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► **To cite this version:**

Athanase Papadopoulos. Yuri Ivanovich Manin (1937-2023): Mathematics, philosophy, poetry. 2023.
hal-04101235v2

HAL Id: hal-04101235

<https://cnrs.hal.science/hal-04101235v2>

Preprint submitted on 20 Sep 2023

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**YURI IVANOVICH MANIN (1937-2023):
MATHEMATICS, PHILOSOPHY, POETRY**

ATHANASE PAPADOPOULOS

ABSTRACT. This is a personal view of and commentary on Yuri Manin's relationship with mathematics, philosophy, literature, poetry, art, and culture. His life epitomized those of the great natural philosophers of yore. The paper will appear as a chapter in the *Handbook of the History and Philosophy of Mathematical Practice*, edited by Bharath Sriraman (Springer, 2023).

KEYWORDS. Keywords: Yuri Manin, biography, linguistics, language, philosophy of mathematics, proof, history of computer science, evolution of mathematical ideas, Andrei Tarkovsky, cosmism, mathematics and poetry.

AMS CODES. AMS codes: 01A70, 91F20, 97M80, 68-03

1. INTRODUCTION

Yuri Manin passed away a few months ago. Such a loss, even if in principle is to be expected, always comes as a shock, and in Manin's case, it is a collective shock. He was an important editorial advisor of the present Handbook. Bharath Sriraman, the chief editor, asked me to write an article on Manin, and at the same time, he told me that it was Manin who had recommended me as co-editor. I am happy to render him this small tribute. It goes without saying that I would have preferred not to have to write it, and that, instead, Manin would have been able to see this Handbook.

I will not make the praise of Manin as a mathematician, because, first, it is well known that he was a first-order mathematician whose works on algebraic geometry, number theory, mathematical physics, complexity theory, computing, and probably other fields, have had a crucial impact on the development of all these domains, and secondly, and most of all, it is difficult for me not to follow Jules Tannery's aphorism: "Il est des hommes auxquels on ne doit pas adresser d'éloges, si l'on ne suppose pas qu'ils ont le goût assez peu délicat pour goûter les louanges qui viennent d'en bas." (There are men whom we should not praise, unless we assume they have the rather indelicate taste for praises that come from below) [24, p. 102]. Instead, I would like to talk about some aspects of Manin's thinking and life that are less known to those who have only heard of his purely mathematical work.

Manin was thoroughly involved, besides mathematics, in philosophy, linguistics, psychology, sociology, history, literature and poetry. In fact, mathematics, for him, was part of the vast field we call "culture", in which his thought was evolving in several directions at the same time. Even in his mathematical writings, he often addresses social, cultural and philosophical

issues. This chapter will be a quick review of some of Manin’s relationships with all these domains, relationships that he lived on a daily basis. The word mathematics will only appear because it is part of the expression “philosophy of mathematics”.

2. TARKOVSKY

I will begin with a personal recollection: a conversation I had with Manin on July 21, 1994.¹ I was visiting the Max-Planck-Institut in Bonn for a few months. Manin had already joined this institute as a permanent professor,² and it was the first time I talked with him. A few days before, I was in Paris, and I had purchased there a book by Andrei Tarkovsky, his *Diary* of the years 1970-1986, in French translation, published by the *Cahiers du Cinéma* [25]. On that day of July 21, I had just finished reading it. Tarkovsky recounts in his diary his everyday life in Moscow, his troubles with censorship,³ his exile in Italy, his illness and the last days of his life, in Paris. One may also read there, among many other things, Tarkovsky’s reflections on art and literature. Figures like Bergman, Bresson, Dostoevsky, Bach, Pushkin, Shakespeare, Solzhenitsyn, Rostropovich, etc. are present throughout the book. In the first pages, Tarkovsky mentions Manin.

On that day, at the 4 o’clock tea of the MPI, I met Manin. I asked him: “Do you know that Tarkovsky talks about you in his book?” He responded with a question: “Which Tarkovsky?” For a fraction of a second, I was puzzled by his question, but then I realized that Manin, like all educated Russians, knows that there are two Tarkovskys: Andrei, the filmmaker, and his father, the poet Arseni Tarkovsky, whose poems resonate throughout the film *The mirror*. I told him “the filmmaker”, with a hand gesture, that of someone who is shooting a film. He was not aware of this. I had the book in my office. I fetched it and I showed him the spot. He was visibly moved. He leafed through the book, photocopied a few pages and asked me to buy one copy for him when I go back to France. I told him I would.

On the next day, I saw Manin anew, at tea time, and we talked again about Andrei Tarkovsky. He told me that he had met him twice: the first time, at Moscow University, where the latter had come to present his film *Andrei Rublev*, and the second time, at the home of Otar Iosseliani, who (I learned this from Manin) had studied mathematics before starting his career as a filmmaker. We also talked about Krzysztof Zanussi, who (I also learned this from Manin) prior to embarking on a career in cinema, studied physics. I told him that Tarkovsky mentions Zanussi in his memoirs, towards the end, when the latter lent him his apartment in Paris.

Here is what Tarkovsky writes in his diary on September 1, 1970:⁴

¹I can tell the exact date because I keep a diary.

²In fact, Manin was hired as one of the directors. The Max Plank Institutes have no “permanent professors” but only directors. (I thank J.-P. Bourguignon for this clarification.)

³From Tarkovsky’s diary, one gets the impression that these troubles arose not because of an opposition to the authorities, but because of the spiritual themes that are present in his movies.

⁴For all these quotes from Tarkovsky’s book, I am using the English translation which appeared in 1994 [25].

Going through old papers I came across the transcript of a university debate on Rublev. God, what a level. Abysmal, pathetic. But there is one remarkable contribution by a maths professor called Manin, Lenin Prize winner,⁵ who can hardly be more than thirty. I share his views. Not that one should say that about oneself. But it's exactly what I felt when I was making Andrei. And I'm grateful to Manin for that.

Here are now, according to the same diary, an excerpt of what Manin said on that occasion:

“Almost every speaker has asked why they have to be made to suffer all through the three hours of the film. I'll try to reply to that question.

It is because the twentieth century has seen the rise of a kind of emotional inflation. When we read in a newspaper that two million people have been butchered in Indonesia, it makes as much impression on us as an account of our hockey team winning a match. The same degree of impression! We fail to notice the monstrous discrepancy between these two events. The channels of our perception have been smoothed out to the point where we are no longer aware. However, I don't want to preach about this. It may be that without it life would be impossible. Only the point is that there are some artists who do make us feel the true measure of things. It is a burden which they carry throughout their lives, and we must be thankful to them.”

Tarkovsky comments: “It was worth sitting through two hours of rubbish for the sake of that last sentence.”

A few days later, I talked again with Manin about Tarkovsky's memoirs, and this time I wanted to ask him about Djouna, an astrologer-medium-paratherapist who practiced in Moscow, of whom Tarkovsky speaks all throughout his diary. She used to treat cancer and all other kinds of diseases by paramagnetism. Tarkovsky, who speaks of her as “an uneducated and capricious woman, a mountain woman”, was being treated by her, and he used to advise his friends to do the same. At some point, she became ill, and Tarkovsky cared for her, using her own methods. Incidentally, when Tarkovsky arrived in Italy, he met another healer—“of Djouna's type,” he writes—who predicted his future.

I was puzzled by all this and I asked Manin if he had heard of Djouna. He responded: “Of course! everyone in Moscow has heard of her.” I told him that I was surprised by the fact that Tarkovsky was so close to her, and he said: “If you had known Moscow in the 70s and 80s, you would not be surprised.” At that moment, I thought of Mikhail Bulgakov's *Master and Margarita*, with its supernatural, non-conformist, spiritistic side, where you meet witches, black magic, etc. All the Russian mathematicians I know have read it.⁶ In fact, I understood later that I was wrong, that paramagnetism

⁵[My note] The Lenin prize was one of the most prestigious prizes (maybe the most prestigious one) of the Soviet Union. Manin received it in 1967. Among the other recipients, let me mention Andrei Sakharov, Sergei Prokofiev, Dmitry Shostakovich, Sviatoslav Richter, and, among the mathematicians, Vladimir Arnol'd and Andrei Kolmogorov.

⁶I learned later on that Bulgakov was a medical doctor, just as Chekhov was. Incidentally, Bulgakov wrote a book titled *A country doctor's notebook*, in a very classical

and hypnosis have nothing to do with magic, that they are part of science, of medicine.⁷

Manin talked, at several occasions, about Moscow in the 1960s-1980s, which hosted a cultural community consisting of small or larger circles, including mathematicians, artists and poets, a community one could find in Moscow and Leningrad and probably nowhere else in the world in terms of vigor and enthusiasm. In the speech he made at his reception ceremony as a foreign associate at the Paris Academy of Sciences [13],⁸ he recalls:

Moscow of the 1960s-1980s, where I spent the most creative time of my life, was also boiling with humanities studies and quests for meaning of human culture, history, and psychology. I was happily participating in seminars and conferences organized by my friends linguists and philologists, delivered talks and published amateurish papers on subjects ranging from the trickster figure in mythology to the epistemology of Lévi-Strauss.

Manin talks here about the period before he left to the West. From what I know, this form of culture is still bubbling today, in Moscow and Saint Petersburg, in the form he describes. As a matter of fact, Manin was among the Russian mathematicians living in the West who returned regularly to their home country.

Several years later, I thought about my conversation with him about Tarkovsky and magnetism, especially after I watched Tarkovsky's *Mirror*, a film that opens with a scene where a young stuttering child recovers his speech thanks to a therapy that mixes magnetism and hypnosis. I thought about Manin because I knew that language and linguistics were among his favorite research subjects all throughout his life, and that in his theory, hypnosis plays a fundamental role. For example, in his essay *The early development of speech and consciousness (Phylogeny)*, reprinted in [18], he discusses works of several scientists on hypnosis and he describes a number of hypnosis experiments in which the researcher makes, as in Tarkovsky's *Mirror*, verbal suggestions and studies their effects on the patient's behavior. In the same essay, Manin reports on instances of auditory hallucinations, discussing the part played by consciousness in the behavior of an individual. He recalls that "hypnosis became possible only after development of speech" (p. 176 of the English version of [18]). In the same essay, he writes:

For many years I led a seminar in Moscow on psycholinguistics and evolution of mind and consciousness. Among its participants and contributors were linguists, ethnologists, neurobiologists, psychologists and psychiatrists. All of us had different backgrounds and different interests, and we tried to find common viewpoints and problems that might possibly be clarified by throwing together our diverging experiences.

style (unlike *The master and Margarita*), which he thought of as a parallel to Chekhov's medical tales.

⁷I also know that the oncologist Léon Schwartzberg, who was treating Manin and who, together with his partner Marina Vlady had become close to him during the last moments of his life, declared once that he could not do his treatment on Tarkovsky in an efficient manner because the latter refused to take the classical drugs.

⁸The passage is also included in the introduction of [18].

I will say more about Manin's ideas on language and linguistics below.

Almost three decades after my first conversation with him, we talked again about Tarkovsky. It was during covid period. I am extracting from an email Yuri sent me on March 20, 2020, with the subject "Tarkovsky":

[...] We have also stopped listening or reading news, although Xenia once a day browses some *Zeitung* in the Web. I am trying to be quiet and keep my feelings. My ultimate joke about all of it: after all, we have read Gogol, Kafka, etc. [...] Let's hope that this crazy time will soon pass. Yesterday my son from Palo Alto sent me an email with subject "Let us watch Tarkovsky", with the address [...]

Manin sent me the address of a website where one can watch free of charge all of Tarkovsky's movies and which I am happy to use until now. I then reminded him of our 1994 conversation about Tarkovsky's diary. In his answer, he gave me a few more details about his intervention in the debate that took place at the university, after the film projection:

It was the first ever public screening of *Rublev*; it was at the Moscow University Club (a former Orthodox Church, now again). I was then a young math professor at the university, and I was invited to attend it. After the screening, several persons from the public (I do not know who, probably, a party committee required it from them) delivered very critical speeches. I was very angry, ran to the podium and defended Tarkovsky and his creativity. I have long forgotten exactly what I said, but Andrei kept a rendering of my words in his diary.

3. MATHEMATICS AND LANGUAGE

In his Kyoto ICM talk (1990) [11], Manin presents mathematics as "a specialized dialect of natural language, and its functioning as a special case of speech". Language is complex, a complexity which is that of man. In several works, Manin considers mathematics, like language, a *metaphor*. He explains the meaning he gives to this crucial word by referring to a sentence in the book by J. P. Carse, *Finite and infinite games* [3], according to which a metaphor is the "joining of like to unlike such that one can never become the other [...]" At its root all language has the character of metaphor, because no matter what it intends to be about it remains language, and remains absolutely unlike whatever it is about."⁹ Language is also an essential component of the talk Manin gave at his reception to the Paris Academy of Sciences [13]. He declares there: "Language and its functioning, both in contemporary mathematics and mathematical logic, and at its very early pre-historical stages about which we can only speculate, permanently fascinated me."¹⁰ From this talk, we learn that it is in his effort to understand the linguistic dimension of mathematics that Manin was led to an extension of the notion of Kolmogorov complexity, to the initiation of

⁹The joining of like to unlike was at the basis of the philosophies of Heraclitus, Anaximander and the Pythagoreans.

¹⁰The passage is also included in the Introduction to his book *Mathematics as metaphor* [18].

the project of quantum computation, and to his contribution to the theory of error-correcting codes.

Since I mentioned the work of Kolmogorov, and since my article opens with references to Tarkovsky's *Diary*, let me report on the following. By the beginning of 1972, Tarkovsky's third feature-length, *Solaris*, was finished, but it had still to pass several censorship committees. Tarkovsky gives a list of the observations, questions, corrections and recommendations that he received from various bodies: the cultural department of the Central Committee, the ideology department of the same Committee, etc. For instance, recommendation Nb. 5 says: "Cut out the concept of God"; Nb. 7 says: "Cut out the concept of Christianity"; Nb. 8 says: "The conference. Cut out the foreign executives". The motive for Gibarian's suicide (not in Lem) ought to be that he is sacrificing himself for his friends and colleagues". Tarkovsky found most of the recommendations absurd. He writes: "I simply cannot work out what they meant, or rather, what was in their minds, when they gave me those corrections." Nb. 21 says: "Put in some quotations from Kolmogorov (about the finite nature of man)." It probably shows that either there was a mathematician in one of these committees, or that Kolmogorov's theories went deep in the collective body of the society.

Let us return to language.

The question of the relationship between mathematics and language is also addressed in the interview *Good proofs are proofs that make us wiser* [1] in which Manin, answering a question about the cultural role of mathematics, sketches a wide historical perspective on language, showing how natural language, used for the purposes of science and the scientific description of nature, became progressively specialized, ending up with the language of mathematics:

In my opinion, the basis of all human culture is language, and mathematics is a special kind of linguistic activity. Natural language is an extremely flexible tool of communicating essentials required for survival, of expressing one's emotions and enforcing one's will, of creating virtual worlds of poetry and religion, of seduction and conviction. However, natural language is not very well fit for acquiring, organizing and keeping our growing understanding of nature, which is the most characteristic trait of the modern civilization. Aristotle was arguably the last great mind that stretched this capability of language to its limits. With the advent of Galileo, Kepler and Newton, the natural language in sciences was relegated to the role of a high-level mediator between the actual scientific knowledge encoded in astronomical tables, chemical formulas, equations of quantum field theory, databases of human genome on the one hand, and our brains on the other hand. Using the natural language in studying and teaching sciences, we bring with it our values and prejudices, poetical imagery, passion for power and trickster's skills, but nothing really essential for the content of the scientific discourse. Everything that is essential is carried out either by long lists of more or less well structured data, or by mathematics. For this reason I believe that mathematics is one of the most remarkable achievements of culture, and my life-long preoccupation with mathematics in the capacity of researcher

and teacher still leaves me with awe and admiration by the end of every working day. However, I do not believe that I can convincingly defend this conviction in the context of contemporary public debate on science and human values.

Manin's approach to language and linguistics involves ideas on psycholinguistics, social interactions, self-consciousness, the theory of development of the central nervous system, animal behavior, the human psychopathology of language and others. I would like to mention here the relationship he makes between, on the one hand, the history of language and, on the other hand, that of the known cultures, from the prehistoric man who painted on cave walls, passing through the Greeks of the time of Homer, mentioning the Winebago Indians, the biblical world, Sanskrit culture, etc. until modern man. This historical parallel, which is part of Manin's aim to understand the process of language, its structural properties and its evolution, is also an example of the notable analogies that he envisioned, and of which I will say a few more words in what follows.

4. ANALOGIES 1: COGNITIVE DEVELOPMENT, INTERNET AND HUMAN CONDITION

An important analogy Manin makes in his theory of language is the comparison between the linguistic development of the child and the history of speech and language. Among the questions he addresses is the following: Since a child can only learn to speak to the extent that he or she is immersed in a language environment, that is, being in contact with adults who speak with him and with each other, who was then the "adult" for the first men? Among the other problems he points out is the one of "gifted individuals" in the face of language skills: [18] p. 171: "[Why,] despite the democratization of education, such national poets as Pushkin and Dante so far exceeded the average level of language abilities as to become the nation's language teachers for centuries?".

Manin also talked about analogies between the history of languages and that of computers. In the article *Semantic spaces* [19] which he wrote with Matilde Marcolli, the two authors discuss mathematical, and in particular geometric, constructions which help bridging, on the one hand, database, usually associated with computer processing and computer memory, and on the other hand, natural language, which resides in the human brain and functions in human communication. Manin develops an analogous theory in an article titled *Cognitive networks: brains, Internet, and civilizations* he wrote in collaboration with his son Dmitry [10]. In the same paper, the cognitive activity of the nervous system is discussed in analogy with the Internet: neurones and neural networks are compared with computers.

In the paper [10], the two Manins, with the aim of understanding the functioning of the cognitive activities of the brain, explore a comparison between certain neural networks and computers. The whole nervous system is compared with the Internet. Dissociative phenomena like multiple personalities, automatism, fugues and hypnosis are also mentioned in this "brain/Internet

metaphor.” The Saussurean trichotomy *langue/langage/parole* is involved in this study.¹¹

Another analogy of the same kind, between something *a priori* special and something universal, appears right in the Introduction to his book *Mathematics as metaphor*, and it is about mathematics, presented as “a specimen and a metaphor of the human condition.” Here, Manin makes the following comment: “This attitude is not as esoteric as it might sound, if only because through mathematics, as through poetry and philosophy, dozens of generations communicate to each other, often over the heads of their contemporaries, because in each generation the community of mathematicians is small and scattered all over the world.”

I will say more on such analogies after a few remarks on computers and on Manin’s ideas on history.

5. BRAIN, COMPUTERS AND THE INTERNET

Manin, in several works, mentions the brain/Internet metaphor regarding information transmission, and he introduces Kolmogorov complexity in this discussion. In his lecture *Kolmogorov complexity as a hidden factor of scientific discourse* [14], he explains the fact that the characteristic features of scientific theories, from epicycles to the standard model of particles, split into two distinct parts: the first one with relatively small Kolmogorov complexity (laws, basic equations, periodic tables, natural selection etc.), and the second one with indefinitely large Kolmogorov complexity (examples include the initial and boundary conditions, phenotypes, population etc.). More generally, in his works on this topic, he divided scientific theories into two parts, “Kolmogorov simple” and “Kolmogorov indefinitely complicated.”

Manin has always highlighted the fact that computers have a non-negligible impact on pure mathematics, in that they allow us to do experiments. In the interview [6], he declares that they gave to the ordinary mathematician a unique possibility of “doing large-scale experiments in mental reality”. He says that thanks to computers, we are capable of doing things that only mathematicians of the stature of Euler and Gauss were capable of doing without computers. He declares: “By now, what Euler and Gauss could do, any mathematician can do, sitting at his desk. So if he doesn’t have the imagination to distinguish some features of this Platonic reality, he can experiment.”

In his paper *The early development of speech and consciousness (Phylogeny)*, which we already quoted, Manin states a metaphor which expands

¹¹The famous linguist Ferdinand de Saussure (1857-1913), considered as the founder of modern linguistics, in his *Cours de linguistique générale*, makes a distinction between the following three notions: *langue*, the result of a social convention transmitted by society to man and for which the latter does not play any important role, *langage*, as the capacity that allows human beings to communicate and interact with each other, and *parole*, as the personal use of language. Note that English makes no difference between “*langue*” and “*langage*”.

the beautiful expression “the smoke of centuries”.¹² “The sharpness of historical vision drops quickly with distance, and the milestones of subjective past conveniently fall on a logarithmic scale”. I will talk about some of Manin’s ideas on history in the next section.

6. HISTORY

Manin’s vision on history is exemplified in his article *Time and periodicity from Ptolemy to Schrödinger: Paradigm shifts vs. continuity in history of mathematics* [17], in which he discusses Kuhn’s notion of *paradigm shift* applied to the history of mathematics. His argument involves examples from the ancient laws of planetary motion until those of elementary particle physics. He starts by quoting Thomas Kuhn who, in his influential book on philosophy of science, *The structure of scientific revolutions* [8], defended the thesis that history of natural sciences can be divided into periods such that the passage between a period and the next one occurs through a revolution, that is, a radical change in the basic assumptions. Manin maintains that Kuhn’s theory does not apply to pure mathematics, a domain, he says, which is different from the other sciences. He recalls that mathematics “has a position mediating, or bridging, daily life, common sense, philosophy, and physics.” He asserts that only one part of mathematics is sensitive to the so-called revolutions or paradigm shifts, namely, the part that is subject to the “reality tests”,¹³ whereas the rest of mathematics, which we call “pure mathematics”, experiences only a continuous development. He considers that the intergenerational continuity of that field has been neglected by historians. Placing himself from a point of view which is above the vision field of most professional historians of mathematics and science general, he focuses on the example of the mathematical theory of time and of periodicity. To explain what he means, he starts with Ptolemy’s dynamical model of the solar system representing the visible motions, a system which involves complex combinations of circles, with epicycles and secondary epicycles. He then continues with the history of the numbers π and e , the appearance of Fourier sums and Fourier integrals, all the way to the attempts to explain the randomness that appears at the electron level by methods of probability, leading directly to the works of Heisenberg, Pauli and Schrödinger on quantum mechanics, to Feynman integrals and to the contemporary cosmological models. He declares that at the end, quantum interaction in quantum mechanics is essentially described by Fourier sums, series, and integrals in complex spaces equipped with Hermitian metrics replacing real Euclidean spaces with real metrics; thus, we are back to Euclid and Ptolemy,¹⁴ and

¹²Manin says that this expression is in Pushkin. Actually, I found it in Walter Scott’s *Rob Roy* (1817), and also in a poem by Dovid Knut (1900-1955), a Russian-Jewish emigre of the first generation who published his poems in Paris in the 1920s.

¹³Manin puts this expression in quotation marks, and it is not clear whether he intends the literal or the Freudian meaning of this expression; both may fit here (although, from what I know, Manin was not a strong supporter of Freud, maybe because of Freud’s materialistic side).

¹⁴The first terms $\sum_{n=1}^N a_n e^{inx}$ of the Fourier series is a trigonometric polynomial which can be put in parallel with Ptolemy’s superposition of periodic motions.

Manin's conclusion is that, in some sense, "quantum mechanics is a complexification of Ptolemy's epicycles."

Regarding continuity vs. revolution in the history of mathematics, let me also mention a passage from an interview with M. Gelfand which carries the significant title *We do not choose mathematics as our profession, it chooses us* [6]. The passage comes as a response to the question "What will happen [in mathematics] in the next twenty years?". Manin declares:

I don't foresee any revolutionary changes, because in my view there have been none in the last 300 years. Every time new and powerful intuitions arose, mathematics retained its character, in some strange way. This is also a theme of a lecture, one I've not given. I would like to show the development of the idea of the integers from the most remote times to Kolmogorov complexity, and all this can be done almost without appealing to new mathematics. One and the same idea persists. It changes a bit in one era or another, its verbal casing changes. But all the same it stays completely invariant and so lives on. Nothing is forgotten. And so I don't foresee anything extraordinary.

7. ANALOGIES 2: MAN AND COSMOS

A brilliant element of Manin's argument in the paper *Time and periodicity from Ptolemy to Schrödinger: Paradigm shifts vs. continuity in history of mathematics* I just discussed involves a comparison between the evolution of knowledge on the scale of civilisations and on that of the development of cognition in the individual brain of a growing human being. This is part of the general pattern he expanded in several works, some of which I alluded to in §4, making parallels between historical developments, on the one hand, at the civilizational level, and on the other hand, at the human level.

All these analogies remind me of a general metaphor relating the "cosmos" (or "macrocosmos") and the "microcosmos", which goes back to Ancient Greece (Anaximander, Plato, the Stoics, etc.), which was taken up by early Christian writers, according to which man is presented as a "microcosm" recapitulating in him the whole of "macrocosm".¹⁵ Manin strongly felt this relation, as an analogy between the microcosmos and the macrocosmos. I think that this is an aspect of his thought which has never been emphasized, if ever it was noted. This reminds me of cosmism, a philosophical, scientific, literary, spiritual and intellectual movement which was born in Russia in the nineteenth century and which attained its apogee at the beginning of the twentieth. This movement claims an alliance, solidarity and harmony between man and the whole creation,¹⁶ Manin's ideas have a taste of cosmism

¹⁵To quote again Russian authors, I mention Sergei Bulgakov, who writes that man is a "microcosm, and his imprint therefore lies upon the entire world, the macrocosm," and that "by his body man belongs to the creaturely world, which, he, as microcosm, has in himself and unifies and generalizes," [2, p. 173, 279], etc.

¹⁶The founder of cosmism is considered to be Nikolai Fiodorov (1829-1903). The movement was developed especially in the first three decades of the twentieth century, before it was squashed brutally by Stalin. This movement inspired writers, philosophers, composers, theatre directors and artists, like Leo Tolstoy, Alexander Scriabin, Vassily Kandinsky, Casimir Malevic, Vladimir Soloviev and Sergei Bulgakov.

even if they stay at the level of *metaphor*, or *theory*.¹⁷ And if one does not like the term cosmism in Manin’s work, one can talk at least about cosmology, as in the paper *Time and periodicity from Ptolemy to Schrödinger* [17] which I already mentioned. I also would like to mention here his talk at the 2014 Plenary Session of the Pontifical Academy of Sciences, titled *Big Bang, Blow up, and modular curves: Algebraic geometry in cosmology* [15],¹⁸ in which he starts by quoting T. S. Eliot, “In my beginning is my end”, and where he sets his goal as follows: “We are trying to explain how it could be that in my end (that is, in the universe’s end) is my (that is, the universe’s) beginning”. Again, the analogy—here an identification—between man and the universe. Let me also quote another Russian author, on the same topic, Pavel Florensky,¹⁹ from his booklet *Hamlet* [5, p. 24-25], talking about “development” as a result of either universal and supra-individual human reason, or of the individual human reason, which he calls “microcosm”:

[...] Motion and development come either from the universal, supra-individual human reason, or from the common, individual human reason. The latter is more easily observable given its reduced dimensions, and its more or less conscious construction, it is a microcosm. Nevertheless, the microcosm corresponds to the macrocosm, in this case the natural, spontaneous and irresistible dialectics of the universal human consciousness; there is a reciprocal correspondence between phylogeny and ontology, between the philosophical structure of the individual and the historical destinies of the world consciousness.

To return to Tarkovsky’s universe, I would like to mention this scene from the *Sacrifice* in which the character, in the middle of a nuclear accident, knowing that he is ready to perish, instead of making, as one would expect, an evaluation of his life, makes an evaluation of existence in general. He strives for salvation in many forms, frightened by an eschatological feeling. He seeks help, but finds none, and is finally seized by this destructive eschatological feeling, not only for himself after his death, but also for the universe after its disappearance. I can also quote Andrei Tarkovsky again, from his book *Captured time* [26]: “There must be other possibilities of working with sound, which would allow us to be more precise, more faithful to the inner world that we try to recreate on the screen, and not only to the inner world of the artist but to what is inside the world itself, to what is essential to it and independent of us. ” Again a relation between the inner world of man and the world itself. Finally, I can quote a line from Arseni Tarkovsky’s poem *Life, life* (1965): “...I tailored the age to fit me ...”

¹⁷The word *theory* comes from the Greek word “theōria”, which means “contemplation”.

¹⁸The article is a joint work with Matilde Marcolli.

¹⁹The mathematician, theologian, physicist, philosopher and historian of art Pavel Florensky (1882-1937) was dubbed the “Russian Leonardo” and the “Pascal of the Gulag”. Solzhenitsyn said about him that he was “perhaps the most remarkable person devoured by the Gulag”. The translation of his books and articles in French, English, Romanian and other languages has taken an accelerated pace in the last two decades. I wrote an article on him for the present Handbook [21].

I think that this feeling of the cosmos and of a strong bond between man and the universe, exemplified in Manin's writings, is a common feature of all Russian thought.

8. PROOF AND TRUTH, MATHEMATICS AND TARKOVSKY AGAIN

This section consists essentially of five quotes: two by Manin and three by Tarkovsky.

All of us have read (or heard of) Bourbaki's famous sentence, from the introduction to *Set Theory*, the opening treatise of the *Elements*: "Depuis les Grecs, qui dit mathématiques dit démonstration"²⁰. Manin belongs to those who go against the grain. In his Kyoto ICM talk, *Mathematics as metaphor* [11], he writes:

[...] What is relevant is the imbalance between various basic values which is produced by the emphasis on proof. Proof itself is a deviate of the notion of 'truth'. There are a lot of values beside truth, among them 'activities', 'beauty' and 'understanding', which are essential in high school teaching and later. Neglecting precisely these values, a teacher (or a university professor) tragically fails.

Let me complement this discussion about "truth" by some words of Tarkovsky, whose name I associate with that of Manin for reasons I explained at the beginning of my article. This is a quote, again from Tarkovsky's diary, in which he talks at the same time of truth, the cosmos, Euclid, and the infinite, topics which were all dear to Manin. He writes, on September 5, 1970:

[...] Since [truth] is human, it is limited, rigidly confined, in human terms, within the framework of the human milieu. There is no conceivable link between what is human and the cosmos. And the same applies to truth. To attain greatness within our own limits—which are Euclidean and insignificant in juxtaposition with infinite—is to illustrate that we are merely human."

In the next section, I will say a few words about Manin as an artist, and in a later section, I will quote one of his poems. Let me, before that, quote a passage, written in Manin's poetical style, about proof, as a conclusion of his paper *Foundations as superstructure*, [16]:

One metaphor of proof is a route, which might be a desert track boring and unimpressive until one finally reaches the oasis of one's destination, or a foot path in green hills, exciting and energizing, opening great vistas of unexplored lands and seductive offshoots, leading far away even after the initial destination point has been reached.

Finally, let me conclude this section with two quotes on mathematics from Tarkovsky's *Diary*. On March 24, 1982, he writes:

What is expressed in mathematics is not so much objective laws of the world as laws of the human psyche, laws of logic, of the human mind. It's really a kind of mental game, and yet the exact sciences,

²⁰Since the Greeks, talking about mathematics means proving.

as they are called, command extraordinary respect—physics, astronomy, and so on. It's very odd, in fact I would say an amazing, paradoxical misconception.

On April 2 of the same year, he writes:

Our reason cannot reach the other parameters or other dimensions which exist, and so it fantasizes about them in the form of mathematics and physics. Science involves the discovery not so much of objective laws of nature, as of the laws which govern the functioning of our consciousness. A kind of music; image; symbol; sign. A mathematical symbol of truth, corresponding to our capacity for knowing it with our brains.

I am always surprised to see how much the filmmaker understood what mathematics is about, even though he never studied mathematics.

9. PAINTING, THE BRAIN AND REVERSE PERSPECTIVE

I would like to mention an episode on painting recounted in the French version of Manin's book, published in 2021 [18, p. 426-427].²¹ He talks there about a man whose full name he does not give, referring to him only by his initials, YM. In fact, before I read this text of Manin, I had already heard this story from him, as something that happened to him personally. In the book, the episode is part of a discussion regarding the asymmetry of the human brain, the tensions between its two hemispheres, and the main features of the division of its cognitive functions. The story is the following.

In 1973, Manin, 36 years old, who was already a professor at the Lomonosov State University in Moscow, was suddenly affected by a brain disease called *arachnoïdite*, an inflammation of the arachnoid—one of three membranes that surround and protect the spinal cord. This disease causes neurological disorders, and sometimes it attacks the back and the limbs. In Manin's case, the sickness was probably a result of a poorly treated flu. As a consequence, he underwent a treatment with antibiotics that lasted several months. He was not able to do mathematics anymore. One day, in the fourth month of his illness, while riding the subway home on the outskirts of Moscow, he felt a sudden urge to paint, although he had never painted before. He got off at the train stop, took the metro in the backwards direction, that is, towards the city center, and there he stopped at an artists' materials store. He bought brushes, tubes of oil paint and blank canvases. Back home, he started to paint. For several months, painting became an important activity in his life, perhaps the only original activity. In the book [18] (p. 427 of the French version), Manin lists a number of canvases he painted during that period.

In retrospect, after he had returned to normal activity, and after he had stopped drawing and painting, Manin examined his paintings, trying to understand the states through which he passed. Some of his drawings seemed strange to him, until he understood that they satisfy a theory developed by a specialist in functional cerebral asymmetry who works at the Institute

²¹The episode is told in a chapter titled *Spontaneous artistic activity, origin of logograms and mathematical intuition*. This chapter is not contained in the English version of the book.

of Biochemistry and Evolutionary Physiology of the Russian Academy of Sciences. This theory is reported on in an article on artistic activity and cerebral asymmetry [20]. The research is based on clinical observations of drawings made by patients with brain damage. The conclusion of this study is that someone who makes drawings and who is in the state in which Manin was during his illness—a state where the right hemisphere of his brain (or his “right brain”) is predominant—tends to produce images that seem too close to the person looking at them, and this is why they are drawn in reverse perspective. In contrast, the left hemisphere, which (for the majority of human beings) is the verbal and logical hemisphere tends to produce representations of space with the standard proportion scheme and with an ordinary perspective. The above story, which describes Manin’s personal experience, is an illustration of this fact.

Manin explains at the beginning of his article that the asymmetry between the left and right hemispheres of the brain is not only a functional division, but the source of a dynamic competition between the two hemispheres which, when it occurs, leads to a back and forth motion around a certain balance of the brain. The point of his article is to say (again) that such an equilibrium can also be observed in the study of civilizations and their cultures, especially in periods of transition. He discusses the balance of the brain in the context of the origin of writing, its development and its transmission at the level of the cultural human heritage and also in connection with the drawings that survive on the prehistoric caves in the South of France, representing animals and hunting scenes, compared to other forms of realistic representations, talking also about poetry, etc. At the same time, he addresses the case of history of mathematics, noting in particular the tension, in Euclid’s *Elements*, between the use of the natural language to express the *logic* of the proofs, and that of the figures, which express the underlying *imagination*.

Returning to Manin’s drawing experience, let me say that personally, I sometimes had to deal with colleagues who, while we were discussing in front of the blackboard, used to draw awkward pictures; for instance I noticed that some of them, when they draw a cube, the perspective is always reversed, and I always wondered how that could be. I understood much later, when I read Manin’s explanations on the brain and reverse perspective, that what I experienced was something that could be explained, and the drawing, here, is a tool that helps us exploring the neural mechanisms that are at the base of the aesthetic experience of the person who performed it.

There are other explanations. The manner we draw, in reverse or in Renaissance (call it “usual”) perspective, is also a question of how we perceive the world, and I cannot help but mention here Byzantine iconography, of which Russian iconography is the full heir, whereby the drawing is usually done in reverse perspective. In this connection, I would like to quote again Pavel Florensky, from his essay *Reverse Perspective* (1920) [4, p. 219]:

The drawings of children, in their lack of perspective and especially their use of reverse perspective, vividly recall mediaeval drawings, despite the efforts of educators to instill in children the laws of linear perspective. It is only when they lose their spontaneous relationship to the world that children lose reverse perspective and

submit to the schema with which they have been indoctrinated. This is how all children behave, independent of each other. This means that it is not mere chance, or a willful invention by one of them putting on Byzantine airs, but a representational method that derives from a characteristic perceptual synthesis of the world. Since the way children think is not weak thinking but a particular type of thinking which, moreover, is capable of unlimited degrees of perfection, including genius, and indeed is primarily akin to genius, it must be admitted that the use of reverse perspective to depict the world is also far from being an unsuccessful, ill-understood, imperfectly learned linear perspective, and is rather a distinctive grasp of the world that should be reckoned with as a mature and independent representational method.

After he recovered his original functioning of the brain, Manin stopped painting, and he came back to mathematics. He did not stop being an artist; mathematics was also, for him, an art. In his reception speech as foreign associate to the Paris Academy of Sciences [13], he tells how he came back later to mathematical research, and in particular to his favorite field, algebraic geometry. About this field, he writes that he sees the set of solutions of systems of polynomial equations with several variables—an object that is at the heart of algebraic geometry—as “a geometric entity, a shape situated in a multidimensional space, stretching to infinity along some directions and capriciously closing upon itself at others. Variety and complexity of such forms is infinitely richer than anything one could glimpse at modern exhibitions of abstract art, but algebraic geometers managed to find patterns, discover connections and laws of this immense world.”

Manin also talks about mathematics as an art while he gives his point of view on the heritage of Georg Cantor (another Russian-born mathematician). He writes, at the beginning of the paper [12]: “Georg Cantor’s grand meta-narrative, Set Theory, created by him almost singlehandedly in the span of about fifteen years, resembles a piece of high art more than a scientific theory.”

Let me also mention a passage in the interview [6]: “Once I was asked to deliver a lecture for the general public in an art museum, and I decided that the Banach–Tarski paradox is a great subject for the presentation *The Abstract Art of Mathematics*.

Let me mention, to close this subject, that on the dust cover of Manin’s *Mathematics as metaphor*, published by the AMS, there is a picture of a house, with the inscription *Geomètre* over the entrance door; the house is drawn in reverse perspective. I never asked him an explanation.

From visual art and abstract art, let us pass to poetry.

10. A POEM

Manin, in an article he wrote on Georg Cantor [12] has, as an epigraph, the following citation from V. Tasić’s [27]: “God is no geometer, rather an unpredictable poet.”

Like God, Manin was a poet. In the French version of *Mathematics as metaphor*, [18] (p. 536)²² he wrote a poem related to his painting period. The title of the poem is *Je peins À l'huile*. I reproduce it here, in French:

La touche vient toucher la touche
comme la phalange — la page.

La touche vient toucher la touche
comme le canon glissant — la tempe.

La touche vient toucher la touche.
Le cerveau se languit sans langue.

L'hémisphère taiseux travaille
comme fait une avalanche —
le bruit arrive en retard . . .

It would be a redundancy to say that Manin was permanently reading poetry. We once talked about the Greek poet Constantine Cavafy, and he told me he has his works on his shelf, in several editions.

11. PLATONISM

We talked about mathematics as a language, but mathematics is also ideas, and Manin had his own idea about ideas. Let us hear again what he says, in his Reception speech at the Paris Academy of Sciences [13]:²³

[. . .] This Platonic feeling that even most abstruse mathematical ideas are somehow predestined to be in harmony with the physical world, always constituted for me one of the most irresistible attractions of our trade. Stéphane Mallarmé wanted to make us aware that poetry is made of words rather than ideas. To a certain degree, this is true about mathematics as well, but in a more profound sense, this is fundamentally wrong. (I suspect that this is wrong for poetry as well).

At several occasions, Manin expressed his “Platonic feeling” of pre-existing mathematical notions. In the interview *Good proofs are proofs that make us wiser* [1], he declares:

I see the process of mathematical creation as a kind of recognizing a preexisting pattern. When you study something—topology, probability, number theory, whatever—first you acquire a general vision of the vast territory, then you focus on a part of it. Later you try to recognize “what is there?” and “what has already been seen by other people?”. So you can read other papers and finally start discerning something nobody has seen before you.

In his interview with Gelfand [6], he expresses the same idea, slightly differently:

²²Manin was very happy of the fact that the French version of the book, which appeared in 2022, included the poems. The French translation of his book was done by Claire Vajou, but it is certain that the translation of his poems was done with his close collaboration, maybe by him alone (he was completely fluent in French).

²³The passage is also reproduced in the Introduction of *Mathematics as metaphor* [18].

I must explain to you how I imagine mathematics. I am an emotional Platonist (not a rational one: there are no rational arguments in favor of Platonism). Somehow or other, for me mathematical research is a discovery, not an invention. I imagine for myself a great castle, or something like that, and you gradually start seeing its contours through the deep mist, and begin to investigate something. How you formulate what it is you've seen depends on your type of thinking and on the scale of what you have seen, and on the social circumstances around you, and so on.

Staying with Plato, let me mention Manin's book with I. Yu. Kobzarev, *Elementary particles. Mathematics, physics and philosophy* [7], a philosophical dialogue, formally written in the style of Plato's dialogues. There are three characters: a theoretical physicist, an experimental physicist and a philosopher, and the aim of the dialogue is to present the development of elementary particle physics, including its present state. However, unlike in Plato's dialogues, there is no "wise man" involved in this dialogue; such a figure is replaced by that of a scientist. Let me take this opportunity to quote Manin again, from *Mathematics as metaphor* (p. 197):

The central figure of a philosophic dialogue is a wise man, whereas modernity generally and systematically replaces wisdom by training. Wisdom seems to be an inborn faculty slowly ripened by life experience; as such it is rarely met and even more rarely put to any use. Training is a democratic surrogate for wisdom which, in spite of all of its (mainly aesthetic) drawbacks, is superior in one respect: it produces professionals.

Let me also mention, since we are talking about Platonism, that the adjective "Platonic", which is systematically used in such a context (the mathematician discovering pre-existent objects) indeed is part of Plato's teaching.²⁴ The term "platonic" is nevertheless abusive, because the fact referred to is not sufficient to characterize Plato's view on mathematical objects.²⁵

In fact, Plato was absolutely against the idea that mathematical objects may be used to describe the real world: they can only describe immutable

²⁴The idea that abstract mathematical objects exist independently of us and our knowledge of them is one of the basic teachings of Plato, who also considered that knowledge (which, for him is, to a large extent, mathematical) is simply recollection (*anamnesis*) of innate knowledge, about pre-existing things. The standard reference for this theory of *anamnesis* is Plato's *Meno* 81e1-82b8. It is worth noting that in this passage, Plato, to illustrate his idea, brings in Socrates helping a slave boy recollecting a specific knowledge, which, in fact, is a mathematical knowledge: how to construct a square whose area is twice that of another given square. S. Negrepontis explained to me that the method Socrates uses for this recollection turns out to be anthyphaeretic division. Finally, let me recall that "platonism", as a belief in pre-existing objects (outside our thought), is contrary to the intuitionistic trend in mathematics (mostly represented, in the modern period, by Brouwer), which holds that mathematical objects exist only when we can construct them.

²⁵As Plotnitsky's rightly points out in his essay *Continuity and discontinuity in, and between, mathematics and physics* [22]: "mathematical Platonism (which is a twentieth-century conception) should not be confused with Plato's own philosophy." In fact, the term "Platonic" is often used and misused. A Japanese friend told me that in Japan, this word is used exclusively (as a Japanese word) as an adjective for a love relation, to say that it is purely mental.

things, whereas the real world is permanently subject to movement and change. In contrast, Manin has always been interested in mathematics as a means of describing reality. Even in his work on algebraic geometry or number theory, he was resolute in thinking in terms of applications to theoretical thinking. Besides that, he taught us that mathematics is not only the language of nature, but it is also an adequate language for talking about art, literature and the deep human psyche. More lyrically, he writes that mathematics is a “novel about Nature and Humankind” and he adds that his wish is “to restore a certain balance between the technological and the humanitarian sides of mathematics” [18, p. 28].

It has often been said (by mathematicians) that there are two kinds of mathematicians: those who build theories and those who solve problems. If this is true, then what we recalled in this article says that Manin belongs definitely to the first category. But I think that this distinction is rather secondary, and even irrelevant, in Manin’s case. For him, mathematics was a language, and he was interested in language much more than in mathematics. According to Aristotle, language is what distinguishes man from the rest of the animals, as we have been reminded above. I would like to quote, closer to us, Emmanuel Levinas : “[...] l’Infini se passe dans le Dire [...]”.²⁶

I started this article with a personal recollection, and I will also finish it with a personal recollection.

12. IN GUISE OF A CONCLUSION

While I was writing this article, I often regretted not having had more conversations with Manin about all of this. In one of our last contacts, an email dated September 14, 2022, answering an invitation I made for him and Xenia to visit us in Strasbourg, he wrote me that he was tired and that he did not travel anymore. I regret not having had the reflex to tell him: “In this case I will come to see you in Bonn.”

Acknowledgement. I am especially grateful to Arkady Plotnitsky and Susumu Tanabashi who kindly made several interesting comments on a previous version of this chapter. This work is supported by the Interdisciplinary Thematic Institute CREEA, as part of the ITI 2021-2028 program of the University of Strasbourg, CNRS and Inserm (IdEx Unistra ANR-10-IDEX-0002), and the French Investments for the Future Program.

²⁶ “[...] The Infinite occurs in the Saying [...]” [My bad translation], from his book *Autrement qu’être ou au-delà de l’essence* [9, p. 230].



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Strasbourg, September 13, 2019

REFERENCES

- [1] M. Aign and V. A. Schmidt, Good proofs are proofs that make us wiser, Interview with Yuri I. Manin, *The Berlin Intelligencer*, Springer Verlag, 1998, p. 16-19. Also published in [18], p. 207-213.
- [2] S. Bulgakov, *The Bride of the lamb*, transl. B. Jakim, Eerdmans Publishing Company, Grand Rapids, MI, 2008
- [3] J. P. Carse, *Finite and infinite games*, Free Press, New York, 2013.
- [4] P. A. Florensky, *Reverse Perspective (1920)*, In: *Beyond vision, Essays on the perception of art*, compiled and edited by Nicoletta Misler, transl. Wendy Salmond, Raktion books, London, 2002.
- [5] P. A. Florensky, *Hamlet*, translated from the Russian by Evdokiya Sichov, Ed. Allia, Paris, 2006.
- [6] M. Gelfand, We do not choose mathematics as our profession, it chooses us: Interview with Yuri Manin, *Notices of the AMS*, Vol. 56, n. 10, p. 1268-1274.
- [7] I. Yu. Kobzarev and Yu. I. Manin, *Elementary particles. Mathematics, physics and philosophy. Ser: Fundamental Theories of Physics. 34.* Dordrecht etc.: Kluwer , 1989.
- [8] T. S. Kuhn, *The Structure of Scientific Revolutions*, University of Chicago Press, Chicago, 1962, several later editions.
- [9] E. Levinas, *Autrement qu'être ou Au-delà de l'essence*, Librairie générale française, Paris, 1990.
- [10] D. Yu. Manin and Yu. I. Manin. *Cognitive networks: brains, Internet, and civilizations.* In: *Humanizing Mathematics and its Philosophy*, ed. Bh. Sriraman, Springer International Publishing AG, 2017, p. 85-96
- [11] Yu. I. Manin, *Mathematics as metaphor*, *Proceedings of the International Congress of Mathematicians*, Kyoto, Japan, 1990, Vol. II, p. 1665-1671. Also published in [18], p. 27-32.
- [12] Yu. I. Manin, *Georg Cantor and his heritage.* *Proc. Steklov Inst. Math.* 246, 195-203 (2004) Also in [11], p. 45-54.
- [13] Yu. I. Manin, *À la recherche du temps perdu (In search of lost time)*, Académie des Sciences, Réception des Associés étrangers élus en 2005, 12 décembre 2006.
- [14] Yu. I. Manin, *Kolmogorov complexity as a hidden factor of scientific discourse*, lecture given at the Pontifical Academy of Sciences, Plenary Session on *Complexity and Analogy in Science: Theoretical, Methodological and Epistemological Aspects*, Vatican City, 5-7 November 2012.
- [15] Yu. I. Manin, *Big Bang, Blow up, and modular curves: Algebraic geometry in cosmology*, lecture given at the Pontifical Academy of Sciences, Plenary Session on *Evolving Concepts of Nature*, Vatican City, 24-28 October 2014.
- [16] Yu. I. Manin, *Foundations as superstructure*, In: *Prozorov, Oleg (ed.), Proceedings of the international conference on philosophy, mathematics, linguistics: aspects of interaction (PhML-2012)*, Euler International Mathematical Institute St. Petersburg, St. Petersburg, Russia, May 22-25, 2012. London: College Publications. *Stud. Log. (Lond.)* 70, 101-115 (2017).
- [17] Yu. I. Manin, *Time and periodicity from Ptolemy to Schrödinger: paradigm shifts vs continuity in history of mathematics.* *Geometry in history*, S. G. Dani and A. Papadopoulos, ed., Springer, Cham, 2019, p. 129-138.
- [18] Yu. I. Manin, *Mathematics as metaphor. Selected essays of Yuri I. Manin.* With foreword by Freeman J. Dyson. Providence, RI: American Mathematical Society, 2007. French version: *Les Mathématiques comme Métaphore (Essais choisis)*, Translated by Claire Vajou, Les Belles lettres, Paris, 2021.
- [19] Yu. I. Manin and M. Marcolli, *Semantic spaces*, *Math. Comput. Sci.* 10, No. 4, 459-477 (2016).
- [20] N. N. Nikolaenko, *Artistic thinking and cerebral asymmetry*, *Acta Neuropsychologica*, 1.2, p. 48-67.
- [21] A. Papadopoulos, *Pavel Florensky and his world*, In: *Handbook of the History and Philosophy of Mathematical Practice*, ed. Bharath Sriramann Springer, 2023.

- [22] A. Plotnitsky, Continuity and discontinuity in, and between, mathematics and physics. In: Essays in geometry – Dedicated to Norbert A’Campo (ed. A. Papadopoulos), EMS Press, European Mathematical Society, Berlin, 2023.
- [23] G.-C. Rota, Indiscrete thoughts. With forewords by R. Hersh and R. Sokolowski. Edited, with notes and an epilogue by F. Palombi. First ed. 1997, Modern Birkhäuser Classics. Basel, new ed. 2008.
- [24] J. Tannery, Pensées, in: En souvenir de Jules Tannery (ed. É. Boutroux and É. Borel) Imprimerie Crété, Corbeil, 1912.
- [25] A. Tarkovsky, Journal, 1970-1986, French translation from the Russian by Anne Chilikov, Cahiers du Cinéma, Atelier 3, 1993. Revised and augmented edition, December 2004, definitive edition, 2017. English edition: Time Within Time—The Diaries 1970-1986, translated by Kitty Hunter-Blair, Faber and Faber, London, Boston, 1994.
- [26] A. Tarkovsky, Le temps scellé : De l’*Enfance d’Ivan* au *Sacrifice*, transl. from the Russian by A. Kichoïlov and Ch. H. de Brantes, Philippe Rey, Paris, 2004.
- [27] V. Tasić, Mathematics and the roots of postmodern thought. Oxford UP, 2001.

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