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# Multipolar bases of equivalent magnetic sources on arbitrary surfaces

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**Decreasing Spherical Harmonic Functions (DSHF) provide a natural base for the magnetic scalar potential far enough from the sources. This multipolar expansion orders the magnetic field according to its decreasing rate and spatial periodicity. Unfortunately, this representation is not valid close to the sources. In this paper we will construct a basis for double-layer potential distributions on a surface, as close to the sources as needed, which is valid next to the sources while exhibiting a multipolar expansion property.**

**Index Terms**—Double-layer potential, multipolar expansion, near magnetic field computation, spherical Harmonic Functions.

## I. INTRODUCTION

**O**UTSIDE any surface enclosing magnetic sources, if the exterior volume is simply connected, the magnetic field derives from a scalar potential  $\phi$  that verifies Laplace equation  $\Delta\phi = 0$ . In spherical coordinates, far enough from the sources, we can express this potential (and the associated field) in their multipolar expansion using the Decreasing Spherical Harmonic Functions (DSHF) [1]:

$$\phi(r, \theta, \varphi) = \sum_{k=1}^{\infty} \sum_{m=-k}^k a_{km} Y_k^m(\theta, \varphi) / r^{k+1}. \quad (1)$$

For a chosen center of expansion, the spherical harmonic coefficients  $a_{km}$  depend only on the source,  $Y_k^m(\theta, \varphi) / r^{k+1}$  being the DSHF of order  $k$  and degree  $m$  with the typical spatial decreasing law  $1/r^{k+1}$ . Because of this decreasing law any desired level of precision can be achieved at any distance by truncation of the DSHF basis.

Due to its expression with only a few degrees of freedom, this representation is commonly used to solve inverse problem [2] or to get a compact model of complex sources in the context of numerical methods coupling [3], [4].

However, this representation is only valid outside the Brillouin Sphere (BS), the smallest sphere which encloses all sources [5]. As a consequence, the BS limits the use of the DSHF representation when the field must be determined close to sources that are located in a volume with a dimension smaller or greater than the others [3], [4].

In this paper we propose a method to construct a basis of distributions for double-layer potentials on an arbitrary surface closer than the BS which produce a valid field inside the BS while producing a field with similar decreasing property as the DSHF outside the BS.

## II. A MULTIPOLAR BASIS ON ARBITRARY SURFACES

### A. Layer potentials

Let  $S$  be any arbitrary surface which encloses all the sources. We can find a double-layer potential  $\tau$  (equivalent to a normal dipoles distribution) on this surface that produces the same magnetic field (and scalar potential) outside the surface [1].

The scalar potential is then expressed at any exterior point  $P$  using Green function's normal derivative  $\partial_n G$ , with:

$$\phi(P) = \int_{M \in S} \tau(M) \partial_n G(P, M) dS \quad (2)$$

The Green's function for any two points  $M$  and  $P$  in 3D is:

$$G(P, M) = \frac{1}{4\pi} \frac{1}{\|\mathbf{PM}\|}. \quad (3)$$

### B. Construction of the multipolar basis

From the harmonic decomposition of (2), we choose an initial basis for the double-layer potential defined by the projection of the base of increasing spherical harmonic functions on the surface  $S$ :

$$\tau_{km}^0(r, \theta, \varphi) = r^k Y_k^m(\theta, \varphi) \quad (4)$$

If the surface  $S$  has some regularities, the family of distribution  $\tau_{km}^0$  remains a basis of the distributions for the double layer potential [6].

This basis produces the same field as the DSHF in ([?]) if  $S$  is a sphere, but has no noticeable multipolar properties in other cases.

With this starting basis, we use the inner product defined in [7]:

$$\langle \tau_1 | \tau_2 \rangle = \int_{M \in S} \sigma[\tau_1](M) \tau_2(M) dS \quad (5)$$

Where  $\sigma[\tau_1]$  is the single-layer potential on the surface  $S$  that produces the same field as  $\tau_1$  outside of  $S$ .

Using the inner product (5) we orthonormalize the basis  $\tau_{km}^0$  to get the new the basis  $\tau_{km}$  using Gram-Schmidt method. The distributions of  $\tau_{km}^0$  are orthonormalized from the lowest order  $k$ .

This orthonormalization process induces an orthogonality property between the initial basis and the orthonormal one, the distribution of the new orthonormal basis are orthogonal to all the previous distributions of the initial basis.

As the spherical harmonic coefficients  $a_{k'm'}$  of the multipolar expansion of any distribution  $\tau_{km}$  are given by:

$$a_{k'm'} = \langle \tau_{k'm'}^0 | \tau_{km} \rangle, \quad (6)$$

All the spherical harmonic coefficients  $a_{k'm'}$  of order  $k'$  strictly lower than  $k$  are set to 0. Thus, the scalar magnetic

potential produced by the new distribution  $\tau_{km}$  presents no contribution of order strictly inferior to  $k$ , meaning that the spatial decreasing law is at least  $1/r^{k+1}$ . This property is illustrated in 2D on Fig 1, using the order  $k$  spatial periodicity of the circular harmonic field.

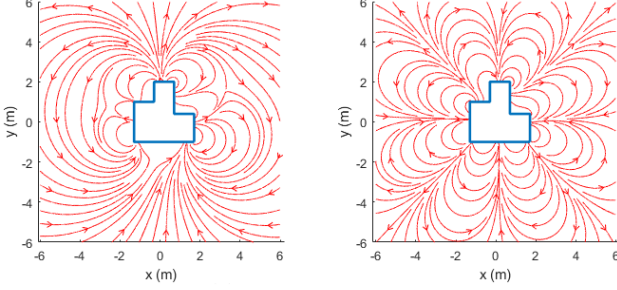


Fig. 1. 2D spatial representation of a fourth order source vector ( $k = 4$ ), field lines (red) produced by the distribution on a non-circular surface  $S$  (blue), before the orthonormalization (left) and after (right).

In the extended paper, further theoretical development on the validity of this basis will be presented as well as another basis for multipolar single-layer potential. Both multipolar bases properties will be compared.

### III. NUMERICAL APPLICATION

We consider a numerical model of a simplified ship hull mock-up, with one dimension higher than the others ( $4.4 \text{ m} \times 0.65 \text{ m} \times 0.6 \text{ m}$ ). The magnetic hull, placed in a low uniform vertical field ( $-15.92 \text{ A/m}$ ), is magnetized and produces its own field in the surroundings (magnetic anomaly).

We enclose the mock-up in a box ( $4.5 \text{ m} \times 0.75 \text{ m} \times 0.7 \text{ m}$ ) which defines the surface  $S$  where the double-layer potential is located. The expected distribution (or reference distribution), shown in figure 2, can be determined using (2) on the surface  $S$ . We will then project this expected equivalent double-layer potential of the anomaly on the new layer harmonics basis.

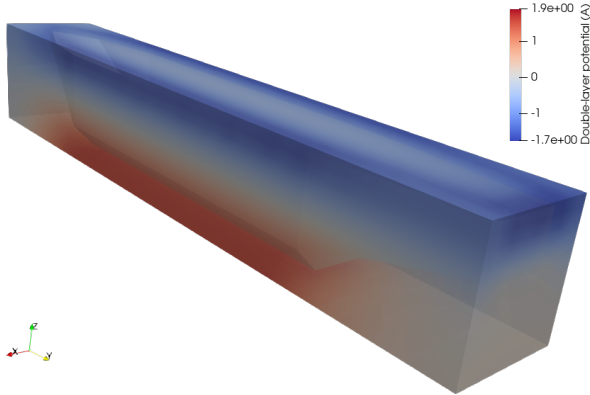


Fig. 2. Mock-up and visualisation of the reference distribution for double-layer potential on the box.

The box is meshed with 2400 quadrangles (2402 nodes) so the reference distribution is represented with 2401 degrees of freedom by linear nodal shape functions. For the multipolar basis the number of degrees of freedom is  $K(K + 2)$  with

$K$  the order for the truncation, here the maximum amount of degrees of freedom is 168 ( $K = 12$ ).

For the results, the relative error presented is the absolute difference between the module of the field produced by the reference distribution and the one produced by the layer harmonic truncated basis, divided by the maximum (on the computation surface) of the module of the field produced by the reference distribution.

#### A. Results outside the Brillouin Sphere

We compute the field produced by the layer harmonics on a sphere of radius 12.5 m. As expected, only the lowest orders are needed to correctly describe the field. In particular, the first order of the surface bases reconstructs the field with an error lower than 3%, with the associated decreasing law of a magnetic dipole.

#### B. Results inside the Brillouin Sphere

We compute the field produced by the layer harmonics on a rectangle of dimensions  $2 \text{ m} \times 7 \text{ m}$  along  $x$  and  $y$  axis and located 1 m below the box. Results (Table I) show that the field close to the mockup, inside the BS, is obtained with a precision lower than 1% with a maximum truncation order for the surface basis, superior or equal to 8. This means that only 80 layer harmonic coefficients must be determined to describe the field so close to the mock-up, which is very convenient for a future purpose of identification of these terms by solving an inverse problem from close field measurements.

TABLE I  
MAXIMUM RELATIVE ERROR ON THE PLANE INSIDE THE BS AGAINST THE MAXIMUM ORDER OF TRUNCATION

Order for the truncation	maximum relative error (%)	Order for the truncation	maximum relative error (%)
1	22.6	7	1.6
2	16.3	8	0.8
3	8.5	9	0.9
4	6.3	10	0.8
5	5.4	11	0.6
6	3.9	12	0.4

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