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A Level-Set Based Topology Optimization for Maximizing the Torque of Switch Reluctance Machines

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This paper presents a topology optimization algorithm based on a Level-Set method and continuum sensitivity analysis in a 2D magnetostatic finite element context. The novelty is the use of a fully analytical expression for the sensitivity analysis based on the virtual work principle and applied to a Level-Set implementation. The numerical study of a switch reluctance machine and the virtual work method to compute the torque are presented. Then, the Level-Set method is detailed in a general context. The corresponding continuum sensitivity analysis using the adjoint method to maximize the torque is expressed. Finally, the application of this method to the optimization of the rotor of a switch reluctance machine is shown.

Index Terms—Topology Optimization, Torque Sensitivity Analysis, Level-Set method, Switch Reluctance Machines.

I. INTRODUCTION

SWITCH RELUCTANCE MACHINES (SRM) have many assets. They offer interesting performances and do not need the use of permanent magnets which are rare and expensive. However, their torque is mainly determined by the topology of their rotor. This topology can be hard to guess and requires an expensive design time for engineers. Thus, topology optimization seems to be a really interesting tool to discover new designs because it completely avoids the need of any prior intuition on the optimal design. Some topology optimization algorithms use density-based methods [1] or a Level-Set (LS) method [2], [3], [4] which all need the sensitivity analysis (SA) of the optimization objective with respect to the optimization variables. The most widely used method for the computation and the SA of the torque is the Maxwell stress tensor [1], [2]. The virtual work method (VWM) [5] is an alternative to this method. It is known to have a better numerical behavior and is not dependent on the choice of a surrounding computation path. In a previous work [6], a discrete SA based on the VWM has been presented and detailed in the free shape optimization case. The contribution of this paper is the presentation of a continuous version of this SA in a topology optimization context and its application on a LS optimization algorithm. In the first section, the 2D magnetostatic numerical analysis and torque computation via the VWM of a SRM are presented. Then, the complete LS method and SA is presented. Finally, the last section presents the application of this method to the optimization of a SRM.

II. NUMERICAL MODEL

A. Finite element model

A numerical model of a SRM is made of distinct parts. The stator Ω_S is made of iron and current sources. It is the fixed part of the machine. The rotor Ω_R is the most important part for the design of the SRM since its topology mainly determines the performances of the motor. It is made of iron. The goal of topology optimization is precisely to find the best design of the rotor. The last parts are the air gap Ω_A and the shaft Ω_T . The air gap is numerically important since the torque is computed in this area. The complete domain of study $\Omega_S \cup \Omega_R \cup \Omega_A \cup \Omega_T$ is denoted Ω .

The performances of the depend on the angular position of the rotor. This rotation is discretized using $N = 120$ angular positions $(\theta_i)_{1 \leq i \leq N}$ over $[0, 2\pi]$. The motor is studied under 2D nonlinear magnetostatic conditions. This nonlinearity leads to N independant nonlinear equations to solve. The Newton-Raphson (NR) algorithm is used to solve the N positions. The iterative NR assembly is built from the following classical weak formulation of the 2D magnetostatic at each angular position i :

$$\forall A' \in \mathbf{H}_0^1(\Omega), \int_{\Omega} [\nabla \times (\nu(|\nabla \times A_i|)\nabla \times A_i) - \mathbf{J}] \cdot A' d\Omega = 0$$

$$\Leftrightarrow h_i(\nu(A_i), A_i, \mathbf{J})(A') = 0, \quad (1)$$

with $1 \leq i \leq N$, A_i the 2D magnetic vector potential such that $\mathbf{B}_i = \nabla \times A_i$ with \mathbf{B}_i the magnetic induction (T), ν the reluctivity following the nonlinear $\mathbf{B}(|\mathbf{H}|)$ laws in the iron parts and $\nu = \nu_0$ the reluctivity in air parts and \mathbf{J} the supply current density ($A.m^{-2}$).

B. Computation of the torque

The main target of the optimization is to improve the torque of the SRM. It is computed at angular position i using the VWM [5], [6]:

$$T_i = \sum_e \int_{\Omega_e} \nu \mathbf{B}_i^T M_e \mathbf{B}_i d\Omega \quad (2)$$

$$\text{with } M_e = -G_e^{-1} \frac{\partial G_e}{\partial \omega} + \frac{1}{2|G_e|} \frac{\partial |G_e|}{\partial \omega} I,$$

where G_e is the Jacobian matrix of the transformation from the mesh element Ω_e to the corresponding reference element Δ_e [5], $|G_e|$ its determinant, I the identity matrix and ω the virtual rotation of the rotor around its rotation axis. Note that the support of this formula is in the mesh element layer surrounding Ω_R . From the torque T_i at each position, we define global quantities T_{mean} , T_{max} and T_{min} which are respectively the average, maximum and minimum torque values over the whole rotation. The average torque is an interesting measure of the overall performance of the SRM. The torque ripple $T_r = (T_{max} - T_{min})/T_{mean}$ is correlated to the SRM noise and has to be constrained during the optimization.

III. TOPOLOGY OPTIMIZATION

A. The Level-Set method

The principle of the LS method is to track the design of the rotor using an implicit definition of the design boundaries. The LS method presented in [3] is used. A Level-Set function $\phi : \mathbb{R} \times \Omega_R \mapsto \mathbb{R}$ is defined such that:

$$\begin{aligned} \phi(t, x) &< 0 \text{ if } x \text{ in air,} \\ \phi(t, x) &> 0 \text{ if } x \text{ in material,} \\ \phi(t, x) &= 0 \text{ if } x \text{ at boundary,} \\ \phi(0, x) &\text{ matches with the input design,} \end{aligned}$$

where t is a pseudo-temporal variable used to advect the design. Using a smooth Heaviside operator \mathcal{H} to project ϕ on mesh elements, it is possible to link ϕ with the reluctivity ν in a similar way to a more classical element density method:

$$\nu(x) = \nu_0 + (\hat{\nu}(x) - \nu_0)\mathcal{H}(\phi) \quad (3)$$

where $\hat{\nu}$ is the reluctivity of the considered nonlinear material and $\mathcal{H}(\phi)$ acts as an element density. Based on [3], ϕ is discretized in a basis of radial functions $(\psi_i(x))_{1 \leq i \leq n}$ defined on the n nodes of the mesh of Ω_R . Thus, ϕ can be written as:

$$\phi = \sum_{i=1}^n \alpha_i \psi_i. \quad (4)$$

ϕ is determined by the coefficient $\alpha = (\alpha_i)_{1 \leq i \leq n}$. Thus, the optimization variable is α . The following optimization problem is considered:

$$\begin{aligned} &\underset{\alpha}{\text{maximize}} \quad T_{\text{mean}}(\mathbf{A}, \alpha) \\ &\text{subject to} \quad T_r(X_i) \leq T_r^{\text{max}} \end{aligned} \quad (5)$$

where T_r^{max} is a given maximum torque ripple value and \mathbf{A} is the magnetic potential vector solution of (1). \mathbf{A} depends implicitly on ϕ through equation (3). The advection process of ϕ , detailed in [3], requires the sensitivity of the objective with respect to the objective T_{mean} and constraint T_r^{max} . Since they are both functions of the T_i values, the key point of the method is to express the sensitivity $\frac{dT_i}{d\alpha}$. This kind of sensitivity analysis under magnetostatic equation constraint (1) has become a classic problem. It is well-known that the adjoint method can be used [1], [6], [7], which is presented in the next section.

IV. SENSITIVITY ANALYSIS OF THE TORQUE

A. Length scale control and projection

Before the smooth projection step \mathcal{H} , a length scale control operator \mathcal{F} similar to [4] is introduced on the α_i to avoid small features such as small holes or hinges, and enforce numerical stability of the method. Let's introduce $\mathbf{v} = \mathcal{F}(\alpha)$. This filtering process, presented in [4], will be detailed in the full paper. Thus, the relation between $\mathcal{H}(\phi)$ and the α_i holds:

$$\rho = \mathcal{H}(\mathcal{F}(\alpha)) = \mathcal{H}(\mathbf{v}), \quad (6)$$

and the sensitivity can be expressed using the chain rule:

$$\frac{dT_i}{d\alpha} = \frac{dT_i}{d\rho^T} \frac{d\rho}{d\mathbf{v}^T} \frac{d\mathbf{v}}{d\alpha}. \quad (7)$$

Since operators \mathcal{H} , \mathcal{F} and their derivatives are known, the only unknown term is $\frac{dT_i}{d\rho^T}$. It is presented in the next section.

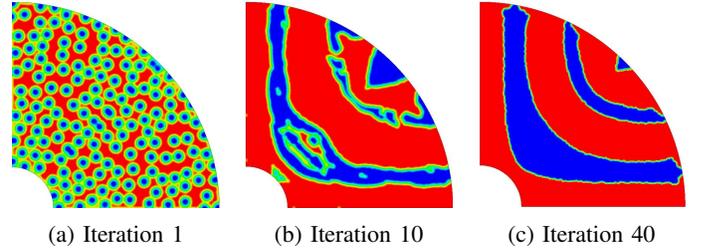


Fig. 1: Initial, intermediate and final views of the rotor (red: $\phi > 0$ (material), blue: $\phi < 0$ (air), green: $\phi \approx 0$ (boundaries))

B. Sensitivity with respect to the density

The term $\frac{dT_i}{d\rho}$, is similar to the SA computed in more classical density based methods [1], [7]. The adjoint method is used. The novelty of this work, aside from the use of the LS, is the expression and use of a continuous version of the SA presented in our recent work [6] in a topology optimization context. The sensitivity in a direction $k \in \mathbf{L}^\infty(\Omega_R)$ reads:

$$\left\langle \frac{dT_i}{d\rho}, k \right\rangle = \int_{\Omega} (\hat{\nu} - \nu_0) \nabla \times \mathbf{A} \cdot \nabla \times \boldsymbol{\lambda} \cdot k \, d\Omega \quad (8)$$

$$\forall u \in \mathbf{H}_0^1(\Omega), h_i \left(\nu(\mathbf{A}), \boldsymbol{\lambda}, \frac{\partial T_i}{\partial \mathbf{A}}(\mathbf{A}, \rho) \right) (u) = 0 \quad (9)$$

$$\left\langle \frac{\partial T_i}{\partial \mathbf{A}}(\mathbf{A}, \rho), u \right\rangle = \int_{\Omega} \nu \mathbf{B} (M_e + M_e^T) \nabla \times u \, d\Omega, \quad (10)$$

where \mathbf{A} is the physical solution to (1), $\mathbf{B} = \nabla \times \mathbf{A}$ and $\boldsymbol{\lambda}$ the adjoint state solution to the linear problem (9). Proofs of expressions (8),(9) and (10) will be given in the full paper.

V. APPLICATION

The LS method III-A and sensitivity (7)-(10) of the torque have been implemented using Altair FluxTM and OptistructTM. The optimization problem (5) is considered to design the rotor Ω_R of a SRM model. At each optimization iteration, the sensitivity is used to slightly advect the current design using the LS method. The process iterates starting from this new design until an optima is found. Figure 1 shows rotor design for different optimization iterations. The final design satisfies the optimization objectives and is similar to existing SRM designs. However, the final design has poor mechanical strength. This issue could be tackled by adding mechanical constraints to (5).

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