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# Use of state circuit solver and model order reduction for solving inductive and capacitive unstructured PEEC formulation

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Computational strategies improvements for the inductive and capacitive unstructured PEEC formulation are presented in order to address efficiently a large frequency range of electromagnetic problems. Good accuracy on results is ensured thanks to the use of an adaptive Gauss integration procedure while multi-threaded Adaptive Multi Level Fast Multipole Method (AMLFMM) matrix compression algorithm allows to speed-up far interactions computation. This article focuses on the following two points: on the one hand, allowing good convergence for iterative linear systems at any frequency by the choice of an efficient preconditioner coupled with a suitable PEEC circuit solver and, on the other hand, the use of Model Order Reduction (MOR) techniques for solving multi frequency problems, allowing fast computing time. An example illustrates the performances of these approaches.

**Index Terms**—Electromagnetic problem, Unstructured Partial Element Equivalent Circuit method, State circuit solver, Model Order Reduction.

## I. INTRODUCTION

THE Partial Element Equivalent Circuit (PEEC) method has been shown to be well suited for the analysis of many electromagnetic devices, such as busbars, PCBs, integrated circuit interconnects, packaging and others [1]. Since the PEEC method is an integral volume method, it is not necessary to mesh the air surrounding the conductive parts. Furthermore, an unstructured PEEC method has been developed [2], [3], removing the limitation of simple geometries, thus allowing the modeling a wider range of applications. The major drawback of full matrix of interactions has been overcome thanks to efficient matrix compression algorithms like AMLFMM. Moreover, accurately integration of the interactions using adaptive and efficient strategies has been developed [4].

In this paper, inductive and capacitive unstructured PEEC formulation is considered (the propagation phenomena are neglected), by using compression technics and adaptive integration. But a challenge remains in term of performance computing when a large range of computation have to be done, for instance to obtain impedance matrix representation of an electromagnetic system on a wide frequency range. To overcome these difficulties, this work proposes the use of circuit state solvers to achieve two objectives:

- obtain good convergence of large iterative linear systems at any frequency,
- use of Model Order Reduction when solving a multi-frequency problem, allowing fast computation time.

## II. UNSTRUCTURED PEEC FORMULATION

We consider a set of massive conductive volume regions. From Maxwell equations and with the use of Lorentz gauge, the electric field  $E$  can be expressed with integrals depending of the current density  $J$  [5]. By discretizing the region with a finite element mesh and interpolating  $J$  with facet elements (Whitney 2-form [4]) the problem, for frequency  $f$ , can then be

represented as an equivalent electric circuit, whose nodes and branches are coming from the dual mesh [3]:

$$\begin{bmatrix} R + j\omega L & 0 \\ 0 & \frac{P}{j\omega} \end{bmatrix} \{I\} = \{\Delta V\} \quad (1)$$

with  $\omega = 2\pi f$ .  $R$  is a resistive sparse finite element matrix,  $L$  an inductive integral matrix and  $P$  an elastance (inverse of capacitance) integral matrix.  $I$  represents the branch currents and  $\Delta V$  the potential differences on these same branches. The capacitive branches that connect the outer faces of the domain to infinity are distinct from the  $R+L$  branches coming from the conductive regions. Note that integral matrices  $L$  and  $P$  are not depending of the frequency, which allows to integrate them only one time when solving multi-frequency problem.

## III. CIRCUIT SOLVER AND PRECONDITIONING

Voltage sources  $U_{su}$  or current sources  $I_{si}$  as well as passive external resistances, inductances and capacitances components can be easily connect to the previous circuit and complete previous  $R$ ,  $L$  and  $P$  matrices. Then the global circuit can be solved by using a standard circuit solver, based on independent loops or independent nodes technics. In this work we have developed circuit solvers based on a state representation, where the system of equations comes in the generic form:

$$[j\omega[E] + [A]]\{x\} = [B]\{u\} \quad (2)$$

Where  $\{u\}$  is an input source vector, in our case composed with the voltage sources  $U_{su}$  and current sources  $I_{si}$ . An advantage is that there is no term multiplied by  $1/j\omega$  and this can be useful for better convergence at any frequency. A nodal state solver, which uses the nodal potential  $V$  and a node-branch matrix of incidence  $N$  such that  $N^t \cdot \{V\} = \{\Delta V\}$  can be written as:

$$\left[ j\omega \begin{bmatrix} 0 & 0 & N_p & 0 \\ 0 & L & 0 & 0 \\ N_p^t & 0 & P & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & N_{RL} & 0 & N_U \\ N_{RL}^t & R & 0 & 0 \\ 0 & 0 & 0 & 0 \\ N_U^t & 0 & 0 & 0 \end{bmatrix} \right] \begin{bmatrix} V \\ I_{RL} \\ I_p/j\omega \\ I_{su} \end{bmatrix} = \begin{bmatrix} N_f \cdot I_{si} \\ 0 \\ 0 \\ U_{su} \end{bmatrix}$$

$N_{RL}, N_p, N_U, N_f$  are partial incidence matrices respectively for conductive region branches, capacitive branches, voltage sources branches and current source branches.  $I_{RL}, I_p, I_{su}, I_{si}$  are the corresponding branch currents. Similar system of equations using incidence mesh-branches matrices can be built for an independent loops state solver.

Our preconditioning strategy consist to find approximate sparse matrices for  $[A]$  and  $[E]$  and to process LU decomposition on the sparse global matrix  $j\omega[E_{sparse}] + [A_{sparse}]$  for each frequency of solving. In practice we keep the sparse finite element matrix  $R$  or its diagonal and conserve all sparse incidence matrices. For integral matrices  $L$  and  $P$ , two strategies can be applied : keep diagonal terms or get near-interactions terms coming from compression algorithm. This last approach allows to reduce the number of iterations, but can lead to prohibitive time computation for LU decomposition in presence of large systems.

Finally, the use of preconditioned state solvers makes it possible to obtain a reduced number of iterations compared to standard circuit solvers, in particular when taking account of capacitive effects at low frequencies.

#### IV. MODEL ORDER REDUCTION (MOR)

Since the circuit solver is written as a state system, the global system of equations can be represented in a reduced state form [6]. For this, in addition to the matrices  $[A][E][B]$  and the source vector  $\{u\}$ , we have to define an output vector  $\{y\}$  and build a matrix  $[C]$  relying this output to the unknowns of the system such as  $\{y\} = [C] \{x\}$ . In our case, the output vector  $\{y\}$  is composed with the currents of voltage sources  $I_{su}$  and the voltages of the current sources  $U_{si}$ . The model order reduction algorithm allows building a reduced state system  $[A^r], [E^r], [B^r], [C^r]$  which can be obtained by solving the global system on a adapted number of frequencies [6]. The size of the reduced system is depending of the number of sources but remains small in comparison with the initial system. Its resolution allows the computation of output quantities  $\{y\}$  for a large number of frequencies almost instantaneously, by solving the reduced system:

$$[j\omega[E^r] + [A^r]]\{x^r\} = [B^r]\{u\} \quad \{y\} = [C^r] \{x^r\} \quad (3)$$

#### V. EXAMPLE

We consider two volumic conductors fed by sources voltage (Figure 1), with capacitances circuit add in serie on the two sources, in order to create some resonances. Conductive regions are meshed with 35 000 tetrahedra. Simulations have been made on 100 frequencies between 100 Hz et 10 kHz. An iterative GMRES solver preconditioned with the diagonals of the integral matrices is used. The average number of iterations for solving the 100 linear systems is about 12 for a nodal state

independent approach (97 000 unknowns). The use of Model Order Reduction needs the solving of 6 linear systems for build a reduced system with 8 unknowns. Figure 2 compare the real and imaginary current flowing in one of the two sources and shows that the two simulation results are very closed, the relative difference of current being  $<0.5\%$  for all frequencies. Table I compares the simulation times of the two approaches.

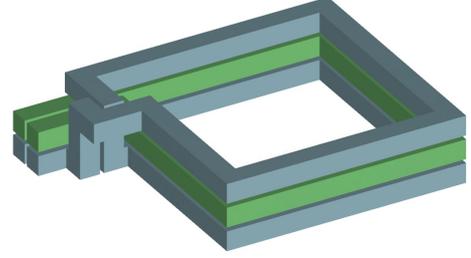


Fig. 1. Problem solve with 100 frequencies from 100Hz to 10KHz.

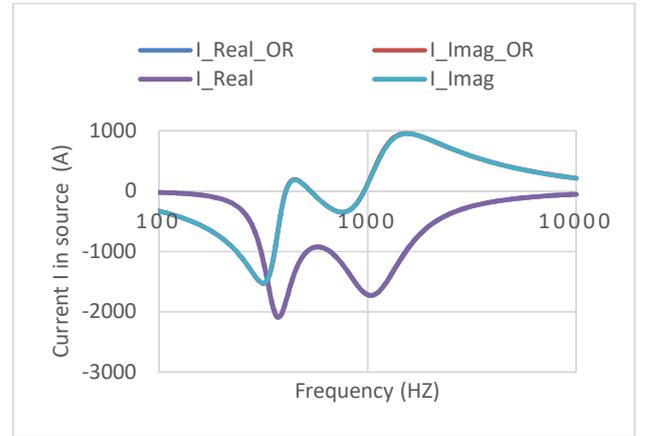


Fig. 2. Current in source comparison: with or without Order Reduction (OR)

TABLE I  
COMPUTATION PERFORMANCES FOR SOLVING 100 FREQUENCIES

	Integration time	Number of linear systems solved	Linear systems total time	Precond LU decomposition total time
standard	140 s	100	1250 s	102 s
MOR	140 s	6 + 100 reduced	149 s	7 s

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