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## ► To cite this version:

Mayra Hernandez, Gérard Meunier, Loïc Rondot, Olivier Chadebec, Jean-Michel Guichon, et al.. A Time-Stepping Volume Integral Method for Nonlinear Field-Circuit Coupled Problems. 24th International Conference on the Computation of Electromagnetic Fields (Compumag'2023), May 2023, Kyoto, Japan. hal-04115430

**HAL Id: hal-04115430**

**<https://cnrs.hal.science/hal-04115430v1>**

Submitted on 2 Jun 2023

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# A Time-Stepping Volume Integral Method for Nonlinear Field-Circuit Coupled Problems

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A B-conforming time stepping volume integral formulation (VIM) for nonlinear 3D field-circuit coupled problems is presented in this paper. The advantage of the VIM with respect to the finite element method (FEM) is that only the ferromagnetic regions have to be discretized, thus avoiding to mesh the air. It is an appealing approach because it avoids the numerical errors that can arise from modeling the air. An application to a current transformer is presented and compared to the FEM to validate its accuracy.

*Index Terms*—Equivalent circuit, nonlinear time-stepping formulation, volume integral method, field-circuit coupled problem.

## I. INTRODUCTION

VOLUME integral method (VIM) is a powerful approach to solve magnetic problems. Its main advantage is that only ferromagnetic regions have to be discretised, thus avoiding to mesh the air as it happens in the finite element method (FEM). VIM is known since 1970s but it wasn't popular because it leads to full matrices that are computationally expensive. Nevertheless, the last decade it became more accessible as a result of matrix compression algorithms such as the fast multipole method [1], advanced integral formulations like [2] and the enlargement of computers RAM memory.

The VIM performs well with a coarse mesh and it is very accurate in the calculation of remote electromagnetic interactions, for instance, when the magnetic flux through external coils is needed. This flux calculation is central in the framework of coupled field-circuit approaches where a distinction between electrically conductive and non-conductive media is made. Thus, it seems natural to implement the field-circuit coupling with a VIM method to treat magnetic phenomena and a classical circuit method to treat coils electrically coupled with current or voltage sources or more complex lumped electric elements.

This paper therefore presents an original extension of the magnetostatic formulation [4] to a general time-stepping formulation for nonlinear magnetic problems coupled with circuit equations for external coils.

## II. VIM TRANSIENT FORMULATION

Let us consider a problem composed of a non-conductive and non-linear magnetic region  $\Omega_m$  surrounded by an air region which includes circuit coils  $\Omega_b$  (Fig. 1). The circuit coils are coupled with lumped elements such as current sources, voltage sources or resistive circuit elements. For the magnetic field part, Maxwell's equations are considered and more precisely the Maxwell Ampere equation written as:

$$\mathbf{H} = -\text{grad}\mathbf{V}_m + \mathbf{H}_b \quad (1)$$

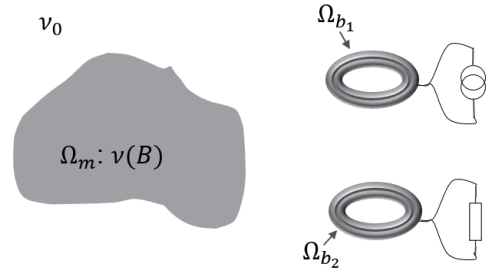


Fig. 1: Problem with a magnetic region and two circuit coils.

where  $\mathbf{H}$  is the magnetic field intensity,  $\mathbf{V}_m$  is the reduced magnetic scalar potential and  $\mathbf{H}_b$  is the magnetic field created by the coils expressed as  $\mathbf{H}_b = \sum_k \mathbf{h}_{b_k} \mathbf{I}_{b_k}$  where  $\mathbf{h}_{b_k}$  is the magnetic field created by coil  $k$  with current 1A and  $\mathbf{I}_{b_k}$  is the current of coil  $k$ .

In this paper, the field formulation is based on [4]. We consider the magnetostatic integral equation with the constitutive relation  $\mathbf{H} = \nu(\mathbf{B})\mathbf{B}$  and facet elements  $\mathbf{w}_i$  (Whitney 2-form [5]) as interpolation functions for the magnetic induction  $\mathbf{B}$  and as Galerkin projection test functions. The resulting discretized integral system reads:

$$\mathbf{R}_m \Phi + \mathbf{P}_m \Phi + \mathbf{L}_{mb} \mathbf{I}_b = \Delta \mathbf{V}_m \quad (2)$$

with

$$\mathbf{R}_{m_{ij}} = \int_{\Omega_m} \mathbf{w}_i \cdot \nu \mathbf{w}_j d\Omega_m, \quad \mathbf{L}_{mb_{ik}} = - \int_{\Omega_m} \mathbf{w}_i \cdot \mathbf{h}_{b_k} d\Omega_m,$$

$$\mathbf{P}_{m_{ij}} = \frac{1}{4\pi} \int_{\Gamma_{ext_j}} \frac{1}{s_i} \left( \int_{\Gamma_{tot}} \frac{1}{s_j} \frac{\delta \nu_j}{r} d\Gamma_{tot} \right) d\Gamma_{ext},$$

where  $\Phi$  is the magnetic flux flowing through each facet,  $\Delta \mathbf{V}_m$  is a vector containing the  $\mathbf{V}_m$  difference between two adjacent elements,  $\nu$  is the reluctivity of the magnetic region,  $\delta \nu$  is the reluctivity jump between two adjacent elements,  $r$  is the distance between the integration point and the evaluation point,  $\mathbf{n}$  is the normal of a facet,  $\Gamma_{tot}$  is the ensemble of facets of the problem and  $\Gamma_{ext}$  refers only to the facets that are on

the boundary of the magnetic region.  $P_m$  is written using the assumption of  $\nu$  being constant per element, which is true in case of simplex mesh elements such as tetrahedrons.

For the circuit coupling, an additional circuit relation for each coil is needed, linking the magnetic flux, the current, the internal resistance and the voltage [6]:

$$\Delta V_b = R_b I_b + \frac{d}{dt} L_b I_b + \mu_0 \frac{d}{dt} \int_{\Omega_m} \mathbf{h}_b \cdot \mathbf{M} d\Omega_m \quad (3)$$

where  $R_b$  is the diagonal resistance matrix of the coils,  $L_b$  is the self and mutual inductance matrix of the coils in the vacuum,  $\mu_0$  is the vacuum permeability and  $M$  is the magnetization, that can be expressed as  $M = (\nu_0 - \nu)B$ .

Assembling the magnetic field and the circuit, the final coupled matrix system reads:

$$\begin{bmatrix} \mathbf{R}_m + \mathbf{P}_m & \mathbf{L}_{mb} \\ \frac{d}{dt} \mathbf{L}_{bm} & \mathbf{R}_b + \frac{d}{dt} \mathbf{L}_b \end{bmatrix} \begin{Bmatrix} \Phi \\ \mathbf{I}_b \end{Bmatrix} = \begin{Bmatrix} \Delta V_m \\ \Delta V_b \end{Bmatrix} \quad (4)$$

where  $L_{bmkj} = \mu_0 \int_{\Omega_m} \mathbf{h}_{b_k} \cdot (\nu_0 - \nu_j) \mathbf{w}_j d\Omega_m$  is a coupling term. External circuit elements could be easily added to the latter matrix system to consider external electrically coupled current sources, voltage sources or other additional lumped circuit elements.

### III. RESOLUTION STRATEGY

The equivalent electric circuit system (4) is solved using either Kirchoff's mesh rule or Kirchoff's node rule. This procedure ensures the free divergence of  $B$ .

In practice, the resolution is carried out with a  $\theta$  schema time-stepping method with a imbricated Newton-Raphson (NR) loop to solve the non linear term. In theory, the jacobian matrices of the system have to be computed at each NR iteration. To avoid the computation of the integral matrix  $P_m$  at every time step, since the hypothesis of constant reluctance per element is set, we can integrate the matrix  $Q_m = \frac{1}{4\pi} \int_{\Gamma_{ext_j}} \frac{1}{s_i} (\int_{\Gamma_{tot}} \frac{1}{s_j} d\Gamma_{tot}) d\Gamma_{ext}$  just once, which only depends on the geometry and multiply it by  $\delta\nu_j$  at each NR iteration.

In order to speed the computations, a matrix compression technique can be used, such as fast multiple method on matrix  $P_m$ . A GMRES iterative solver is then considered and it is accelerated with a LU type preconditioner on the finite element matrix  $R_m$ .

### IV. APPLICATION

The proposed formulation has been applied to a current transformer to validate its accuracy. The reference is the solution obtained with a FEM commercial software with a mesh that has converged.

The current transformer is composed of a magnetic region, a primary coil and a secondary coil as Fig. 2 shows. The primary coil has 1 turn and an imposed current of  $300 \sin(2\pi ft)$  where  $t$  is the time and  $f$  is the frequency, 50 Hz. The secondary coil has 980 turns and it is connected to a resistance of 63  $\Omega$ . The simulation is done for three periods, therefore  $t \in [0, 0.06]$  seconds, with 50 time steps per period. The magnetic region is non linear following an isotropic analytic saturation of

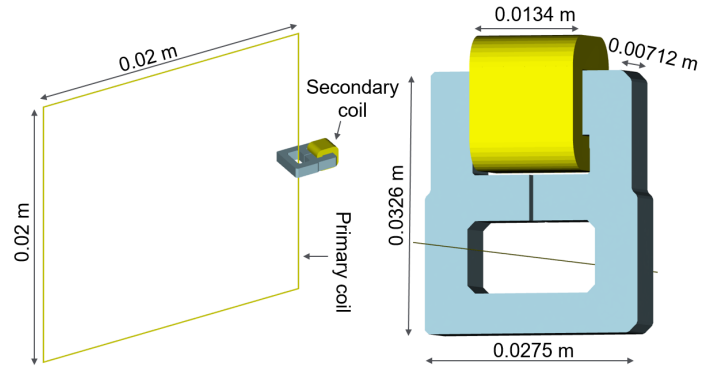


Fig. 2: Current transformer geometry.

arctangent type with two coefficients:  $\mu_r = 100$  and saturation magnetization = 1.2 T.

The converged mesh has 52,581 tetrahedral elements in the magnetic part of the current transformer for VIM. For FEM, the mesh of the magnetic part is similar and there are 261,863 additional elements in the air region.

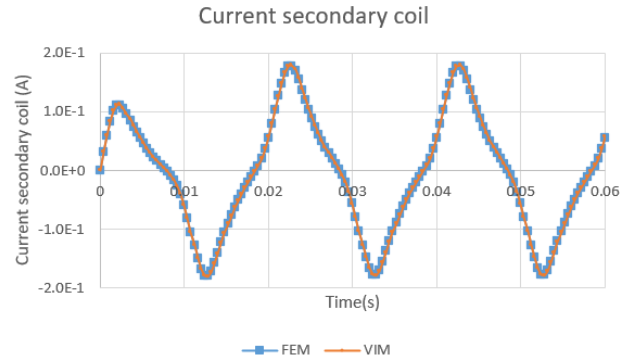


Fig. 3: Current secondary coil for VIM and FEM.

Fig. 3 shows the current of the secondary coil for FEM formulation and the developed formulation. The average relative difference between the two methods is 0.36 % thus validating the VIM developed.

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