled values is reduced up to 70%,  
 33 and the frequency dependence of the imaginary part brought by the ocean dynamical  
 34 response is confirmed. A systematic effect with respect to the values modelled by Ray  
 35 and Erofeeva (2014) has been detected and hints a defect of this model. Moreover, the  
 36 role of land water and associated sea level variation is notable at the semi-annual period.

37 **Plain Language Summary**

38 This study is devoted to the determination of the admittance coefficients describ-  
 39 ing the Earth rotational response to zonal tide components. In contrast of the former  
 40 kindred studies, it both addresses the discrepancy of the results brought by various EOP  
 41 series and the hydro-atmospheric corrections to be applied on LOD. It forwards the com-  
 42plementary corrections brought by the ocean, the land water and sea level changes. Re-  
 43moving the atmospheric-oceanic excitation from LOD allows to much better constraint  
 44the admittance complex coefficients  $\kappa$  than applying the atmospheric correction only. The  
 45role of land water and associated sea level variation is notable at the semi-annual period.  
 46This study also aims at synthesising scattered theoretical results pertaining to the mod-  
 47elling of the length of variation caused by zonal tides and concurrent effect of the mass  
 48transports taking place in the surface fluid layer. Finally, a systematic effect with respect  
 49to the values modelled by Ray and Erofeeva (2014) has been detected and hints a de-  
 50fect of this model.

51 **1 Introduction**

52 The objective of this paper is to gain knowledge on the rotational response to the  
 53 periodic components of the zonal tide between 7 days and one year. This is an upgrade  
 54 of past studies by considering all the perturbations brought by hydro-atmospheric trans-  
 55 ports according to up-to-date global circulation models.

56 Recall the underlying physics to underline the interest of such a study. By raising  
 57 an axisymmetric mass redistribution within the Earth, the lunisolar zonal tides modify  
 58 its axial moment of inertia; **by** virtue of the angular momentum balance, the rotation  
 59 speed  $\omega$ , equivalently the length-of-day (LOD), changes accordingly. At the zonal tide  
 60 frequency  $\sigma$ , the relative fluctuation of the angular velocity  $m_3^g(t)$  can be related to the  
 61 corresponding component of the tidal generating potential through an admittance co-  
 62efficient  $\kappa(\sigma)$  (Agnew & Farrell, 1978), of which the definition is recalled in Section 3.  
 63 As those admittance coefficients depend on the rheology of the solid Earth, on the ocean  
 64 behaviour, on the coupling between the fluid core and the mantle (Wahr et al., 1981),

65 their empirical estimates are of primary importance for understanding the Earth rhe-  
66 ology.

67 The effect of the zonal tides in UT1 was first proposed and modelled by Jeffreys  
68 (1928), but at that times remains speculative. In the 1960's and the 1970's, the increase  
69 in time keeping precision has given rise to refined theoretical model, that we shall not  
70 itemize here, see e.g. (Pariiskii & Pertsev, 1973; Pil'nik, 1974).

71 The first estimates were performed in the 1960-1970's, as soon as optical astrogeode-  
72 tic observations became precise **enough** to detect the largest periodic terms raised by  
73 the zonal tide in UT1-TAI or in the **LOD** anomaly. After the non conclusive attempt  
74 of Markowitz (1959), Guinot (1970) succeeded in determining the prominent lunar waves  
75 from BIH optical data, namely Mf (13.66 days) and Mm (27.56 days) - both reaching  
76 0.8 ms in UT1 (respectively 0.4 ms and 0.2 ms in LOD) (see standard IERS values in  
77 Table 1) - with errors of about 0.2 ms. In the 1970's the UT1 uncertainty dropped to  
78 0.5 ms (0.2 ms on LOD), allowing to estimate smaller terms below 35 days, especially  
79 Mtm (9.13 days), MSf (14.77 d), and MSm (31.82 days) with a precision of about 0.01  
80 ms on UT1 terms (Pil'nik, 1974).

81 Whereas the stochastic contribution of the atmospheric transports competes with  
82 the tidal effect, its continuous background spectrum below 0.01 ms for LOD in the 7-  
83 35 day range did not exceed the uncertainty of the astrogeodetic determination of that  
84 epoch. From lunar laser ranging observations **Yoder and Williams (1981)** fitted the  
85 basic parameters theorising the changes of the Earth angular velocity, and **built** an ex-  
86 haustive model of the zonal tide effect on LOD containing 62 terms. As the modelling  
87 of the zonal tidal winds and its contribution to the atmospheric angular momentum had  
88 improved, the removal of the subsequent effect on LOD (over the period 1976-1982) al-  
89 lowed Merriam (1984) to better estimate the admittance coefficients for fortnightly Mf  
90 and monthly Mm tides. In the 1990's, thanks to VLBI observations, the LOD was de-  
91 termined with a precision of about 0.02 ms. Removing the atmospheric correction from  
92 LOD, McCarthy and Luzum (1993) concluded that dynamical ocean tides, as modelled  
93 by Dickman (1993) or Wuensch and Busshoff (1992), are more relevant than equilibrium  
94 ones for calculating the LOD oscillations at zonal tide periods, and Chao et al. (1995)  
95 determined the admittances of 11 zonal tides between 2 and 35 days with an uncertainty  
96 smaller than 0.05. In order to account for these progresses, Defraigne and Smits (1999)  
97 slightly improved the model of Yoder and Williams (1981), addressing a three layered  
98 Earth (an anelastic inner core, an inviscid fluid core and an anelastic mantle) and the  
99 dynamical effect of the ocean tides. To our knowledge, the more recent model, account-  
100 ing for both dynamical response of the ocean and anelasticity of the mantle, was build  
101 by Ray and Erofeeva (2014).

102 **For** three decades, ocean circulation models have strikingly progressed, and the LOD  
103 measurements have benefited from the progresses achieved in Astrogeodesy, in partic-  
104 ular in global navigation satellite system. Since the 2000's, the LOD uncertainty has drop  
105 down to 0.01 ms, so that, in the range 2-365 days, the reconstructed oceanic excitation,  
106 presenting a standard deviation of about 0.03 ms against 0.30 ms for the atmosphere,  
107 could be detected (values obtained from [https://eoc.obspm.fr/index.php?index=excitative&](https://eoc.obspm.fr/index.php?index=excitative&lang=en)  
108 **lang=en** with the atmospheric model ECMWF and ocean model MPIOM over the pe-  
109 riod 2000-2019, see Section 4). Therefore, it is relevant to estimate the observed admit-  
110 tance  $\kappa(\sigma)$  in light of the combined atmospheric and oceanic corrections reconstructed  
111 by contemporaneous global circulation models. To our knowledge, such an estimation  
112 has been done only for fortnightly tides by Ray and Egbert (2012). In this paper, we aim  
113 at extending this calculation to all prominent zonal tides over the last two decades (2000-  
114 2019). And last but not least, we also take into account **of** the correction brought by the  
115 land water transports and corresponding sea level changes.

116 Currently, as  $m_3(t)$  is determined with an error of  $10^{-10}$ , and components of the  
 117 zonal tidal potential are even given with a smaller error, corresponding admittance co-  
 118 efficients provide the best quantification of the overall Earth response to a zonal poten-  
 119 tial of degree 2. As the geophysical excitation is much less precise than the tidal poten-  
 120 tial, the corrected LOD reflects the uncertainty of the geophysical forcing, and spoils the  
 121 estimates of the admittance. Another source of error comes from the astro-geodetic mea-  
 122 surements of the LOD itself. In particular, for LOD series resulting from satellite tech-  
 123 niques, the LOD fluctuations at zonal tide periods can be **correlated with** the corre-  
 124 sponding tidal effect affecting the satellite orbits. At zonal tidal periods in the range 7–  
 125 365 days, geophysical excitation is quite well modelled by the atmospheric transports;  
 126 along with the zonal tides they explain up to 99% of the LOD change in the range of 2–  
 127 365 days. At longer time scale, the only relevant zonal tide is associated with the pre-  
 128 cession of the lunar orbital plane at the 18.6 year period. But, at this time scale, the geo-  
 129 physical excitation is dominated by the fluid core-mantle interaction, of which the un-  
 130 certain modelling at a few ms level prevents **extracting** the tidal term in LOD, and thus  
 131 **determining** the admittance confidently.

132 First, in order to better grasp the physical significance of the admittance coefficient,  
 133 it is **relevant to explicit the theoretical basis of the problem, that is** scattered  
 134 in various works (Sections 2, 3). Then we come up to the adjustment of  $\kappa$  in Section 4  
 135 for the dominant zonal tides. We shall see that the correction brought by ocean and land  
 136 water transports, added to the atmospheric one, allows to improve this fit (Section 5).  
 137 The estimated values are confronted to the theoretical estimates of Ray and Erofeeva  
 138 (2014), accounting for both dynamical response of the ocean, anelasticity of the man-  
 139 tle and fluid core effect (Section 5). In the concluding section, we address the problem  
 140 of systematic shift between **estimated and modelled values**.

## 141 2 Modelling mass redistribution effects on length of day

### 142 2.1 Inertial decoupling of the mantle from the core

143 What is measured through the astro-geodetic techniques is the angular velocity vec-  
 144 tor of a terrestrial frame tied to the Earth crust. At subsecular time scale, it is commonly  
 145 admitted that the lithosphere and the mantle present the same rotation changes with  
 146 respect to a celestial frame, whereas the fluid core can rotate in a slight different way.  
 147 In the terrestrial frame  $Gxyz$ , in which the rotation axis does not deviate more than  $0.5''$   
 148 from the  $Gz$  axis, the angular velocity vector reads  $\Omega(m_1, m_2, 1+m_3)$  where  $\Omega = 7.292115 \cdot 10^{-5}$   
 149  $\text{rad s}^{-1}$  and  $(m_1, m_2, m_3)$  are relative perturbations not exceeding  $10^{-6}$  for the two first  
 150 ones and  $10^{-8}$  for the last. It can be easily **shown** that the angular velocity, namely the  
 151 module of this vector, is approximated at the first order by  $\Omega(1+m_3)$ . Equivalently the  
 152 relative variation of the length of day is

$$\Delta LOD / LOD_0 = -m_3 \quad (1)$$

153 with  $LOD_0 = 86400 \text{ s SI}$ .

154 For the axial change  $m_3$ , it is even assumed that the mechanical system composed  
 155 of the solid Earth (mantle and lithosphere), and of its *surface fluid layer*, also called *hydro-*  
 156 *atmospheric layer*, presents a full inertial decoupling from the fluid core. So, consider a  
 157 mass redistribution within the extended mantle, accompanied by an axial moment of in-  
 158 ertia change  $c_{33}^m$  and a relative angular momentum in the terrestrial frame  $Gxyz$ ; the ax-  
 159 ial part of the linearised Liouville equations then reduces to **(Munk & MacDonald,**  
 160 **1960)**

$$m_3 = -\frac{c_{33}^m}{C_m} - \frac{h_3}{C_m \Omega} \quad (2)$$

161 where  $C_m = 7.1225 \cdot 10^{37}$  kg m<sup>2</sup> is the mean axial inertia moment of the mantle (Dziewon-  
162 ski & Anderson, 1981). More generally, we have

$$m_3 = -\alpha_1 \frac{c_{33}^m}{C} - \alpha_1 \frac{h_3}{C\Omega} , \quad (3)$$

163 where  $\alpha_1 = C/C_m = 1.129$  ( $C = 8.0365 \cdot 10^{37}$  kg m<sup>2</sup>) (Dickman, 2003; Nam & Dick-  
164 man, 1990) tends towards 1 when the coupling with the fluid core increases.

## 165 2.2 Case of a surface mass redistribution

166 In the case of a surface mass redistribution, as the one resulting from the hydro-  
167 atmospheric circulation, the load change produces a deformation of the underneath lithosphere-  
168 mantle, in turn it raises the moment inertia variation  $c_{33}^m = c_{33}^l(1 + k_2^m)$ , where  $c_{33}^l$  is  
169 the pure contribution of the load and  $k_2^m$  is the load Love number pertaining exclusively  
170 to the mantle. According to Nam and Dickman (1990),  $k_2^m$  is related to the load Love  
171 number of the entire Earth  $k_2^l$  through  $k_2^m = \alpha_3 k_2^l$  with  $\alpha_3 = 0.792$  and  $k_2^l = -0.3075$   
172 (Petit & Luzum, 2010). So, (3) becomes

$$m_3 = -\alpha_1 \frac{(1 + \alpha_3 k_2^l) c_{33}^l}{C} - \alpha_1 \frac{h_3}{C\Omega} , \quad (4)$$

173 If the fluid core tends to be inertially coupled with the mantle,  $\alpha_3$  will increase towards  
174 1.

175 This equation puts forward the effective *axial angular momentum function* (AMF)  
176 of the fluid layer

$$\chi_3^l = \alpha_1 \frac{(1 + \alpha_3 k_2^l) c_{33}^l}{C} + \alpha_1 \frac{h_3}{C\Omega} , \quad (5)$$

177 composed of the matter term  $\frac{c_{33}^l}{C}$  and motion term  $\frac{h_3}{C\Omega}$ .

## 178 2.3 Case of a volume potential of degree 2

179 A volume potential, as the one of the zonal tides, produces deformations within the  
180 solid Earth and the core, and in turn a variation of the axial moment of inertia. Such  
181 a potential also produces a sea level variation, in turn an axial inertia moment  $c_{33}^o$ , as  
182 well as currents, that yield a relative angular momentum  $h_3^o$ . Both  $c_{33}^o$  and  $h_3^o$  can be de-  
183 termined from an ocean tidal model, and their effect on LOD can be derived according  
184 to (4).

185 **Assess** that the ocean-less Earth undergoes a zonal tide potential of degree 2, at  
186 the Earth surface taking the form

$$W_2(t, \theta) = W_0(t) P_{20}(\theta) , \text{ equivalently } W_2(t, \theta) = W_0^*(t) Y_2^0(\theta) , \quad (6)$$

187 with the normalised spherical harmonic function

$$Y_2^0 = \sqrt{\frac{5}{4\pi}} P_{20}(\cos \theta) , \quad P_{20}(\cos \theta) = (3 \cos^2 \theta - 1)/2 , \quad (7)$$

188 **assuming** an elastic deformation of the solid Earth and hydrostatic response of the in-  
189 ternal fluid part of the Earth, then the geopotential change can be described through  
190 a unique real body Love number  $k_2$ :  $\Delta U(\theta) = k_2 W_2(t, \theta)$ . This geopotential variation  
191 is accompanied by an axial inertia moment change of the entire Earth, expressed by

$$c_{33}^v = -k_2 \frac{R_e^3}{3G} \sqrt{\frac{5}{\pi}} W_0^*(t) . \quad (8)$$

192 (see demonstration in Subsection 3 in the general case of a complex frequency depen-  
 193 dent response, in particular Eq. 35). In the same vein, the corresponding mantle defor-  
 194 mation is characterised by the Love number  $k_2^m$ , yielding the inertia moment increment  
 195  $c_{33}^{v,m} = -k_2^m R_e^3 / (3G) \sqrt{5/\pi} W_0^*(t)$ . So, we have

$$\frac{c_{33}^{v,m}}{c_{33}^v} = \frac{k_2^m}{k_2} = \alpha_2, \quad (9)$$

196 where the quantity  $\alpha_2$  has been introduced by Dickman (2003). On the other hand,  $c_{33}^{v,m}$   
 197 and  $c_{33}^v$  are related by

$$\frac{c_{33}^{v,m}}{C_m} = \alpha'_2 \frac{c_{33}^v}{C}, \quad (10)$$

198 with  $\alpha'_2 = 0.886$  (Wahr et al., 1981). It results in

$$\alpha_2 = \frac{\alpha'_2}{\alpha_1} = 0.785, \quad c_{33}^{v,m} = \alpha_2 c_{33}^v. \quad (11)$$

199 The resulting rotational change of the extended mantle is governed by an equation  
 200 similar to (3) where  $c_{33}^m$  is replaced by  $c_{33}^{v,m} = \alpha_2 c_{33}^v$ , and the relative angular momen-  
 201 tum drops:

$$m_3 = -\alpha_1 \frac{c_{33}^{v,m}}{C} = -\alpha_1 \alpha_2 \frac{c_{33}^v}{C}. \quad (12)$$

202 As for the surface load excitation, we define the volume excitation

$$\chi_3^v = \alpha_1 \alpha_2 \frac{c_{33}^v}{C}. \quad (13)$$

203 The combined volume (v) and surface load (l) effects, possibly associated with a  
 204 relative angular momentum  $h_3$ , yields the Liouville equation

$$m_3 = -(\chi_3^l + \chi_3^v) = -\alpha_1 \left( \frac{(1 + \alpha_3 k_2^l) c_{33}^l}{C} + \frac{h_3}{C\Omega} \right) - \alpha_1 \alpha_2 \frac{c_{33}^v}{C}. \quad (14)$$

205 Here we have to keep in mind that the coefficient  $\alpha_2$  and  $c_{33}^v$  pertain to tidal mass  
 206 redistribution excluding the oceanic one, that is treated as a loading effect.

#### 207 2.4 Influence of the centrifugal effect

208 A more rigorous approach should account for the **feedback** effect of the rotational  
 209 inertia change  $c_{33}^{(r)}$  resulting from the centrifugal deformation of the Earth when  $m_3$  varies.  
 210 The variation of the centrifugal potential at the Earth surface ( $r = R_e$ ) as a function  
 211 of the colatitude  $\theta$  is

$$W^{(r)}(t, \theta) = W_0^{(r)}(t) (-1 + P_{20}(\cos \theta)) \quad (15)$$

212 with

$$W_0^{(r)}(t) = -\frac{2\Omega^2 R_e^2}{3} m_3(t). \quad (16)$$

213 The term  $W_0^{(r)}$  of degree 0 produces a pure radial deformation. The resultant ef-  
 214 fect on the axial moment of inertia is (Dahlen, 1976)

$$c_{33}^{(r,0)}(t) = n_0 \frac{\Omega^2 R_e^5}{3G} m_3(t), \quad (17)$$

215 with  $n_0 = 0.155$ .

216 At the Earth surface, neglecting dissipation, the term  $W_{20}^{(r)} = W_0^{(r)}(t) P_{20}(\cos \theta)$   
 217 of degree 2 causes the geopotential variation

$$\Delta U^{(r)}(t, \theta) = k_2 W_{20}^{(r)}(t, \theta) = -k_2 \frac{2\Omega^2}{3} R_e^2 m_3(t) P_{20}(\cos \theta), \quad (18)$$

218 where  $k_2$  is the Love number of degree 2, including the hydrostatic deformation of the  
 219 ocean raised by  $m_3$ . For the dominant zonal tide Mf,  $k_2 \approx 0.3350$  according to body  
 220 Love number model reported in Petit and Luzum (2010) and FES 2004 ocean tidal model  
 221 (Table 6 of Williams and Boggs (2016)).

222 So, in the spherical harmonic development of the geopotential, namely

$$U = \frac{GM_{\oplus}}{r} [1 + C_{20}P_{20}(\cos \theta) + \dots] , \quad (19)$$

223 the Stokes coefficient  $C_{20}$  varies by

$$\Delta C_{20}^{(r)}(t) = -k_2 \frac{2\Omega^2 R_e^3}{3GM_{\oplus}} m_3(t) . \quad (20)$$

224 To  $\Delta C_{20}^{(r)}$  **corresponds** a change of the diagonal inertia moments  $c_{ii}^{(r)}$  according to

$$\Delta C_{20}^{(r)} = \frac{1}{M_{\oplus} R_e^2} \left( -c_{33}^{(r)} + \frac{c_{11}^{(r)} + c_{22}^{(r)}}{2} \right) . \quad (21)$$

225 As the trace of the inertia matrix is constant for degree 2 **excitation** potential (Pari-  
 226 iskii & Pertsev, 1973; Rochester & Smylie, 1974; Dahlen, 1976), that is  $c_{33}^{(r)} + c_{11}^{(r)} + c_{22}^{(r)} =$   
 227 0, we have

$$\Delta C_{20}^{(r)} = \frac{1}{M_{\oplus} R_e^2} \left( -c_{33}^{(r)} - \frac{c_{33}^{(r)}}{2} \right) = -\frac{3}{2M_{\oplus} R_e^2} c_{33}^{(r)} . \quad (22)$$

228 From (22) and (20), the centrifugal geopotential variation of degree 2 yields the change  
 229 of axial inertia moment

$$c_{33}^{(r)} = -\frac{2}{3} M_{\oplus} R_e^2 \Delta C_{20}^{(r)} = k_2 \frac{4\Omega^2 R_e^5}{9G} m_3 . \quad (23)$$

230 In comparison to the effect induced by the radial centrifugal deformation given by (17),  
 231 this term is  $4k_2/(3n_0) \sim 3$  times larger.

232 Notice that, introducing the normalised spherical harmonic  $Y_2^0$  given by (7),  $W_{20}^{(r)}(t, \theta)$   
 233 reads

$$W_{20}^{(r)}(t, \theta) = W_0^{(r)}(t)P_{20} = W_0^{*(r)}(t)Y_2^0 , \quad (24)$$

234 and the expression (23) becomes

$$c_{33}^{(r)}(t) = -k_2 \frac{R_e^3}{3G} \sqrt{\frac{5}{\pi}} W_0^{*(r)}(t) . \quad (25)$$

235 Whereas the corresponding inertia moment variation for the extended mantle is given  
 236 through the coefficient  $\alpha_2$  in (11), the radial change  $c_{33}^{(r,0)}$  is associated with **another** co-  
 237 efficient, denoted  $\alpha_2^0$ , since the exciting potential is now of degree 0. In turn, separa-  
 238 ting the centrifugal contribution in the Liouville equation (14) from the other volume ex-  
 239 citation  $\chi_3^v$ , we obtain

$$m_3 = -\chi_3^l - \chi_3^v - \alpha_1 \left( \alpha_2 \frac{c_{33}^{(r)}}{C} + \alpha_2^0 \frac{c_{33}^{(r,0)}}{C} \right) . \quad (26)$$

240 According to (17) and (23), the former equation becomes

$$m_3 \left[ 1 + \alpha_1 \frac{\Omega^2 R_e^5}{3GC} \left( \frac{4}{3} \alpha_2 k_2 + \alpha_2^0 n_0 \right) \right] = -\chi_3^l - \chi_3^v , \quad (27)$$

241 or

$$m_3 (1 + \eta) = -\chi_3^l - \chi_3^v \quad \text{with} \quad \eta = \alpha_1 \frac{\Omega^2 R_e^5}{3GC} \left( \frac{4}{3} \alpha_2 k_2 + \alpha_2^0 n_0 \right) . \quad (28)$$



**Table 1.** Components  $V_\sigma^{20}$  of the zonal tidal potential for which admittance coefficients are determined. Tidal generating model of Hartmann and Wenzel (1995). Corresponding effects on UT1 and LOD according to the IERS 2010 model Petit and Luzum (2010).

	Delaunay arguments					period (days)	$V_\sigma^{20}$ ( $\text{m}^2 \text{s}^{-2}$ )	UT1 (ms)		LOD (ms)	
	$l$	$l_s$	$F$	$D$	$\Omega$			sin	cos	cos	sin
MSq	0	0	2	2	2	7.096	-0.0200	-0.0123	0.0000	0.0109	0.0000
	1	0	2	0	1	9.121	-0.0518	-0.0411	0.0000	0.0283	0.0000
Mtm	1	0	2	0	2	9.133	-0.1250	-0.0993	0.0000	0.0683	0.0000
MSt	-1	0	2	2	2	9.557	-0.0237	-0.0197	0.0000	0.0129	0.0000
	0	0	2	0	0	13.606	-0.0253	-0.0299	0.0000	0.0138	0.0000
	0	0	2	0	1	13.633	-0.2706	-0.3187	0.0201	0.1469	0.0093
Mf	0	0	2	0	2	13.661	-0.6526	-0.7847	0.0532	0.3609	0.0245
	2	0	0	0	0	13.777	-0.0283	-0.0338	0.0000	0.0154	0.0000
MSf	0	0	0	2	0	14.765	-0.0572	-0.0734	0.0000	0.0312	0.0000
	-1	0	2	0	2	27.093	0.0184	0.0435	0.0000	-0.0101	0.0000
	1	0	0	0	-1	27.443	0.0224	0.0534	0.0000	-0.0122	0.0000
Mm	1	0	0	0	0	27.555	-0.3447	-0.8405	0.0250	0.1917	0.0057
	1	0	0	0	1	27.667	0.0226	0.0544	0.0000	-0.0124	0.0000
Msm	-1	0	0	2	0	31.812	-0.0659	-0.1824	0.0000	0.0360	0.0000
Ssa	0	0	2	-2	2	182.621	-0.3031	-4.9717	0.0433	0.1711	0.0015
Sa	0	1	0	0	0	365.259	-0.0489	-1.5889	0.0153	0.0273	0.0003

242 Assuming  $\alpha_2^0 \sim \alpha_2 \approx 0.785$  and  $\alpha_1 = C/C_m$ , we obtain  $\eta \approx 0.00186$ . We see that the  
 243 rotational effect decreases the influence of AMF by about 0.2%, quite negligible in line  
 244 with the uncertainty affecting the reconstruction of  $\chi_3$ .

245 The modelling of atmospheric (A), oceanic (O), and inland water or hydrological  
 246 (H) transports permit to reconstruct the major part of  $\chi_3$  from days to inter-annual time  
 247 scales. Then,  $\chi_3$ , mostly the atmospheric component, well accounts for  $m_3$  fluctuations  
 248 except the harmonic oscillations ruled by zonal tides, in particular at 13.6 and the 27.3  
 249 days. However, the precision of  $\chi_3^{AOH}$  does not reach a sufficient accuracy to derive  $k_2$   
 250 by fitting variations of  $m_3$  to those of  $\chi_3$  from (12), after the zonal tide effect is removed  
 251 from  $m_3$ . Therefore, in determining  $k_2$ , tidally raised terms in LOD are still the most  
 252 pertinent.

### 253 3 Modelling zonal tide effects

#### 254 3.1 For an ocean-less Earth

255 The zonal tide potential is factorised in time part and spatial part according to (see  
 256 e.g. Eq. 5.205 in Dehant and Mathews (2015))

$$W(t, \theta, r) = \Re [W_0^*(t, r)] Y_2^0(\theta) \quad \text{with} \quad W_0^*(t, r) = \frac{r^2}{R_e^2} \sum_{\sigma > 0} V_\sigma^{20} e^{i\theta_\sigma(t)}. \quad (29)$$

257 where  $V_\sigma^{20}$  and  $\theta_\sigma(t)$  are respectively the real coefficients of and phases of the zonal tidal  
 258 potential, as reported in Table (1) for each prominent component between 7 and 365.26  
 259 days. The phase  $\theta_\sigma(t)$  is given by a linear combination of the Delaunay luni-solar argu-  
 260 ments, of which the integer coefficients are given in the same table. The coefficients  $V_\sigma^{20}$

261 provide the tidal heights  $V_\sigma^{20}/g$  in the hydrostatic approximation. So we have

$$W(t, \theta, r) = \frac{r^2}{R_e^2} Y_2^0 \sum_{\sigma>0} V_\sigma^{20} \Re \left[ e^{i\theta_\sigma(t)} \right] \quad (30)$$

262 At the Earth surface  $r \approx R_e$

$$W(t, \theta, r = R_e) = \Re [W_0^*(t, R_e)] Y_2^0 . \quad (31)$$

263 If the mantle was perfectly elastic, the corresponding variation of the gravitational po-  
 264 tential of the entire Earth would be  $\Delta U(t, \theta) = k_2 W(t, \theta, R_e)$  where  $k_2$  would be a real  
 265 body Love number. But the anelastic response of the mantle produce out-of-phase terms  
 266 with respect to the tidal phase  $\theta_\sigma(t)$  and makes  $k_2$  frequency-dependent. Notwithstand-  
 267 ing its complexity, this response can be described through a frequency dependent com-  
 268 plex Love number  $k_\sigma$ , so that, at a given tidal frequency

$$\Delta U = \Re [k_\sigma W_0^{*,\sigma}(t, R_e)] Y_2^0 , \quad (32)$$

269 with

$$W_0^{*,\sigma}(t, R_e) = V_\sigma^{20} e^{i\theta_\sigma(t)} . \quad (33)$$

270 Identifying  $\Delta U_\sigma$  with the term  $(GM_\oplus/R_e) \Delta C_{20}^* Y_2^0$ , we get

$$\Delta C_{20}^* = \frac{R_e}{GM_\oplus} \Re [k_\sigma W_0^*(t, R_e)] . \quad (34)$$

271 Eq. (22) pertains to any **driving** zonal potential of degree 2, so that the zonal tide at  
 272 frequency  $\sigma$  produces the Earth moment inertia change

$$c_{33}^v = -\frac{2}{3} M_\oplus R_e^2 \sqrt{\frac{5}{4\pi}} \Delta C_{20}^* = -\frac{R_e^3}{3G} \sqrt{\frac{5}{\pi}} \Re [k_\sigma W_0^{*,\sigma}(t, R_e)] . \quad (35)$$

273 Equivalently, accounting for (33),  $c_{33}^v$  has the sinusoidal form

$$c_{33}^v = \frac{-R_e^3}{3G} \sqrt{\frac{5}{\pi}} \Re [k_\sigma V_\sigma^{20} e^{i\theta_\sigma(t)}] . \quad (36)$$

274 According to the expression (13), the resulting volume excitation of the LOD is

$$\chi_3^v = -\alpha_1 \alpha_2 \Re [k_\sigma \gamma V_\sigma^{20} e^{i\theta_\sigma(t)}] . \quad (37)$$

275 with the coefficient  $\gamma$  introduced in Nam and Dickman (1990):

$$\gamma = \sqrt{\frac{5}{\pi}} \frac{R_e^3}{3GC} . \quad (38)$$

### 276 3.2 Admittance coefficient

277 Any tidally coherent component of the anomaly  $\Delta LOD$  can be expressed through

$$\frac{\Delta LOD}{LOD_0} = \Re [K_\sigma e^{i\theta_\sigma(t)}] = K_\sigma^c \cos(\theta_\sigma(t)) - K_\sigma^s \sin(\theta_\sigma(t)) , \quad (39)$$

278 with  $K_\sigma = K_\sigma^c + iK_\sigma^s$ . In absence of ocean dynamical effects or anelastic deformations  
 279 of the solid Earth,  $K_\sigma^s$  would be zero, as long as we apply ideal hydro-atmospheric cor-  
 280 rections.

281 So, from (28), we have

$$\Re [K_\sigma e^{i\theta_\sigma(t)}] (1 + \eta) = -\alpha_1 \alpha_2 \Re [k_\sigma \gamma V_\sigma^{20} e^{i\theta_\sigma(t)}] , \quad (40)$$

282 yielding

$$K_\sigma(1 + \eta) = -\alpha_1\alpha_2k_\sigma\gamma V_\sigma^{20} . \quad (41)$$

283 Actually, for a complete theoretical picture, we have to consider the dynamical ocean  
 284 tide raised by tidal potential ( $W_0^{*,\sigma}(t) = V_{20}e^{i\theta_\sigma t}$ ). It results in an inertia moment change  
 285  $c_{33}^o$  and a relative angular momentum  $h_3^o$ . They can be described as terms proportional  
 286 to  $W_0^{*,\sigma}$  through two supplementary coefficients determined from an ocean tidal model.  
 287 Then,  $c_{33}^o$  and  $h_3^o$  are treated as a surface excitation in the Liouville equation (14). In  
 288 turn, the actual expression of  $K_\sigma$  with respect to  $V_\sigma^{20}$  is more complicated, depending  
 289 of the ocean tidal coefficients and the hypotheses pertaining to the Earth rheology through  
 290 coefficients  $\alpha_1, \alpha_2, \alpha_3$  (see Eq. 41). Thus, the body Love number  $k_\sigma$  cannot be deter-  
 291 mined directly, and one favours the determination of the pure empirical coefficient  $\kappa_\sigma$   
 292 defined as the ratio of the relative angular velocity change  $m_3^\sigma = -K_\sigma$  to the tidal po-  
 293 tential  $\gamma V_{20}^\sigma$ :

$$K_\sigma = -\kappa_\sigma\gamma V_\sigma^{20} . \quad (42)$$

294 By analogy to the ratio of a pulsed electric current to the generating electric potential,  
 295  $\kappa_\sigma$  is called *admittance*, and was first coined by Agnew and Farrell (1978) (p. 177 with  
 296  $V_0 = -V_\sigma^{20}$ , see also Eq. 1 of Chao et al. (1995), Eq. 9 of Defraigne and Smits (1999),  
 297 and Ray and Erofeeva (2014) p. 1499).

#### 298 4 Hydro-atmospheric correction and estimation of the admittance

299 At annual and semi-annual periods, the atmospheric forcing, given through the at-  
 300 mospheric angular momentum function (AAM), dominates the tidal effects by one or-  
 301 der of magnitude (500  $\mu$ s induced AAM effect against 30  $\mu$ s for the Sa tidal term, 20  $\mu$ s  
 302 against 17  $\mu$ s for the Ssa tidal term). For the other dominant zonal tides, below 32 days,  
 303 the situation reverses: the atmospheric contribution appears as a continuous spectrum,  
 304 showing power defect with respect to the LOD spectrum at the zonal tidal frequencies.  
 305 For instance, for the fortnightly tide, the tidal effect is 360  $\mu$ s, against 5  $\mu$ s for the at-  
 306 mospheric effect and to a much lesser extent for the oceans. Notice, that, in contrast to  
 307 the oceanic angular momentum (OAM), the AAM also includes tidally induced varia-  
 308 tions, for the pressure and the winds assimilated in the atmospheric circulation models  
 309 mix both the pure thermodynamic transports induced by the solar heating and tidal ef-  
 310 fects.

311 The period chosen for the estimation of the largest tidal terms listed in Table 1 is  
 312 a trade-off between precision of LOD data, availability of hydro-atmospheric angular mo-  
 313 mentum series, and good decorrelation between close frequency components of the zonal  
 314 tide, whose minimum separation is 1/18.6 cycle/year. So, we select the period 2000–2019.  
 315 Then, considering the IERS C04 combined solution, the adjustment of the largest tidal  
 316 terms ( $K_\sigma^c, K_\sigma^s$ ) in  $\Delta LOD$  can be determined with a formal error in the range 2–6  $\mu$ s  
 317 (see Table 1 for the tidal arguments). That uncertainty corresponds to the spectral dis-  
 318 persion - at periods of the dominant zonal tides (27.5 d, 13.6 d, 9.1 d) - with other LOD  
 319 series, well representing the state of the art:

- 320 • SPACE combined solution produced at JPL from GNSS (Global Navigation Satel-  
 321 lite System), VLBI (Very Long Baseline Interferometry), SLR (Satellite Laser Rang-  
 322 ing), and LLR (Lunar Laser Ranging) observations (Ratcliff & Gross, 2015),
- 323 • Intra-technique GNSS combined solution of the International GNSS Service (IGS),
- 324 • CODE GNSS solution of the Bern University,
- 325 • Intra-technique VLBI combined solution of the International VLBI Service (IVS)

326 As shown hereafter in Table 3, the results obtained with the combined series of the In-  
 327 ternational Laser Ranging Service (ILRS) combined series are at least two less precise.  
 328 All considered LOD series can be downloaded or compared from the Earth orientation  
 329 WEB site <https://eoc.obspm.fr/index.php?index=operational&lang=en>.

330 So, at such a precision level, the hydro-atmospheric excitation intervenes at zonal  
 331 tide periods, and for isolating accurately the tidal contribution, the LOD should be free  
 332 from the surface layer forcing (except the tidal ocean circulation). To our knowledge, the  
 333 atmospheric correction has been applied for extracting zonal tides in LOD in many stud-  
 334 ies (Merriam, 1984; Defraigne & Smits, 1999; Chao et al., 1995), but the smaller oceanic  
 335 correction has only be considered for the Mf component by Ray and Egbert (2012), and  
 336 hydrological correction have never been applied.

337 Two groups of hydro-atmospheric corrections, resulting from coupled models are  
 338 tested:

- 339 • NCEP AAM paired to ECCO-MIT OAM, that is forced through the NCEP model  
 340 (available from the WEB site of the IERS Global Geophysical Fluid Center <http://loading.u-strasbg.fr/GGFC>);
- 341 • ECMWF AAM paired to the OAM from the Max Plank Institute Ocean model  
 342 (MPIOM), as derived by GFZ; to the former ECMWF+MPIOM excitation we add  
 343 the Hydrological Angular Momentum (HAM) and the Sea Level angular Momen-  
 344 tum (SLM) , as calculated by GFZ from the Land Surface Discharge Model (LSDM)  
 345 (downloadable from <http://rz-vm115.gfz-potsdam.de:8080/repository>).

347 We consider 7 cases:

- 348 1) LOD series without corrections
- 349 2) LOD with AAM-NCEP corrections
- 350 3) LOD with AAM-NCEP and OAM-ECCO corrections
- 351 4) LOD with AAM-ECMWF corrections
- 352 5) LOD with AAM-ECMWF and OAM-MPIOM corrections
- 353 6) LOD with AAM-ECMWF, OAM-MPIOM and HAM-LSDM corrections
- 354 7) LOD with AAM-ECMWF, OAM-MPIOM, HAM-LSDM, and SLM-LSDM cor-  
 355 rections

356 First, considering the set composed of the 15 dominant zonal tides reported in Ta-  
 357 ble 1 (tidal height  $V_{20}/g$  above 1.8 mm, effect on the LOD larger than 10  $\mu$ s), except the  
 358 annual term, the  $K_\sigma$  coefficients are fitted in LOD or corrected LOD by least-squares  
 359 method according to the observation relation (39). Then, the corresponding admittance  
 360 coefficients are computed by applying (42). This derivation is based upon the coefficients  
 361  $V_\sigma^{20}$  corresponding to the Hartmann-Wenzel tide generating model (Hartmann & Wen-  
 362 zel, 1995), available in the ETERNA package provided by the International Geodynam-  
 363 ics and Earth Tide Service ([http://igets.u-strasbg.fr/soft\\_and\\_tool.php](http://igets.u-strasbg.fr/soft_and_tool.php)).

364 This calculation is achieved not only for the 7 **kinds** of corrections listed above but  
 365 also for each of the 6 LOD series that we mentioned, thus providing a total of  $6 \times 7 =$   
 366 42 estimated sets of admittance.

367 In Table 2 we report the estimated coefficients in the case of AAM-ECMWF and  
 368 OAM-MPIOM corrections (case 5) applied to SPACE LOD series; it provides the best  
 369 fit with respect to the theoretical models of Ray and Erofeeva (2014), as shown in the  
 370 next section.

## 371 5 Analysis of the results

372 In contrast to the constant contribution of an elastic mantle ( $\kappa_e = 0.2639$ ), ocean  
 373 dynamics and anelasticity of the mantle cause a frequency dependent response. The cor-  
 374 responding theoretical modelling, that had been initiated in the 1980's, was improved  
 375 7 years ago by Ray and Erofeeva (2014), that calculated a set of admittance coefficients

**Table 2.** Estimated admittances for the LOD (SPACE series) corrected from atmospheric (ECMWF), oceanic (MPIOM) excitations. Comparison with the values of the Ray and Erofeeva model.

	period (days)	$\kappa_{re}$		$\kappa_{im}$			
		estimates	model	estimates	model		
MSq	7.096	0.2968	$\pm 0.0589$	0.3005	-0.0137	$\pm 0.0589$	-0.0238
x	9.121	0.3051	$\pm 0.0227$	0.3060	-0.0230	$\pm 0.0227$	-0.0232
Mtm	9.133	0.3073	$\pm 0.0094$	0.3060	-0.0220	$\pm 0.0094$	-0.0231
MSt	9.557	0.2888	$\pm 0.0495$	0.3070	0.0043	$\pm 0.0496$	-0.0229
x	13.606	0.3182	$\pm 0.0465$	0.3136	-0.0170	$\pm 0.0465$	-0.0199
x	13.633	0.3118	$\pm 0.0043$	0.3136	-0.0208	$\pm 0.0043$	-0.0199
Mf	13.661	0.3116	$\pm 0.0018$	0.3137	-0.0193	$\pm 0.0018$	-0.0199
x	13.777	0.3121	$\pm 0.0417$	0.3138	-0.0067	$\pm 0.0417$	-0.0198
MSf	14.765	0.3146	$\pm 0.0206$	0.3148	-0.0284	$\pm 0.0206$	-0.0190
x	27.093	0.3147	$\pm 0.0640$	0.3205	-0.0107	$\pm 0.0640$	-0.0129
x	27.443	0.3138	$\pm 0.0527$	0.3206	-0.0080	$\pm 0.0527$	-0.0128
Mm	27.555	0.3177	$\pm 0.0034$	0.3206	-0.0144	$\pm 0.0034$	-0.0128
x	27.667	0.3335	$\pm 0.0520$	0.3207	-0.0228	$\pm 0.0520$	-0.0128
Msm	31.812	0.3221	$\pm 0.0178$	0.3214	0.0006	$\pm 0.0178$	-0.0116
Ssa	182.621	0.3331	$\pm 0.0039$	0.3261	-0.0140	$\pm 0.0039$	-0.0047
Sa	365.259	-1.1525	$\pm 0.0241$	0.3274	1.0632	$\pm 0.0240$	-0.0042

for all zonal tidal components with periods stretching from 4.6 days to 18.6 years. With growing frequency the ocean response becomes dynamical, and this strongly affect the admittance value. From Ray and Erofeeva (2014), the contribution of the ocean to  $\kappa_r$  decreases from 0.050 at 182.6 d (Ssa) down to 0.025 at 7.1 d (Msq), whereas, for the same frequency interval,  $k_i$  varies from  $-0.002$  to  $-0.021$  meaning a frequency growing phase lag of the oceanic tide, as shown in Table 2 and Figure 3 of Ray and Erofeeva (2014). The anelasticity of the mantle induces variation **to a lesser extent**: from 1/182 cpd to 1/7.1 cpd, the real part decreases of about 0.005, and the imaginary part increases by about 0.007 (Ray & Erofeeva, 2014). In the end, those coefficients better account for the LOD oscillation at the zonal tidal frequencies.

So, for each selected EOP series, we are inclined to compare the estimated values  $\kappa_{re}(\sigma)$  and  $\kappa_{im}(\sigma)$  with the Ray and Erofeeva (2014) values shown in Table 2. In this respect, for each of the 42 estimated set, we calculated the weighted mean value and the weighted standard deviation of the differences with respect to the modelled values. The weight of a tidal component is given by  $1/E_\kappa^2$ , where  $E_\kappa$  is the formal error of  $\kappa_{re}$  or  $\kappa_{im}$ . These statistics are limited to the 14 admittances between 7 and 32 days. Indeed the Ssa and Sa components are excluded, as the corresponding hydro-atmospheric correction is not reliable **enough** in the seasonal band, and introduces a too large uncertainty. As shown in Table 2, the value of the admittance obtained for Sa is clearly erroneous.

It appears that SPACE series yield the smallest dispersions. The corresponding estimates according to the 7 correction options are represented in Figure 1. **Analogous** plots are done for the imaginary part  $\kappa_{im}$  in Figure 2. As shown more than two decades ago (Merriam, 1984; McCarthy & Luzum, 1993; Chao et al., 1995; Defraigne & Smits, 1999), the error bars and the dispersion are strikingly squeezed by considering the atmospheric correction (case 2/4). The improvement resulting from the additional ocean

**Table 3.** Weighted mean and weighted standard deviations of the offsets between the 14 estimated admittances ( $\kappa_{re}$ ,  $\kappa_{im}$ ) in the range 7-32 days and the modelled values ( $\kappa_{re}^{mod}$ ,  $\kappa_{im}^{mod}$ ) of Ray and Erofeeva (2014).

series	correction	$\kappa_{re} - \kappa_{re}^{mod}$		$\kappa_{im} - \kappa_{im}^{mod}$	
		mean	std	mean	std
C04	no	-0.0070	0.0101	-0.0010	0.0089
	NCEP	-0.0040	0.0044	-0.0020	0.0035
	NCEP+ECCO	-0.0044	0.0035	-0.0015	0.0030
	ECMWF	-0.0045	0.0037	-0.0021	0.0030
	ECMWF+MPIOM	-0.0049	0.0016	-0.0017	0.0017
	ECMWF+MPIOM+LSDM	-0.0050	0.0015	-0.0017	0.0018
	ECMWF+MPIOM+SLSDM	-0.0049	0.0017	-0.0017	0.0015
CODE	no	-0.0089	0.0104	0.0018	0.0098
	NCEP	-0.0060	0.0044	0.0008	0.0035
	NCEP+ECCO	-0.0064	0.0033	0.0012	0.0030
	ECMWF	-0.0065	0.0045	0.0006	0.0038
	ECMWF+MPIOM	-0.0069	0.0031	0.0011	0.0030
	ECMWF+MPIOM+LSDM	-0.0069	0.0030	0.0011	0.0031
	ECMWF+MPIOM+SLSDM	-0.0068	0.0029	0.0011	0.0029
IGS	no	-0.0072	0.0099	0.0020	0.0089
	NCEP	-0.0042	0.0043	0.0011	0.0039
	NCEP+ECCO	-0.0046	0.0031	0.0014	0.0042
	ECMWF	-0.0047	0.0039	0.0010	0.0034
	ECMWF+MPIOM	-0.0051	0.0014	0.0014	0.0029
	ECMWF+MPIOM+LSDM	-0.0052	0.0013	0.0013	0.0029
	ECMWF+MPIOM+SLSDM	-0.0051	0.0016	0.0014	0.0028
IVS	no	-0.0041	0.0093	0.0011	0.0081
	NCEP	-0.0011	0.0048	0.0001	0.0041
	NCEP+ECCO	-0.0015	0.0032	0.0005	0.0045
	ECMWF	-0.0016	0.0045	-0.0001	0.0041
	ECMWF+MPIOM	-0.0020	0.0023	0.0004	0.0040
	ECMWF+MPIOM+LSDM	-0.0021	0.0022	0.0004	0.0040
	ECMWF+MPIOM+SLSDM	-0.0020	0.0024	0.0004	0.0041
SPACE	no	-0.0042	0.0096	0.0009	0.0085
	NCEP	-0.0013	0.0041	-0.0001	0.0023
	NCEP+ECCO	-0.0016	0.0027	0.0003	0.0024
	ECMWF	-0.0017	0.0037	-0.0003	0.0019
	ECMWF+MPIOM	-0.0021	0.0010	0.0001	0.0018
	ECMWF+MPIOM+LSDM	-0.0022	0.0009	0.0001	0.0019
	ECMWF+MPIOM+SLSDM	-0.0021	0.0010	0.0001	0.0018
ILRS	no	-0.0129	0.0163	-0.0053	0.0181
	NCEP	-0.0100	0.0118	-0.0063	0.0159
	NCEP+ECCO	-0.0103	0.0110	-0.0059	0.0166
	ECMWF	-0.0104	0.0116	-0.0064	0.0154
	ECMWF+MPIOM	-0.0108	0.0104	-0.0060	0.0166
	ECMWF+MPIOM+LSDM	-0.0110	0.0104	-0.0060	0.0166
	ECMWF+MPIOM+SLSDM	-0.0108	0.0104	-0.0060	0.0166

**Figure 1.** Real part  $\kappa_{re}$  of the admittance coefficient determined from the 15 largest zonal tidal components in LOD (SPACE series). Red points: rough LOD. Green points: LOD minus atmospheric excitation. Blue points: LOD minus atmospheric and oceanic or LOD - atmospheric and oceanic and continental waters. Two set of coupled hydro-atmospheric models are considered: ECMWF-MPIOM-LSDM and NCEP-ECCO. We also indicate the corresponding tidal heights [in cm] for the largest tidal terms symbolised by letters.

**Figure 2.** Same as Fig. 1 but for the imaginary part  $\kappa_{im}$  of the admittance coefficient.

**Table 4.** Coefficients used in various studies for computing effective axial angular momentum of a surface fluid layer.

	$\alpha_3$	$\alpha_{ma}$	$\alpha_{mo} = \alpha_1$
(Eubanks, 1993)		0.849	0.126
(Dobslaw & Dill, 2018)		0.843	0.125
This study		0.854	0.129

401 correction can be noticed for the real part at many tidal periods, where oceans bring the  
 402 estimated value towards the mean (13.78 d, MSf, 27.44 d, 27.68 d, MSm , Ssa) despite  
 403 a large uncertainty for some waves. This reduction of the dispersion is more evident in  
 404 the statistics of Table 3 (excluding Ssa) whatsoever the EOP series.

405 For  $\kappa_{re}(\sigma)$ , it appears that adding oceanic corrections (case 3 and 5) divides the  
 406 standard deviation with respect to the theoretical values by about a factor 1.5 (ECCO)  
 407 or 3.7 (MPIOM). A visual proof that the oceanic correction allows better evidence the  
 408 frequency dependence of  $\kappa_{re}$  coefficient is given in Figure 3 where we display together  
 409 modelled and estimated values in the case of atmospheric (case 4), atmospheric+oceanic  
 410 (case 5), and total hydro-atmospheric correction (case 7).

411 For  $\kappa_{im}(\sigma)$ , the oceanic correction decreases the dispersion with respect to the mod-  
 412 elled values by 5% and only in the case of ECMWF and MPIOM corrections.

413 If the land water correction does not reduce the standard deviation of the offsets  
 414 in the band 7-32 days, it influences the Ssa component. Addressing the SPACE series,  
 415  $\kappa_{Ssa} = 0.333(4) - i 0.014(4)$  for the ECMWF+MPIOM correction becomes  $\kappa_{Ssa} =$   
 416  $0.307(4) - i 0.017(4)$  for ECMWF+MPIOM+LSDM (see Table 2), and finally  $\kappa_{Ssa} =$   
 417  $0.331(4) - i 0.019(4)$  by adding the additional sea level correction, which best matches  
 418 the real part given by the Ray and Erofeeva model (0.326).

419 The cases 4) and 5) associated with ECMWF and ECMWF+MPIOM respectively  
 420 give smaller dispersions than NCEP and NCEP+ECCO (cases 2 and 3).

421 In the case of Mf, our estimates corresponding to ECMWF+MPIOM correction,  
 422 namely  $(0.3116 \pm 0.0018, -0.0193 \pm 0.0018)$ , are consistent with the values obtained by  
 423 Ray and Egbert (2012) over the period 1985-2009 of SPACE series after having applied  
 424 the correction derived from the ECMWF and OMCT angular momentum functions (OMCT  
 425 is the model that has preceded MPIOM), namely  $(0.3125 \pm 0.0005, -0.0193 \pm 0.0005)$ .  
 426 However, our formal error is 4 times larger than the one reported by those authors, and  
 427 the reason for this has not been elucidated.

## 6 Discussion and conclusions

We showed that removing the combined oceanic-atmospheric correction on LOD allows to better constraint the admittance coefficients  $\kappa_\sigma$  for the 14 largest zonal tides below 32 days than applying the atmospheric correction only.

The role of land water and associated sea level variation is noticeable at the semi-annual period.

A **statistical study confirms** the frequency dependence of the admittance coefficient, as determined by Ray and Erofeeva (2014): as the period grows,  $\kappa_{re}$  increases, as expected from growing anelastic behaviour of the mantle, whereas the imaginary coefficient  $\kappa_{im}$  decreases, as the dynamical response of the ocean fades.

Though, the weighted averages of the offsets  $\kappa_{re} - \kappa_{re}^{mod}$  (over the 14 components between 7 and 32 days) present negative systematic biases.

First we notice that without hydro-atmospheric corrections, the smallest biases are obtained for LOD values derived from VLBI observations alone (case of IVS) and from the combined GNSS-SLR-LLR-VLBI processing (SPACE), allowing to increase the temporal resolution of the LOD while insuring its consistency with UT1 (Ratcliff & Gross, 2015) ( $\bar{\kappa}_{re} \sim -0.004$ ,  $\bar{\kappa}_{im} \sim 0.001$ ). For series associated with satellite techniques - IGS, ILRS, and combined C04 based on IGS values (Bizouard et al., 2018), the biases are about two times larger, probably in link to the contamination of LOD by orbital artifacts at tidal periods.

Whatever the LOD series, the fluid correction reduces significantly the biases, by about 0.002 for  $\kappa_{re}$  and 0.001 for  $\kappa_{im}$ . In the case pertaining to SPACE, where the dispersion are the smallest, the bias on  $\kappa_{re}$  value, of about  $-0.002$  (ECMWF+MPIOM correction), is two times larger than the standard deviation with respect to the model. Representing also 1% of  $\kappa_{re}$  value, it has the order of magnitude of the anelastic frequency variability between 7 and 32 days. For  $\kappa_{im}$ , the corresponding bias of about 0.0001 is much less significant, and represents about 0.5% of  $\kappa_{im}$  or 1% of the modelled variability of  $\kappa_{im}$  between Msq (7.1 days) and MSm (31.8 days) periods.

Such systematic effects are too important to originate from the values of  $C$ ,  $G$ ,  $R_e$ ,  $V_{20}$  determining  $\kappa$  through (41), of which the relative uncertainties are smaller than 0.01%.

Thus, we tested the hypothesis according to which those biases could be explained by a partial core-mantle coupling, that is a lower value of  $\alpha_1$  determining the coefficients  $\alpha_{ma} = \alpha_1(1 + k'_2\alpha_3)$  and  $\alpha_{mo} = \alpha_1$  of the hydro-atmospheric corrections (14). As indicated in Table 5,  $\alpha_{ma}$  and  $\alpha_{mo}$  present relative differences up to 4% for three studies, including ours, and this order of magnitude can be proposed for the uncertainty pertaining to  $\alpha_1$ . For SPACE series, decreasing  $\alpha_1 = 1.129$  by 5% of the adopted value, that is diminishing the fluid layer correction, increases the absolute value of the biases by the same relative amount. This hypothesis is therefore not **sound**. So, we could suppose in contrast that the values of the hydro-atmospheric angular momentum ( $\Omega_{c33}$ ,  $h_3$ ) are systematically underestimated, as proposed by Chao and Yan (2010) and should be increased by a 10% factor. But, that change decreases the absolute value of the real bias by 0.0002 only.

Thus, we can rule out the hydro-atmospheric correction, and incriminate the Ray and Erofeeva (2014) model as the main cause of that systematic effect, as far as SPACE/IVS series give accurate LOD values. In this respect, we can wonder whether the spherical Earth approximation in the theoretical model matters, but the answer to this question deserves complex developments out of the scope of the present study.

Finally, despite the pitfall encountered, we can conclude that the search for admittance coefficients advances the Earth rheological models, global hydro-atmospheric cir-



**Figure 3.** Estimates of the admittance coefficients  $\kappa_{re} + i\kappa_{im}$  corresponding to LOD SPACE series and to ECMWF, ECMWF+MPIOM and ECMWF+MPIOM+SLSDM corrections. Comparison to the modelled value of Ray and Erofeeva (2014). The error bars, shown in the case of ECMWF+MPIOM corrections, are similar for ECMWF+MPIOM+SLSDM.

477 culation, and astrogeodetic determination of the length of day change at microsecond  
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491 The time series of the length of day changes are those gathered by the Earth Ori-  
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495 (ECMWF / MPIOM / LSDM); corresponding url links are given in Section 4.

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