



# Shock formation in flowing plasmas by temporally and spatially smoothed laser beams

J D Ludwig, S Hüller, H A Rose, C Bruulsema, W Farmer, P Michel, A Milder, G F Swadling, W Rozmus

## ► To cite this version:

J D Ludwig, S Hüller, H A Rose, C Bruulsema, W Farmer, et al.. Shock formation in flowing plasmas by temporally and spatially smoothed laser beams. *Physics of Plasmas*, 2024, 31, pp.032103. 10.1063/5.0189559 . hal-04255663v2

**HAL Id: hal-04255663**

**<https://cnrs.hal.science/hal-04255663v2>**

Submitted on 12 Feb 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Shock formation in flowing plasmas by temporally and spatially smoothed laser beams

\*J. D. Ludwig,<sup>1</sup> S. Hüller,<sup>2</sup> H. A. Rose,<sup>3</sup> C. Bruulsema,<sup>1</sup> W. Farmer,<sup>1</sup> P. Michel,<sup>1</sup> A. L. Milder,<sup>4</sup> G. F. Swadling,<sup>1</sup> and W. Rozmus<sup>4</sup>

<sup>1</sup>*Lawrence Livermore National Laboratory, 7000 East Avenue, Livermore, California 94551, USA<sup>a)</sup>*

<sup>2</sup>*Centre de Physique Théorique CPHT, CNRS, Ecole polytechnique, Institut Polytechnique Paris, 91128 Palaiseau Cedex, France*

<sup>3</sup>*Los Alamos National Laboratory, Los Alamos, New Mexico, USA 87545*

<sup>4</sup>*Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 2E1*

(Dated: 24 January 2024)

The cumulative impact of multiple laser speckles on a supersonic plasma flow across optically smoothed laser beams is investigated. The bending of laser beams caused by ponderomotive laser-plasma coupling, together with flow, lead to plasma a momentum conserving response that results in a deceleration of the flow. Once the flow velocity decreases to a subsonic level, the action of the laser beams can generate a shock within the plasma. This scenario has been predicted theoretically and confirmed by hydrodynamic simulations. The conditions of shock generation are given in terms of the ponderomotive pressure, speckle size and the flow velocity. The nonlinear properties of the shocks are analyzed using Rankine-Hugoniot relations. According to linear theory, temporally smoothed beams exhibit a higher threshold for shock generation. Numerical simulations with beams that are smoothed by spectral dispersion compare well with the linear theory results, diverging only in the nonlinear regime. The conditions necessary for shock generation and their effects on the laser plasma coupling in the inertial confinement fusion experiments are also discussed.

Keywords: Laser Shock, beam bending, laser plasma interaction of optically smoothing laser beams

## I. INTRODUCTION

Inertial Confinement Fusion (ICF) faces challenges with implosion symmetry, energy coupling and variety of laser plasma-interaction (LPI) processes that become experimentally accessible and relevant for the performance of the ICF targets. For example, implosion symmetry is known to be strongly affected by Cross Beam Energy Transfer (CBET). Energy transfer between laser beams occurs when an ion acoustic wave is in resonance with the ponderomotive force due to overlapping laser beams. In the case of intersecting beams with equal frequency, a plasma flow at the ion sound speed can Doppler shifts the ion acoustic frequency and to establish this resonance<sup>1–7</sup>. In a flowing plasma, the process of beam bending in the direction of the flow velocity has been examined in theory and experiments<sup>8–11</sup>. The density perturbations resulting from the ponderomotive force of the laser are skewed by flow, this redirects the laser beam and by momentum conservation introduces a drag on the plasma flow slowing it down. The rate of beam bending is maximized when the plasma flow is at the sound speed, resulting in the largest drag and flow deceleration. As the flow transitions from supersonic to subsonic velocity, conditions conducive to shock generation in a plasma are formed. Such shocks and related density and velocity perturbations have not been examined so far and are the main focus of our work.

While a shock can be generated by a coherent laser beam,

much smaller beam intensity and energy are required for shock formation in speckled beams, as produced by the use of a random phase plate (RPP). This effect is attributed to the stronger ponderomotive force exerted on the plasma by the small-scale speckles generated in such beams. Even though local flow perturbations on the scale of individual speckles may be minimal, their cumulative effect, especially when the flow interacts with large-scale ICF beams, will lead to shock formation<sup>12–14</sup>. Furthermore, shock generation can be intensified in the region where beams intersect. Crossing beams results in a reduction in the effective size of the speckles. This reduction amplifies the ponderomotive force on the flow, which is further augmented by the increased laser intensity due to beam overlap. In ICF experiments, temporal smoothing techniques like smoothing by spectral dispersion (SSD) are applied to large-scale laser beams. Our investigation into the effects of SSD on shock generation, through linear theory calculations, has revealed an enhancement of threshold conditions and an increase in the required penetration length of the plasma flow across the laser beam.

The shocks discussed in our article originate gradually within the plasma as it traverses a speckled laser beam as shown in the geometry of Fig. 1. The laser induces perturbations in both density and flow velocity. Upon separation from the laser, the perturbations take on the form of a bow shock enveloping the whole beam, extending beyond the laser and potentially influencing other LPI processes. The underlying physical mechanisms for shock formation are common in ICF experiments. However, detecting freely propagating shocks in experiments may necessitate specific experimental configurations and precise plasma parameters. For instance,

---

<sup>a)</sup>Electronic mail: ludwig8@lnl.gov

our theoretical model of an isothermal plasma, displays a relatively slow velocity of nonlinear shock propagation, thereby amplifying the energy requirements for the laser beam by increasing its required time duration.

The rest of this article is organized as follows: in sections II and III we present the theoretical basis for beam deflection and its action on the plasma flow. In section IV, results from numerical simulations are presented and analyzed. Section V concludes the article with a discussion of implications for ICF experiments.

## II. TRANSVERSE PLASMA FLOW AND BEAM DEFLECTION

In the limit of low-frequency plasma response for cold ions in a collisionless laser-produced plasma, the equations of the fluid model for density and momentum conservation,

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{v}_i) = 0, \quad \frac{\partial \vec{v}_i}{\partial t} + (\vec{v}_i \cdot \nabla) \vec{v}_i = -\frac{Ze}{m_i} \nabla \phi, \quad (1)$$

can be simplified by assuming the case of an isothermal plasma<sup>15–17</sup>. Thus the electrostatic potential  $\phi$  together with the ponderomotive potential  $U$  of the laser quiver motion relate to the electron density  $n_e$  via the Boltzmann factor

$$n_e = n_0 \exp(e\phi/T_e - U/T_e), \quad (2)$$

with  $T_e$  and  $n_0$  denoting the electron temperature and the equilibrium density, respectively.

The ponderomotive potential  $U$  is defined in terms of the laser electric field  $\vec{\mathcal{E}} = \frac{1}{2}[\vec{E}e^{-i\omega_0 t} + c.c.]$  with its amplitude  $\vec{E}$  as

$$U = e^2 |\vec{E}|^2 / (4m_e \omega_0^2). \quad (3)$$

The electromagnetic field amplitude  $\vec{E}$  satisfies the time enveloped wave equation,

$$2i \frac{\partial \vec{E}}{\partial t} + (c^2/\omega_0) \nabla^2 \vec{E} + \omega_0(1 - n_e/n_c) \vec{E} = 0. \quad (4)$$

The model presented above, based on the ion fluid equations (1) coupled to the electromagnetic field through the ponderomotive force, the Boltzmann electron distribution Eq. (2) incorporating the ponderomotive potential  $U$ , and the high-frequency enveloped wave equation (4), conserves the total momentum. This includes both the momenta of the flowing plasma and of the radiation field<sup>17</sup>. This theoretical framework serves as the basis for our study, wherein we investigate how the slight deflection of the electromagnetic momentum flux associated with beam bending induces an opposing change in momentum by decelerating the plasma flow.

The coupling between the laser beam and the plasma motion scales with the magnitude of the ponderomotive potential in terms of  $U/T_e$  as in Eq. (2), which can be given in practical units as  $U/T_e \equiv 0.09 \langle I \rangle \lambda^2 (10^{15} \text{ W } \mu\text{m}^2/\text{cm}^2) / (T_e/\text{keV})$ , with  $\langle I \rangle$  the laser intensity and the wave length  $\lambda$ .

In a rough preliminary estimate that disregards the details of laser-plasma coupling physics, we can compare a small change in the average laser field momentum flux due to beam deflection to the change in the momentum flux of the flowing plasma. For an angular deflection in terms of the wave vector ratio,  $k_\perp/k_0$ , the change in the field momentum can be expressed as  $(k_\perp/k_0)\langle I \rangle/c$ , with  $\langle I \rangle = c\epsilon_0 \langle E^2 \rangle/2$  denoting the laser intensity, and can be compared with  $n_i m_i c_s^2 \delta M$  which results from the change in the ion momentum flux by the change in the flow velocity normalized to the speed of sound  $\delta M$ . In the proximity of the sonic velocity layer where  $v_i \approx c_s$ , this results in an effective reduction of the flow. Comparing the two momentum changes yields  $\delta M = (k_\perp/k_0)(\langle I \rangle/5 \times 10^{14} \text{ W/cm}^2)(10^{20} \text{ cm}^{-3}/n_i)(1 \text{ keV}/T_e)$ . Considering a deflection comparable to the angular aperture  $k_\perp/k_0 \sim 1/(2f)$ , the corresponding flow reduction  $\delta M$  would be  $\sim 0.125$  for  $\langle I \rangle \approx 10^{15} \text{ W/cm}^2$ ,  $n_i = 10^{20} \text{ cm}^{-3}$ ,  $f = 8$  and  $T_e = 1 \text{ keV}$ . Despite the rough nature of this estimate, this results shows the relevance of the process for the regime of high-energy NIF beams.

When transverse fluctuations in a laser field are large compared to fluctuations along the propagation axis, the hydrodynamic model may be simplified to two dimensions, so that only the fluid momentum component perpendicular to the laser axis is effected. The direction of the propagation of a laser is along the z-axis and the flow velocity will be in the x-y plane (cf. Fig. 1).

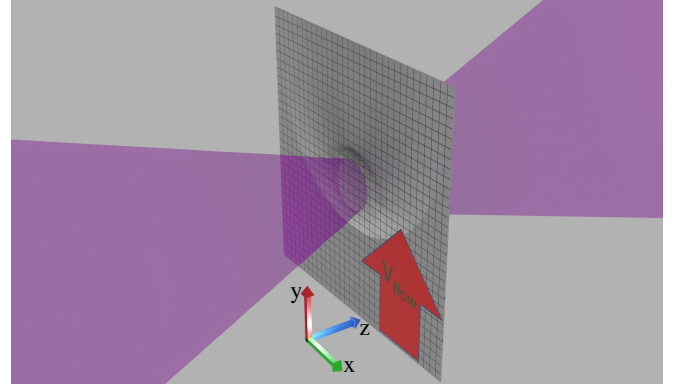


FIG. 1. Sketch of the geometry in numerical simulations: the laser is propagating along z and is passing through the plasma located around the focal region. This is represented in the simulations by a two-dimensional (2D) domain (plane bisecting laser axis) in the x-y plane of the laser beam cross section at best focus. The plasma flow is in the vertical direction (arrow) within the simulation plane and perpendicular to the laser propagation direction.

By denoting  $\vec{p}_\perp = \rho \vec{v}_\perp$  as the plasma fluid momentum density in the direction perpendicular to the laser propagation and  $c_s = (ZT_e/m_i)^{1/2}$  as the sound speed, with  $\rho \equiv n_e/n_0$  the isothermal fluid equations (1) with Eq. (2) read,

$$\frac{\partial \vec{p}_\perp}{\partial t} + \nabla_\perp \cdot (\vec{v}_\perp \vec{p}_\perp) = -c_s^2 \rho \nabla_\perp \left( \ln \rho + \frac{U}{T_e} \right), \quad (5)$$

$$\frac{\partial \rho}{\partial t} + \nabla_\perp \cdot \vec{p}_\perp = 0. \quad (6)$$

In absence of flow, a stationary equilibrium can be reached via balance between the local density and the ponderomotive

potential, namely  $\rho_0 = (n_e/n_0)_{v_\perp=0} \equiv \exp(-U/T_e)$ . It has, however, been shown that flow can considerably modify the response of the plasma fluid<sup>8,9,18-20</sup>.

To obtain analytical results it is useful to consider in the fluid equations (5),(6) a small velocity perturbation and to linearize the momentum equation about background flow velocity,  $\vec{v}_0$  in the x - y plane,  $\vec{v}_\perp = \vec{v}_0 + \delta\vec{v}_\perp$ . This leads to the following set of equations<sup>9</sup>

$$\left(\frac{\partial}{\partial t} + \vec{v}_0 \cdot \nabla_\perp\right) \ln \rho + \nabla_\perp \cdot \delta\vec{v}_\perp = 0, \quad (7)$$

$$\left(\frac{\partial}{\partial t} + \vec{v}_0 \cdot \nabla_\perp\right) \delta\vec{v}_\perp + 2v_{ia}\delta\vec{v}_\perp = -c_s^2 \nabla_\perp \left(\ln \rho + \frac{U}{T_e}\right) \quad (8)$$

where  $v_{ia}$  is a spatial convolution operator approximating Landau damping of ion acoustic perturbations. The time independent solution of Eqs. (7), (8) in Fourier space reads<sup>9</sup>

$$(\ln \rho)_{k_\perp} = \frac{(\ln \rho_0)_{k_\perp}}{1 - (k_y/k_\perp)M(k_y M/k_\perp - 2i\hat{v})}, \quad (9)$$

where the background flow  $\vec{v}_0$  is chosen along the y-axis, the Mach number  $M = v_0/c_s$  and the normalized damping operator  $\hat{v} = v_{ia}/(kc_s)$ . The density perturbation Eq. (9) displays a skewed profile due to the flow compared to  $\rho_0$  in the stationary plasma and the ponderomotive potential  $U$  of a single laser speckle. This asymmetry in density perturbations averaged over speckles of the randomized laser beam leads to beam bending. In the small angle approximation the beam bending can be quantified by the average angular deflection rate<sup>9</sup>

$$\frac{\partial \langle \theta \rangle}{\partial z} = \frac{\partial}{\partial z} \left\langle \frac{\vec{k}_\perp}{k_0} \right\rangle_k \cdot \frac{\vec{v}_0}{|\vec{v}_0|} = -\frac{1}{2} \frac{n_0}{n_c} \left\langle \nabla_\perp \frac{\delta n}{n_0} \right\rangle_r \cdot \frac{\vec{v}_0}{|\vec{v}_0|}, \quad (10)$$

where the small density perturbation  $\delta n/n_0 = \rho - 1$  or alternatively  $\ln \rho \approx \delta n/n_0$ . The average in the two dimensional k-space of any function  $h(\vec{k}_\perp)$  is calculated according to the following expression

$$\langle h \rangle_k(z) = \frac{\int d^2 k_\perp |E(\vec{k}_\perp, z)|^2 h(\vec{k}_\perp)}{\int d^2 k_\perp |E(\vec{k}_\perp, z)|^2}, \quad (11)$$

while the spatial average of any function  $g(\vec{r}_\perp)$  reads

$$\langle g \rangle_r(z) = \frac{\int d^2 r_\perp |E(\vec{r}_\perp, z)|^2 g(\vec{r}_\perp)}{\int d^2 r_\perp |E(\vec{r}_\perp, z)|^2}. \quad (12)$$

The rate of beam deflection Eq. (10) and the averaging defined in Eqs. (11), (12) involve the electric field amplitude  $\vec{E}$  obtained from the wave equation (4) in the paraxial approximation.

### III. DRAG ON PLASMA FLOW DUE TO BEAM DEFLECTION

Analytical results in this Section are derived using an expression for the beam deflection rate, Eq.(10). Furthermore

we use the results from section V.A of Ref. 13, in particular Eq. (157), which was derived for the ponderomotive potential assuming a stationary, speckled field pattern with weak amplitude representing a beam generated by a random phase plate (RPP). Equation (10) yields then the following expression for  $M > 1$

$$\frac{\partial}{\partial z} \left\langle \frac{\vec{k}_\perp}{k_0} \right\rangle_k \cdot \frac{\vec{v}_0}{|\vec{v}_0|} = \frac{2}{\ell_{\text{RPP}}} \frac{\langle n \rangle}{n_c} \frac{\langle U \rangle}{T_e} f(M, \hat{v}), \quad (13)$$

where the brackets  $\langle \rangle_k$  [Eq. (11)], indicate an average over a speckle ensemble,  $\ell_{\text{RPP}} = \frac{45}{64} F \lambda$  denotes the speckle correlation length of an RPP beam<sup>13</sup>, and  $n_c$  the critical density. Similar expressions in the context of beam bending<sup>8,11,20</sup> have been derived, with the dependence on the plasma density  $n$ , the average ponderomotive potential  $\langle U \rangle$ , and the speckle (or beam) width  $\sim F \lambda$ , with  $F$  denoting the beam optics F-number. Note that the spatially averaged plasma density  $\langle n \rangle$  and the average ponderomotive potential  $\langle U \rangle/T_e$  normalized to the electron temperature can be assumed to vary slowly on the speckle's width  $F \lambda$  and response time  $F \lambda/c_s$ . The ponderomotive potential in the above expression is related to the density fluctuations for the case of a stationary plasma, namely  $(\delta n/\langle n \rangle)_{v_\perp=0} = 1 - \rho_0 \approx U/T_e$ .

Following Ref. 13, the function  $f(M, v_{ia})$  is given by:

$$f(M, v_{ia}) = \frac{2}{\pi} \int_0^{\pi/2} \frac{M v_{ia} \cos^2 \theta}{(1 - M^2 \cos^2 \theta)^2 + 4M^2 v_{ia}^2 \cos^2 \theta} d\theta, \quad (14)$$

where  $v_{ia}$  is the ion acoustic damping rate normalized to the ion acoustic frequency and  $\vec{k}_\perp \cdot \vec{v}_0 = |\vec{k}_\perp| |\vec{v}_0| \cos \theta$ . The integral represents a uniform disk of transverse wavenumber Fourier amplitudes with radius  $\pi/F \lambda$ .

The case of the  $M = 1$  singularity and its vicinity is integrable, so that if  $M$  varies with  $z$  and goes through a sonic point the total angular deflection is finite.

In the limit of small damping  $v_{ia} \rightarrow 0$  for supersonic flow,  $M > 1$ , the function  $f(M, v_{ia})$  can be approximated by:

$$\lim_{v_{ia} \rightarrow 0} f(M, v_{ia}) \approx \tilde{f}(M) \equiv \frac{1}{2M\sqrt{M^2 - 1}}, \quad (15)$$

The cumulative action of the ponderomotive force from numerous speckles can be evaluated by separating the mean values and the fluctuation of the fluid quantities, namely  $\rho = \langle \rho \rangle + \delta \rho$  and  $\vec{p}_\perp = \langle \vec{p}_\perp \rangle + \delta \vec{p}_\perp$ , in Eqs. (5) and (6). Following Refs. 9 and 14, the loss in fluid momentum as a function of time due to the collective action of speckles can be derived by ignoring the averages of fluctuating terms except in the last term on the RHS of Eq. (5). The fluctuation coupling results in equations for the averages  $\langle \vec{p}_\perp \rangle$  and  $\langle \rho \rangle$ ,

$$\frac{\partial \langle \vec{p}_\perp \rangle}{\partial t} + \nabla_\perp \cdot (\langle \vec{p}_\perp \rangle \langle \vec{v}_\perp \rangle) = -\alpha \langle \vec{p}_\perp \rangle - c_s^2 \langle \rho \rangle \nabla_\perp \left( \ln \langle \rho \rangle + \frac{\langle U \rangle}{T_e} \right) \quad (16)$$

$$\frac{\partial \langle \rho \rangle}{\partial t} + \nabla_\perp \cdot (\langle \rho \rangle \langle \vec{v}_\perp \rangle) = 0, \quad (17)$$

with a drag (source) term  $-\alpha \langle \vec{p}_\perp \rangle$ . When  $\langle U \rangle/T_e$  is small, the drag term,  $\alpha \langle \vec{p}_\perp \rangle$ , points along the local momentum direction for the isotropic RPP disk model such that  $\alpha$  is simply



a coefficient. Otherwise,  $\alpha$  must be generalized to a tensor. The drag coefficient,  $\alpha$ , as given by Eq. (28) of Ref. 9, reads

$$\alpha = 2 \frac{\langle U \rangle}{T_e} \frac{n_c}{\langle n \rangle} \frac{c_s}{M} \frac{\partial \langle \theta \rangle}{\partial z} = \frac{4 c_s}{\ell_{\text{RPP}}} \left( \frac{\langle U \rangle}{T_e} \right)^2 \frac{f(M, v_{ia})}{M}. \quad (18)$$

We can now find analytic results for shock formation due to drag from an idealized laser beam whose spatial envelope is a slab, only varying in the  $y$ -direction for incident flow with Mach number  $M > 1$ , specified at  $y = 0$ , with an initial unperturbed density profile. The combined effect of laser speckles, as described by the drag coefficient, leads to the deceleration of supersonic flow, transitioning it from supersonic to subsonic velocities, which in turn signifies the occurrence of shock formation. From Eqs. (13-15,16-18), assuming steady state flow in the positive  $y$ -direction,

$$\langle \rho \rangle \left( \frac{(\langle \vec{v}_\perp \rangle \cdot \nabla_\perp) \langle \vec{v}_\perp \rangle}{c_s^2} + \frac{\nabla_\perp \langle \rho \rangle}{\langle \rho \rangle} + \frac{\alpha}{c_s} \frac{\langle \vec{v}_\perp \rangle}{c_s} \right) = -\langle \rho \rangle \nabla_\perp \frac{\langle U \rangle}{T_e} \quad (19)$$

together with  $\langle \rho \rangle \langle v_\perp \rangle \equiv \langle \rho \rangle M c_s$  and  $\nabla_\perp \rightarrow \partial_y$ , neglecting for simplicity the right-hand-side term in Eq. (19) one obtains

$$\frac{d}{dy} \left( M + \frac{1}{M} \right) = -\frac{\alpha}{c_s}. \quad (20)$$

Then, using the small damping limit Eq. (15) in Eq. (18) for  $\alpha$ , the steady state solutions of Eqs. (16) and (17) imply

$$\frac{d}{dy} \left( M + \frac{1}{M} \right) = -\frac{2}{\ell_{\text{RPP}}} \left( \frac{\langle U \rangle}{T_e} \right)^2 \frac{1}{M^2 \sqrt{M^2 - 1}}. \quad (21)$$

Sonic flow,  $M = 1$  is reached at  $y = y_{\text{sonic}}$

$$\begin{aligned} y_{\text{sonic}} &= \frac{\ell_{\text{RPP}}}{2} \left( \frac{\langle U \rangle}{T_e} \right)^{-2} \int_1^M (u^2 - 1)^{3/2} du \\ &= \frac{45}{1024} y_p \left( M(2M^2 - 5)\sqrt{M^2 - 1} + 3 \ln \left[ 2(\sqrt{M^2 - 1} + M) \right] \right), \end{aligned} \quad (22)$$

wherein  $y_p = F\lambda / (\langle U \rangle / T_e)^2$  is the scaling length. Figure 2 shows  $y_{\text{sonic}}$ , Eq. (22) in comparison to results of numerical solutions of the fluid equations (5), (6). Details of simulations and nonlinear limit of shock formation are discussed next in section IV.

#### IV. NUMERICAL SIMULATIONS AND NONLINEAR SHOCKS

Numerical solutions of Eqns. (5), (6) in the plane normal to the beam propagation (see Fig. 1) have been computed using a two-dimensional (2D) isothermal variant of the Clawpack hydrodynamics package, initially developed by LeVecque et al.<sup>23</sup>. This code has been further enhanced in Ref. 22 by incorporating a ponderomotive force term into the momentum equation. The majority of the simulations have been carried out for an electron density of  $n_e = 0.1 n_c$ , with  $n_c$  denoting the critical density, and for the laser F-number  $F = 8$ . The spatial domain was resolved with 4096x4096 mesh points. The

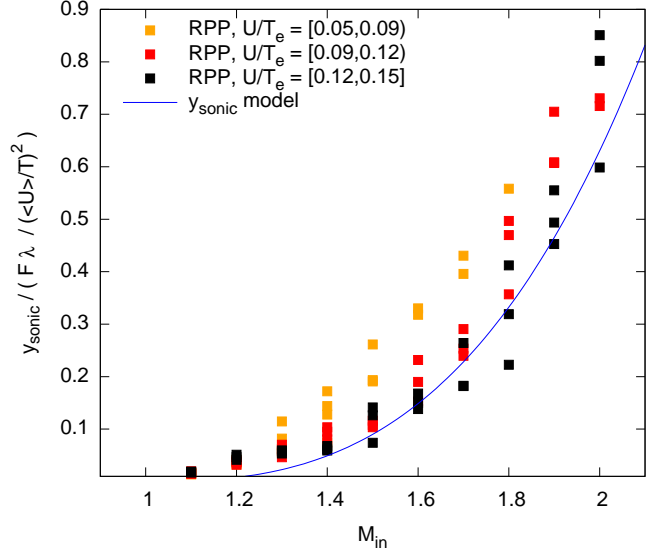


FIG. 2. Position of the point along the  $y$ -axis at which sonic flow is reached,  $y_{\text{sonic}}$ , as a function of the incoming flow Mach number  $M_{\text{in}}$ . The solid line shows the  $y_{\text{sonic}}$  position according to Eq. (22). Data points correspond to hydrodynamic simulations<sup>21,22</sup> including ponderomotive action of the laser beam speckle pattern of a beam generated with RPP. Parameters:  $n/n_c = 0.1$ ,  $F = 8$ . The  $y_{\text{sonic}}$  axis is scaled to  $y_p = F\lambda / (\langle U \rangle / T_e)^2$ . The colour of the data points indicates the range of the average ponderomotive potential, orange for  $0.05 \leq \langle U \rangle / T_e < 0.09$ , red for  $0.09 \leq \langle U \rangle / T_e < 0.12$ , and black for  $0.12 \leq \langle U \rangle / T_e < 0.15$ .

simulations were performed in dimensionless units. Taking a laser wave length of  $\lambda = 0.351 \mu\text{m}$  this corresponds to a spatial resolution of  $dx = dy = 0.4826 \mu\text{m}$  for a domain size with  $L_x = L_y = 1.9768 \text{mm}$ . The laser beam is focused in the center of the simulation box at  $x = 1000 \mu\text{m}$ ,  $y = 1000 \mu\text{m}$ . Times can be converted to  $ps$  by assuming  $ZT_e/A = 1 \text{keV}$  for the evaluation of the sound speed in the conversion with  $\lambda/c_s$ .

##### A. Random phase plate beams

In the RPP top hat model of the intensity distribution each Fourier mode of the laser light electric field  $\vec{E}$  within the  $x$ - $y$  plane of the plasma flow is assigned a random phase  $\phi(\vec{k})$  and the constant amplitude  $|E(\vec{k})|$  for  $|\vec{k}| \leq k_0/(2F)$  where  $k_0$  is the laser wavenumber. The speckle patterns produced by such disk-shaped RPP beams are evident in Fig. 3 via the density and flow velocity perturbations.

In the early stage of shock formation, a position at  $y_{\text{sonic}}$  along the  $y$ -axis can be identified by the location of the earliest transition from the supersonic ( $y < y_{\text{sonic}}$ ) to subsonic flow velocities ( $y > y_{\text{sonic}}$ ). For our simulations, all carried out for ponderomotive potential values  $\langle U \rangle / T_e > 0.015$ , this usually occurs in the vicinity of the earliest intense laser speckle situated close to the edge of the laser beam envelope. Behind this position (in the flow direction, here  $y$ ) the flow may again

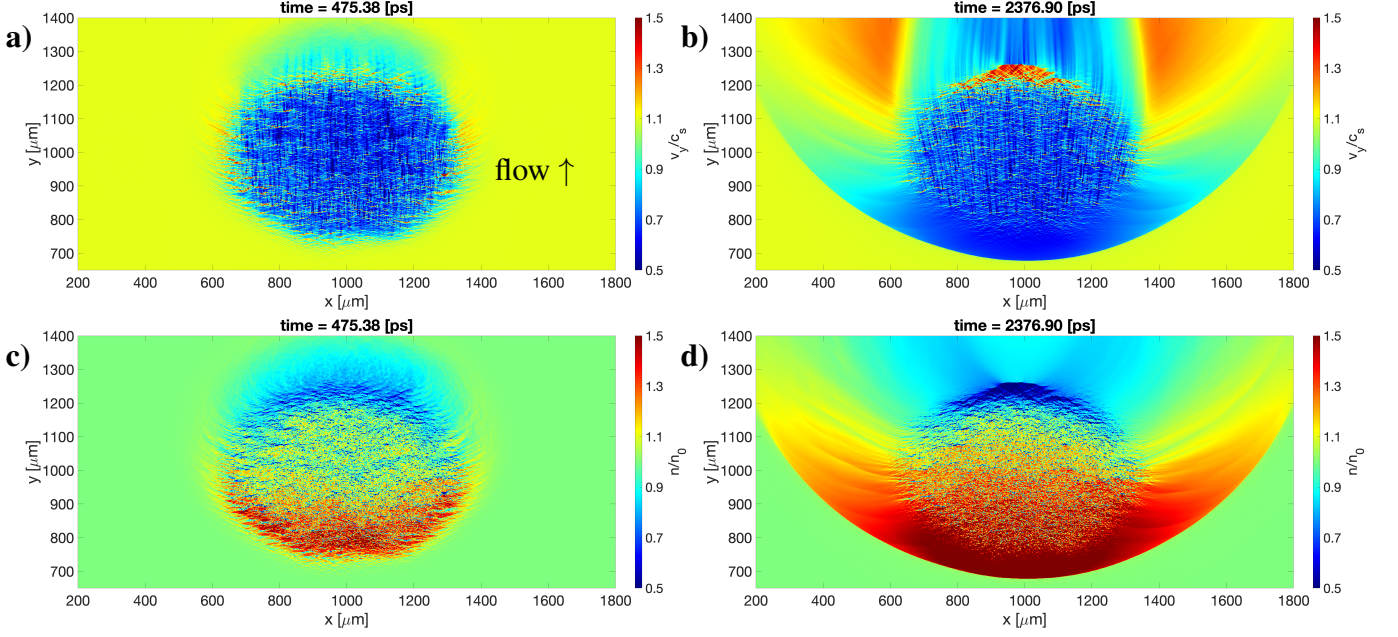


FIG. 3. Plasma flow  $v_y/c_s$  (top, a, b) and density  $n/n_0$  (bottom c, d) cross sections in  $x$  and  $y$  at early time (left, a, c) and late time (right, b, d) of a simulation with incoming flow at  $M_{in} = 1.1$  (from below) and an average normalized laser ponderomotive potential  $\langle U \rangle / T_e = 0.13$ . The early images (left) show the imprint of the laser beam cross section with speckle structure while the late images (right) show a developed shock with a clear front departing from the laser speckle pattern.

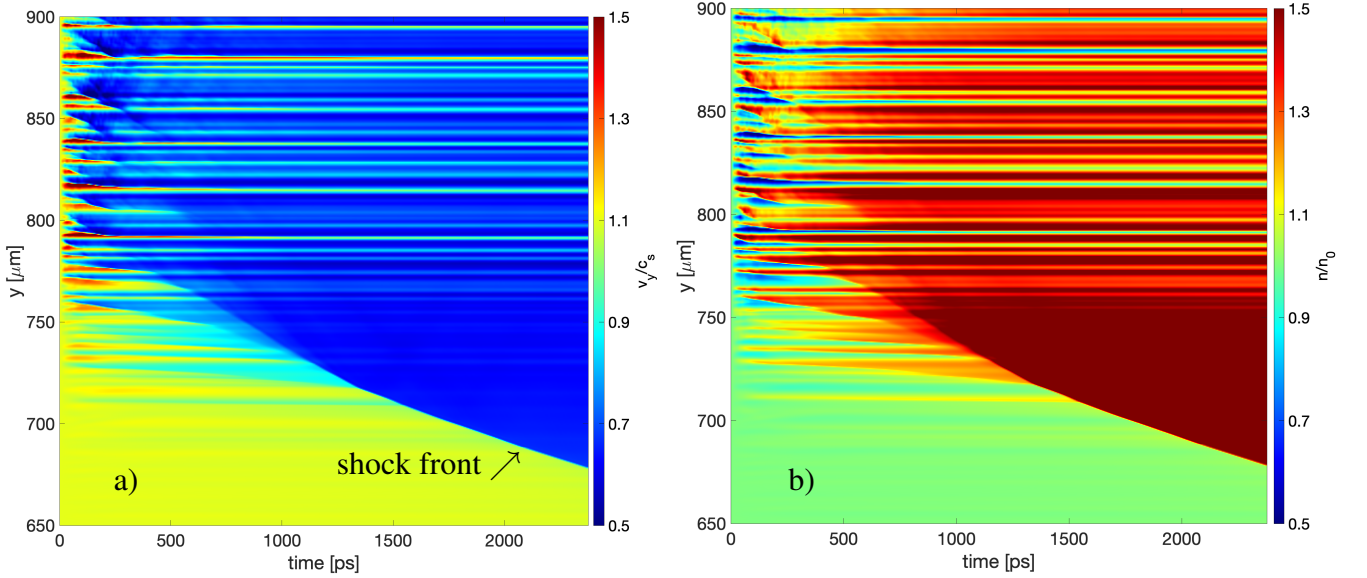


FIG. 4. Plasma flow velocity  $v_y/c_s$  (left, a) and plasma density (right, b) as a function of time on the horizontal axis from a line-out in  $y$  through the center of the simulation box at  $x=1000\mu\text{m}$ . The image corresponds to the same simulation as in Fig. 3 with  $M_{in} = 1.1$  and  $\langle U \rangle / T_e = 0.13$ . Note the sharp transition occurs (in both the flow and density) and moves out of the speckle pattern at a nearly constant speed during late times ( $t > 1300\text{ps}$ ). The position in  $y$  of the transition from super- to sub-sonic flow is, for this case, around  $y_{sonic} = y \simeq 715\mu\text{m}$ . The slope of this line is used to determine the shock speeds later on shown in Figure 7.

exceed  $c_s$  before the next, i.e. a neighbouring and/or subsequent intense speckle slows the incoming flow. The transition from super-sonic to sub-sonic may be repeated several times along the flow direction. It is then the combined action of the speckle ensemble that eventually leads to a shock formation.

We have determined positions  $y_{sonic}$  for a set of simulations by varying the incoming flow Mach numbers  $M_{in}$  and the average ponderomotive potential associated with the smoothed beam. These results are shown in Fig. 2 and are compared to the expression given by Eq. (22), taking into account a

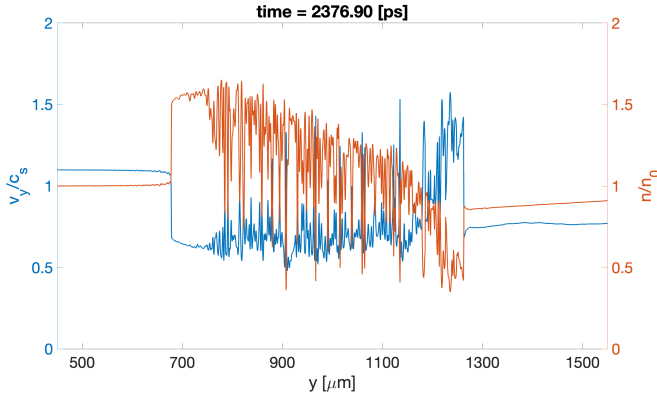


FIG. 5. Plasma density  $n/n_0$  (red line) and flow  $v_y/c_s$  (blue) as a function  $y$  taken from a line-out the center of the simulation box at  $x=1000\mu\text{m}$  at  $t = 2376\text{ps}$ . The image corresponds to the same simulation as in Figs. 3- 4 with  $M_{in} = 1.1$  and  $\langle U \rangle/T_e = 0.13$ , and to the latest line-out in time of Figs. 4. Both quantities show a shock front running against the incoming flow, located around  $y = 700\mu\text{m}$ , well outside the beam.

correction<sup>24</sup> with respect to the simulations due to the gradual rise in the envelope of the average laser intensity value in the wings of the laser beam envelope for a width of  $70\mu\text{m}$  (i.e.  $\sim 200\lambda$ ). The comparison shows good agreement between simulations and the model developed in section III. For the early stage, only the ponderomotively imprinted perturbations in the flow and the density are seen, as illustrated in both subplots in Fig. 3, left column.

Later on, as shown in the right column subplots of Fig. 3, at  $t = 2377\text{ps}$ , a clean bow shock front propagates against the flow coming from below, here with  $M_{in} = 1.1$ .

The space-time evolution of the plasma flow and the gradual shock formation leading to a free shock propagation against plasma flow with a well defined velocity is illustrated in Fig. 4. The values of density and velocity in Fig. 4 are extracted from two-dimensional density and velocity fields  $\rho(x,y), v_y(x,y)$  at the central position in  $x$  with respect to the speckle pattern (here  $x \simeq 1000\mu\text{m}$ ) along  $y$ .

Figure 4 shows that this clear and sharp shock front occurs, here beyond  $t \simeq 1100\text{ps}$ , once the collectively formed perturbation leaves the zone of influence of the laser beam speckle pattern. Behind this front, the flow then remains sub-sonic, except close to intense speckles further inside the beam. Figure 3, for a late time, here  $t = 2377\text{ps}$ , displays a sharp shock front propagating down against the background plasma flow of  $M_{in} = 1.1$ . Figure 5 illustrates, correspondingly, the profiles for density (red curve) and flow (blue) as a function of  $y$  taken from a simulation in the central cut of the laser beam, i.e.  $x \simeq 1000\mu\text{m}$  in Fig. 3, showing clear jumps left of the spatial zone occupied by the laser beam  $750 < y(\mu\text{m}) < 1350$ .

## B. Density and momentum jumps due to shocks

We have determined the jumps in density as a function of the ponderomotive potential  $\langle U \rangle$  for series of simulations

with three different values of the incoming super-sonic flow, namely  $M_{in} = 1.05, 1.1, 1.2$ , and  $1.3$ . The results shown in Fig. 6 correspond to the cases of optically smoothed laser beams using (i) the method with random phase plates, RPP, without temporal incoherence and (ii) the spatio-temporal smoothing method ‘Smoothing by Spectral Dispersion’, SSD. The resulting density jumps increase with the ponderomotive drive, and are stronger for higher incoming flow. It results that the ratio between the density jumps for the same  $\langle U \rangle$  value are almost linearly proportional to  $M_{in}$ . We have examined whether the

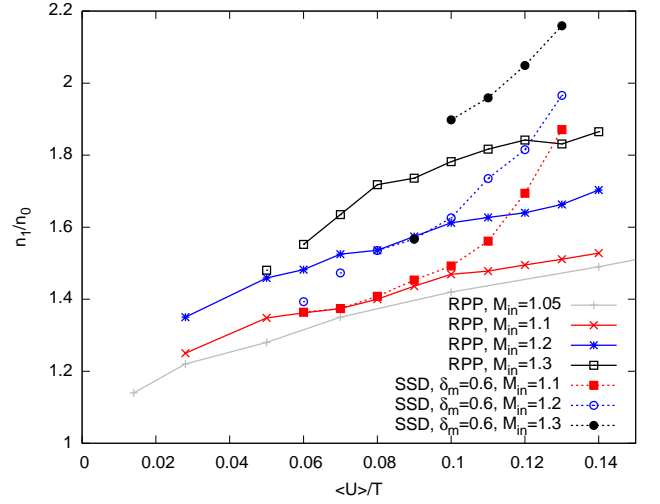


FIG. 6. Density jump,  $n_1/n_0$  over the shock as a function of the average value of the ponderomotive potential  $\langle U \rangle/T_e$  are displayed for both RPP cases (solid lines) and for selected SSD cases (dashed) with phase depth,  $\delta_m = 0.6$ . The lines correspond to different values of the incoming flow,  $M_{in} = 1.05$  (only RPP),  $1.1, 1.2$ , and  $1.3$ .

observed jumps in density and flow velocity fulfill the conditions expected for a shock, namely whether – once far away from the influence of the beam – they fulfill the Rankine-Hugoniot relations resulting from continuity, momentum, and energy balance in the frame of the shock, the details of which are discussed in the Appendix. These relations eventually reveal the jumps in density and flow velocity between both sides of the shock. We denote with  $n_0$  the plasma density in front of the shock, in the unperturbed plasma, and  $n_1$  the plasma density behind the shock, usually assuming  $n_1 > n_0$ . Likewise we define the plasma flows  $v_{y,0}$  and  $v_{y,1}$ , in the laboratory frame.  $M_{in} = v_{y,0}/c_{s,0}$  is the Mach number of the incoming flow;  $c_{s,j}$  denotes the sound speed in front ( $j=0$ ) and behind ( $j=1$ ) the shock front, respectively. The Mach numbers in the Rankine-Hugoniot relations are those of the shock frame which can be determined from  $v_{y,0}/c_{s,0}$  and  $v_{y,1}/c_{s,1}$  via the shock speed in the laboratory frame  $v_{sh}$ , readily given by given by  $M_0 = (v_{y,0} - v_{sh})/c_{s,0}$  and  $M_1 = (v_{y,1} - v_{sh})/c_{s,1}$ .

For the current study we have considered the case of an isothermal plasma (with  $\gamma = 1$ , degrees of freedom  $\rightarrow \infty$ ), being a particular case, for which the relation between the internal energies on both sides of the jump, see Eq. (A.3), can be disregarded. For the isothermal plasma case with  $c_{s,0} = c_{s,1} \equiv c_s$ , the set of Rankine-Hugoniot relations is sim-

plified to the continuity and Euler equations (see Eqs. (A.6)-(A.7)), such that the resulting density jump is then essentially a function of the Mach numbers,  $n_1/n_0 = M_0^2 = M_1^{-2}$  together with  $M_0 M_1 = 1$ .

From our simulations the shock speed in the laboratory frame,  $v_{sh}$ , has been deduced from the slope of the shock front position as a function of time, an example of which is illustrated in Fig. 4. One can consequently determine the Mach numbers  $M_0$  and  $M_1$  on each side of the shock, and  $n_1$ , while  $n_0$  and  $M_{in}$  are known from the unperturbed plasma. For the case of an isothermal plasma it follows from Eq. (A.8),  $-v_{sh}/c_s = \sqrt{n_1/n_0 - M_{in}^2}$ , that the formation of a shock propagating against the incident flow ( $-v_{sh} > 0$ ) can only occur for a sufficiently high density jump, namely  $n_1/n_0 > M_{in}^2$ ; consequently for  $M_{in} = 1.05, 1.1, 1.2$ , and  $1.3$  this means that shocks should not be able to leave the laser beam cross-section and propagate freely unless  $n_1/n_0 > 1.10, 1.21, 1.44$ , and  $1.69$ , respectively. The latter explains the fact that the simulation curves for  $M_{in} = 1.3$  shown in Fig. 6 are partially incomplete for smaller  $\langle U \rangle/T$  values, and that the corresponding data points (black) for  $M_{in} = 1.3$  and  $\langle U \rangle/T_e < 0.08$  have been deduced with large uncertainties from the simulations. On the other hand, as Fig. 6 illustrates, the strength of the shock in terms of the jump conditions increases with  $M_{in}$ , as a consequence of the criterion  $n_1/n_0 > M_{in}^2$ , and increases with the cumulative action of the ponderomotive force in the beam speckles,  $\propto \langle U \rangle/T_e$ .

The time required to observe a freely propagating shock separated from the laser beam cross-section can be very long, sometimes beyond the run time of our simulations. This is subject of a further discussion in the concluding section.

The speed of the emerging shock depends on the ponderomotive force exerted cumulatively by different speckles. Its value, as a function of the average ponderomotive potential  $\langle U \rangle$  can be determined by integrating Eq. (19) from the upstream region of unperturbed plasma over the shock front into the region of the speckle pattern. For the cases of RPP, without temporal smoothing (i.e. not for SSD), the following relation between both sides of the shock results,

$$\frac{(v_{y,0} - v_{sh})^2}{c_s^2} = \frac{(v_{y,1} - v_{sh})^2}{c_s^2} + \ln\left(\frac{n_1}{n_0}\right) + 2\frac{\langle U \rangle}{T_e} \quad (23)$$

which yields a relation between the flow, the density jump  $n_1/n_0$ , and the ponderomotive potential  $\langle U \rangle/T_e$ . With the continuity in the shock frame and with  $M_{in} \equiv v_{y,0}/c_s$  one obtains

$$\left(M_{in} - \frac{v_{sh}}{c_s}\right)^2 = \frac{\ln(n_1/n_0)^2 + 2\langle U \rangle/T_e}{1 - (n_1/n_0)^{-2}}. \quad (24)$$

The evaluation of expression Eq. (24) using numerical results from our hydrodynamic simulations is compared to the directly determined values of  $-v_{sh}/c_s$  as shown in Fig. 7. The jump in the density  $n_1/n_0$  corresponding to the shock, necessary to compute Eq. (24) together with the value of  $\langle U \rangle/T_e$ , are taken from the simulation results summarized in Fig. 6.

Inspecting Fig. 7 it becomes evident that for values of  $\langle U \rangle/T_e < 0.04$  it is difficult to observe a shock outbreak in simulations, i.e. the freely propagating shock that is separated

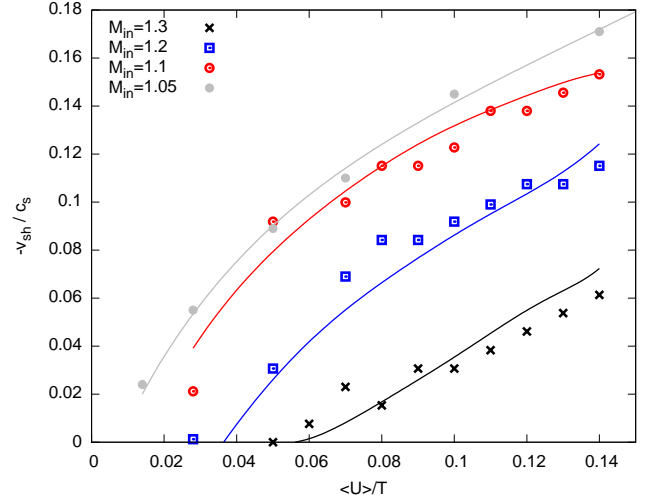


FIG. 7. Shock speed  $v_{sh}$  (in the laboratory frame) related to the sound speed  $c_s$ , as a function of the ponderomotive potential, deduced from hydrodynamic simulations with supersonic flow under the influence of a laser beam smoothed with RPP, for  $M_{in} = 1.05$  (marker in gray), 1.1 (red), 1.2 (blue), and 1.3 (black). Curves in the respective colors show the shock speed values expected from Eq. (24) depending on the average ponderomotive potential  $\langle U \rangle/T_e$  and on the density ratio,  $n_1/n_0$ , for which the values shown in Fig. 6 have been used.

from the laser. The data shown as 'o' symbols in Fig. 7 have therefore particularly low reliability for  $\langle U \rangle/T_e < 0.08$ .

In general, the time to observe the shock outbreak increases considerably with  $M_{in}$ . This is a consequence of the decrease of the drag with increasing  $M_{in}$  as  $1/M_{in}^2$  as shown in Eq. (15). For  $M_{in} = 1.3$  shock outbreak is only observed for higher values  $\langle U \rangle/T_e > 0.05$ . It is also consistent with the criterion that for  $-v_{sh}/c_s > 0$  the density jump in the shock has to be strong enough. This criterion relates  $n_1/n_0$  to  $M_{in}$  via  $-v_{sh}/c_s = M_0 - M_{in}$ , which is for the case of an isothermal plasma  $n_1/n_0 > M_{in}^2$ , and follows from Eq. (A.4) for the general case for any  $\gamma$  value.

An approximate criterion for possible shock outbreak can be obtained by combining Eqs. (A.8) and (24) which yields  $(\langle U \rangle/T_e) > \frac{1}{2}(M_{in}^2 - M_{in}^{-2} - 2\ln M_{in}^2) \simeq \frac{4}{3}(M_{in} - 1)^3$  for weakly supersonic flow. The resulting  $\langle U \rangle/T_e$  values above which shock outbreak can be expected for  $M_{in} = 1.1, 1.2$ , and  $1.3$  are, respectively,  $(\langle U \rangle/T_e) > 0.001, 0.008$ , and  $0.024$ . They are below the values for which we observe shock outbreak in our simulations, e.g. for  $M_{in} = 1.3$  we observe the threshold at twice the magnitude of  $\langle U \rangle/T_e$ . Note that we have used here the criterion that does not account for the gradual transition of  $\langle U \rangle/T_e$  as a function of  $y$  in the simulations.

### C. Beams with temporal smoothing

In the preceding sections the formation of a ponderomotively driven bow shock was considered for the case of an optically smoothed laser beam with merely spatial incoherence due to the introduction of a random phase plate in the near field. Modern laser facilities also include spatio-



temporal smoothing techniques. Smoothing by Spectral Dispersion (SSD) is implemented on all the major ICF and high-energy density laser facilities such as the NIF, Omega and LMJ lasers<sup>25–27</sup>.

The effect of SSD on beam deflection and consequently on the bow shock formation is modified by the non-stationary features of the beam speckle pattern.<sup>12,20,28</sup> SSD is a method

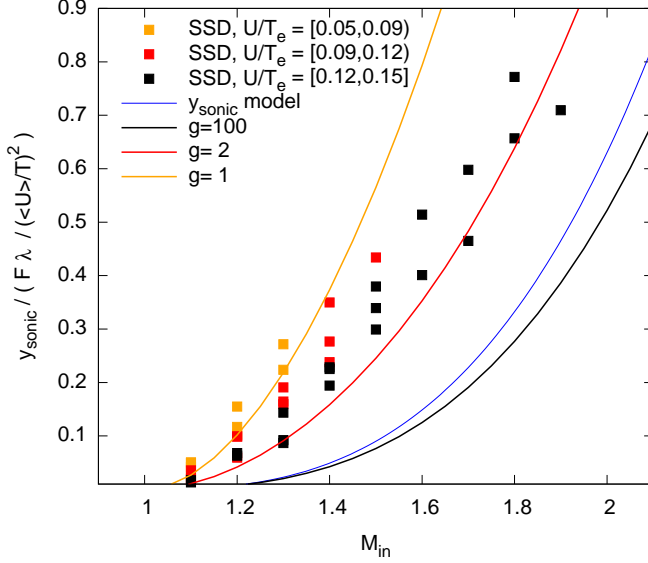


FIG. 8. Similar to Fig. 2, position of the sonic flow as a function of the incoming flow Mach number  $M_{in}$  from hydrodynamic simulations with laser beams smoothed with the SSD method, parameters:  $n/n_c = 0.1$ ,  $F = 8$ , SSD bandwidth 17 GHz and  $\delta_m = 0.6$ . The colour of the data points indicates the range of the average ponderomotive potential, orange for  $0.05 \leq \langle U \rangle / T_e < 0.09$ , red for  $0.09 \leq \langle U \rangle / T_e < 0.12$ , and black for  $0.12 \leq \langle U \rangle / T_e < 0.15$ . The  $y_{sonic}$  values corresponding to the model Eqs. (27–28) are shown as coloured curves for the parameter values  $g = 1, 2$ , and  $100$ . The curve labelled ‘ $y_{sonic}$  model’ corresponds to the Eq. (22) as also shown in Fig. 2.

of using bandwidth in combination with a grating to produce speckle movement. This method was originally proposed<sup>29</sup> for temporally smoothing laser focal spots.

In this article we focus on 1D transverse SSD as is implemented on the NIF. The first stage of SSD is to apply a sinusoidal phase modulation to the pulse, which introduces bandwidth. The beam then passes through a dispersion grating which tilts the pulse front and results in a distribution of frequencies across the lens, typically along one dimension. The resulting electric field at the lens can be expressed as:

$$\mathbf{E}(y, t) = \frac{1}{2} \mathbf{E}_0(y, t) e^{i[\omega_0 t + \delta_m \sin(\omega_m t + \alpha_{SSD} y) + \phi_0]} + c.c., \quad (25)$$

where  $\omega_0$  is the central angular frequency of the laser,  $\phi_0$  is the initial phase,  $\omega_m$  is the modulation frequency, and  $\delta_m$  is the modulation depth. Here  $\alpha_{SSD} = 2\pi N_{cc}/w_y$ , where  $N_{cc}$  is the number of color cycles and  $w_y$  is the width of the beam along the  $y$ -direction at the lens. The total bandwidth added to the laser is  $\Delta\omega \approx 2\delta_m\omega_m$  which is small relative to the laser frequency  $\Delta\omega \ll \omega_0$ . Frequency tripling of the laser after the initial bandwidth is added is common at high energy laser

facilities. To account for such frequency tripling we use a factor of three times higher value of  $\delta_m$ .

#### D. Analytical model of beam bending and shock onset with spatio-temporal smoothing

The characteristic distance  $y_{sonic}$ , Eq. (22), can be obtained from the solution of Eq. (20) with the suitable expression for the drag coefficient  $\alpha$ , Eq. (18), and the beam deflection rate Eq. (10). For an approximate choice of electric field temporal correlation function, the calculation of beam deflection has been reduced in Ref. 12 to evaluation of a two-dimensional integral in the Fourier space. The analytical theory of beam deflection by a flow in Ref. 12 has been derived for the case of a laser beam which is spatially smoothed by an RPP and temporally smoothed by the random phase modulations, similar to SSD Eq. (25), that was used for analytical analysis. Following results of Ref. 12 in the limit of large bandwidth and finite Landau damping, Eq. (45) in Ref. 12, we have for the drag coefficient

$$\alpha = \left( \frac{\langle U \rangle}{T_e} \right)^2 \frac{k_0 c_s}{FM} \Xi(M, v_{ia}, g, N_{cc}), \quad (26)$$

where the two-dimensional integral reads

$$\Xi(M, v_{ia}, g, N_{cc}) = i \int d\phi \int d\kappa \kappa^2 \frac{\hat{c}(\kappa) \cos \phi}{1 - u^2 - i 2v_{ia}u}. \quad (27)$$

In the one-dimensional realization of the SSD where the speckles move along the direction  $y$  of the background plasma flow the argument  $u$  of the integral is given by

$$u = \frac{i}{g\kappa} (1 - e^{-N_{cc}\kappa \cos \phi}) - \frac{\kappa M \cos \phi}{\kappa}, \quad (28)$$

where  $g = k_0 c_s / (F \gamma_c)$  can be understood as the ratio between the correlation time of the electric field,  $1/\gamma_c$ , and the ion acoustic transit time over the width of a speckle,  $F/(k_0 c_s)$ . The function  $\hat{c}(\kappa)$  is given by Eq.(32) from Ref. 12. In the large bandwidth limit  $N_{cc}\gamma_c/\omega_m \gg 1$ . We have used Eq. (26) for the drag coefficient  $\alpha$  to obtain  $y_{sonic}$  from Eq. (20) and results are plotted in Fig. 8 for  $g = 1, 2$ , i.e. for short field correlation times; the case  $g = 100$  is very close to the RPP result, Eq.(22).

#### E. Shock formation by SSD beams

Consequently we have also carried out simulations by using laser beams generated with RPP and SSD, in complement to our study of beams with RPP only. For this study we have used one color cycle  $N_{cc} = 1$ , a modulation depth of  $\delta_m = 0.6$ , and the modulation frequency of  $\omega_m = 2\pi \times 17 \text{ GHz}$ , similar to the NIF laser at the National Ignition Facility.

We generated 128 snapshots of the laser focal spot during one SSD period  $\tau_{SSD} = 2\pi/\omega_m \approx 58.8 \text{ ps}$  so that for each hydro time step the next snapshot was used and the snapshots



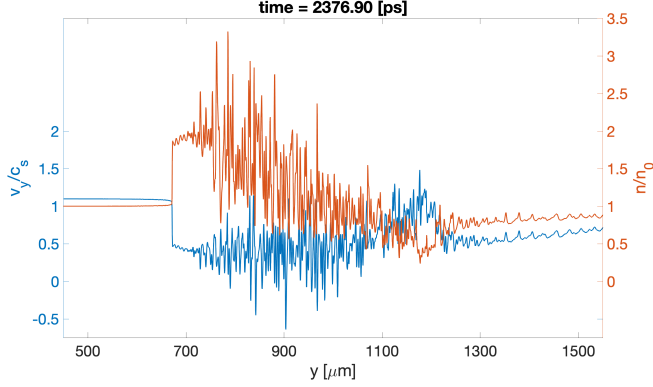


FIG. 9. Plasma density  $n/n_0$  (red line, right axis) and flow  $v_y/c_s$  (blue, left axis) as a function  $y$  taken from a line-out the center of the simulation box at  $x=1000\mu\text{m}$  at  $t = 2376\text{ps}$ . The image corresponds to the same simulation as in Figs. 3, 4, and 5 but with SSD present. Both quantities show a shock front running against the incoming flow, located around  $y=700\mu\text{m}$ , well outside the beam. The jump in both flow and density at the location of the shock is stronger for the SSD case displayed here.

repeated. Results of this method are consistent with speckle velocity theory<sup>30</sup>.

Similar to the results for RPP beams shown in Figs. 2, 4, and 5, we have plotted in Fig. 8 the scaled values of  $y_{\text{sonic}}$  for the simulations with SSD and in Fig. 6 the density jumps  $n_1/n_0$ . Figure 5 summarises the comparison between RPP simulations (solid lines) and the SSD simulations (dashed). Figure 9, similar to Fig. 5 with RPP only, shows the plasma density profile at the central cut ( $x = 1000\mu\text{m}$ ) after the shock outbreak. It illustrates that the density jump in the shock is more pronounced for SSD with respect to RPP for the shown case with  $M_{\text{in}} = 1.1$  at  $\langle U \rangle/T_e = 0.13$ , corresponding to the values reported in Fig. 6.

The action of spatio-temporal smoothing moves the position in the plasma,  $y = y_{\text{sonic}}$ , where the plasma flow is slowed down to  $M = 1$ . While this position was shifted further inside the speckle pattern with respect to the case of only spatial smoothing with RPP, see Fig. 2, the shock density jump for the cases with SSD may be stronger than for the corresponding RPP cases, see Fig. 6, at least for quite pronounced ponderomotive drive with  $\langle U \rangle/T_e > 0.1$ .

We have also performed simulation with SSD with phase modulation in the  $x$ -direction, instead of modulation in  $y$ -direction, i.e.  $\alpha_{\text{SSD}}x$  instead of  $\alpha_{\text{SSD}}y$  in Eq. (25). We cannot observe a remarkable difference between both variants in the jumps in the shock for both  $n_1/n_0$  and  $v_y/c_s$ . Differences can be seen in the flow profiles in the zone behind the beam speckle pattern with respect to the incoming flow, which is of minor importance in the current context.

The stronger density jump for SSD, with respect to RPP, may be attributed to the fact that SSD speckles move. The speckle therefore occupies a greater spatial region and can potentially expel a greater "mass" by the ponderomotive action, reinforcing the shock strength.

## V. CONCLUSIONS AND IMPLICATIONS FOR ICF EXPERIMENTS

Speckled laser beam propagation, in plasma slab or cylindrical geometries, with perpendicular supersonic plasma inflow and laser-plasma coupling through the ponderomotive force always leads to a macroscopic shock formation provided the plasma can propagate into a thick enough laser beam. The characteristic distance of plasma penetration across the randomized laser beam required for the flow to slow down to subsonic velocity and form a shock is given by Eq. (22). This analytical result was derived in the slab geometry and confirmed in numerical simulations where the laser beam of a cylindrical cross-section had a large diameter giving rise to a bow shock. The physics of the shock formation is related to the beam bending processes due to the plasma flow and the plasma's reaction to the momentum change induced by the redirected laser light.

We have performed numerical simulations for laser coupling to the plasma hydrodynamics over the laser beam cross section which clearly show the shock formation. In one of the numerical examples based on the RPP beam, Sec. III and Fig. 2, we take the beam intensity  $I = 2 \times 10^{15} \text{ W/cm}^2$  and electron temperature  $T_e = 2 \text{ keV}$  obtaining the average, normalized ponderomotive potential  $\langle U \rangle/T_e \approx 0.011$ . With  $F\lambda = 1\mu\text{m}$  and an incoming flow velocity corresponding to  $1.05 \leq M_{\text{in}} \leq 1.2$ , the distance  $y_{\text{sonic}}$  to shock formation within the laser is approximately  $0.05\text{--}0.1 F\lambda / (\langle U \rangle/T_e)^2$  (see Figs. 2 and 8), i.e. approximately  $400\text{--}810\mu\text{m}$ . To estimate a time necessary to generate the shock we take this distance and divide it by the sound speed obtaining roughly  $1.3\text{--}2.7 \text{ ns}$ . Thus within the square of  $400\mu\text{m} \times 400\mu\text{m}$  or  $810\mu\text{m} \times 810\mu\text{m}$  a laser beam of average intensity  $I = 2 \times 10^{15} \text{ W/cm}^2$  will require  $9\text{kJ}$  or  $36\text{kJ}$ , respectively, as energy. These are energy requirements that can be realized at the NIF.

The theory describing the deceleration of plasma flow to subsonic speeds, thereby promoting shock generation, has been derived specifically for isothermal plasmas. This formulation is based on beam bending calculations for laser beams that are spatially and temporally smoothed. Consistently, nonlinear shock modeling using hydrodynamic simulations has been conducted, considering the effects of ponderomotive laser-plasma coupling, while disregarding factors such as laser absorption and plasma heating.

## ACKNOWLEDGMENTS

Simulations carried out by SH were granted access to the French HPC resources of IDRIS under the allocations A0100500573 and AD010500573R1 by GENCI, France. SH acknowledges the support by the CPHT computer team. We acknowledge support from the Digital Research Alliance of Canada.

AM and WR would like to acknowledge support from the U.S. Department of Energy Contract number: DE-NA0004144.

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Lab-

oratory under Contract DE-AC52-07NA27344 and by LLNL's WPD, ICF Program's Academic Collaboration Teams's University Program (ACT-UP) under Subcontract No. B645970.

## REFERENCES

- <sup>1</sup>W. L. Kruer, S. C. Wilks, B. B. Afeyan, and R. K. Kirkwood, *Phys. Plasmas* **3**, 382 (1996).
- <sup>2</sup>R. K. Kirkwood, B. B. Afeyan, W. L. Kruer, B. J. MacGowan, J. D. Moody, D. S. Montgomery, D. M. Pennington, T. L. Weiland, and S. C. Wilks, *Phys. Rev. Lett.* **76**, 2065 (1996).
- <sup>3</sup>V. V. Eliseev, W. Rozmus, V. T. Tikhonchuk, and C. E. Capjack, *Physics of Plasmas* **3**, 2215 (1996).
- <sup>4</sup>P. Michel, L. Divol, E. A. Williams, S. Weber, C. A. Thomas, D. A. Callahan, S. W. Haan, J. D. Salmonson, S. Dixit, D. E. Hinkel, M. J. Edwards, B. J. MacGowan, J. D. Lindl, S. H. Glenzer, and L. J. Suter, *Phys. Rev. Lett.* **102**, 025004 (2009).
- <sup>5</sup>P. Michel, L. Divol, E. A. Williams, C. A. Thomas, D. A. Callahan, S. Weber, S. W. Haan, J. D. Salmonson, N. B. Meezan, O. L. Landen, S. Dixit, D. E. Hinkel, M. J. Edwards, B. J. MacGowan, J. D. Lindl, S. H. Glenzer, and L. J. Suter, *Phys. Plasmas* **16**, 042702 (2009).
- <sup>6</sup>P. Michel, S. H. Glenzer, L. Divol, D. K. Bradley, D. Callahan, S. Dixit, S. Glenn, D. Hinkel, R. K. Kirkwood, J. L. Kline, W. L. Kruer, G. A. Kyralla, S. Le Pape, N. B. Meezan, R. Town, K. Widmann, E. A. Williams, B. J. MacGowan, J. Lindl, and L. J. Suter, *Phys. Plasmas* **17**, 056305 (2010).
- <sup>7</sup>A. Colaitis, S. Hüller, D. Pesme, G. Duchateau, and V. T. Tikhonchuk, *Physics of Plasmas* **23**, 032118 (2016).
- <sup>8</sup>D. E. Hinkel, E. A. Williams, and C. H. Still, *Phys. Rev. Lett.* **77**, 1298 (1996).
- <sup>9</sup>H. A. Rose, *Physics of Plasmas* **3**, 1709 (1996).
- <sup>10</sup>J. D. Moody, B. J. MacGowan, D. E. Hinkel, W. L. Kruer, E. A. Williams, K. Estabrook, R. L. Berger, R. K. Kirkwood, D. S. Montgomery, and T. D. Shepard, *Phys. Rev. Lett.* **77**, 1294 (1996).
- <sup>11</sup>B. Bezzerides, *Physics of Plasmas* **5**, 2712 (1998).
- <sup>12</sup>H. A. Rose and S. Ghosal, *Physics of Plasmas* **5**, 775 (1998).
- <sup>13</sup>S. Ghosal and H. A. Rose, *Physics of Plasmas* **4**, 2376 (1997).
- <sup>14</sup>S. Ghosal and H. A. Rose, *Physics of Plasmas* **4**, 4189 (1997).
- <sup>15</sup>J. Candy, W. Rozmus, and V. T. Tikhonchuk, *Phys. Rev. Lett.* **65**, 1889 (1990).
- <sup>16</sup>S. Hüller, *Physics of Fluids B: Plasma Physics* **3**, 3317 (1991).
- <sup>17</sup>H. A. Rose, *Physics of Plasmas* **4**, 437 (1997).
- <sup>18</sup>R. W. Short, R. Bingham, and E. A. Williams, *Phys. Fluids* **25**, 2302 (1982).
- <sup>19</sup>A. J. Schmitt, *Physics of Fluids B* **1**, 1287 (1989).
- <sup>20</sup>C. Ruyer, P. Loiseau, G. Riazuelo, R. Riquier, A. Debayle, P. E. Masson-Laborde, and O. Morice, *Matter and Radiation at Extremes* **8**, 025901 (2023).
- <sup>21</sup>R. J. LeVeque, *Numerical methods for conservation laws (2. ed.)*, Lectures in mathematics (Birkhäuser, 1992) pp. 1–214.
- <sup>22</sup>S. Hüller, P. E. Masson-Laborde, D. Pesme, M. Casanova, F. Detering, and A. Maximov, *Phys. Plasmas* **13**, 022703 (2006).
- <sup>23</sup>R. J. L. Vecque, *J. Comp. Phys.* **131**, 327?353 (1997).
- <sup>24</sup>By separation of variables in Eq. (21), an integral  $\int_{y=0}^{y_{\text{sonic}}} (U(y)/T)^2 dy$  is applied, with  $U(y)$  denoting the local value of the envelope of the ponderomotive potential, with  $U(y=0)=0$  and  $U(y) \rightarrow \langle U \rangle$  for  $y$  inside the beam. This yields a correction to Eq. (22).
- <sup>25</sup>T. R. Boehly, V. N. Goncharov, O. Gotchev, J. P. Knauer, D. D. Meyerhofer, D. Oron, S. P. Regan, Y. Srebro, W. Seka, D. Shvarts, S. Skupsky, and V. A. Smalyuk, *Physics of Plasmas* **8**, 2331 (2001).
- <sup>26</sup>D. H. Kalantar, M. H. Key, L. B. DaSilva, S. G. Glendinning, J. P. Knauer, B. A. Remington, F. Weber, and S. V. Weber, *Phys. Rev. Lett.* **76**, 3574 (1996).
- <sup>27</sup>V. N. Goncharov, S. Skupsky, T. R. Boehly, J. P. Knauer, P. McKenty, V. A. Smalyuk, R. P. J. Town, O. V. Gotchev, R. Betti, and D. D. Meyerhofer, *Physics of Plasmas* **7**, 2062 (2000).
- <sup>28</sup>S. Hüller, G. Raj, M. Luo, W. Rozmus, and D. Pesme, *Phil. Trans. Royal Society A* **378**, 20200038 (2020), DOI:10.1098/rsta.2020.0038.
- <sup>29</sup>S. Skupsky, R. W. Short, T. Kessler, R. S. Craxton, S. Letzring,

and J. M. Soures, *Journal of Applied Physics* **66**, 3456 (1989), <https://doi.org/10.1063/1.344101>.

- <sup>30</sup>F. Walraet, G. Bonnaud, and G. Riazuelo, *Physics of Plasmas* **8**, 4717 (2001).
- <sup>31</sup>J. Spurk and N. Aksel, *Fluid Mechanics* (Springer International Publishing, 2019) Chap. 9.2.3.
- <sup>32</sup>P. Mulser, “Hot matter from high-power lasers,” (Springer International Publishing, 2020) pp. 236–255.

## Appendix: Rankine-Hugoniot shock relations

The Rankine-Hugoniot relations between the zones '0' for the unperturbed plasma and '1' for the zone behind the shock, propagating with  $v_{sh}$ , readily read<sup>31,32</sup>

$$n_0(v_{y,0} - v_{sh}) = n_0 M_0 c_{s,0} = n_1 M_1 c_{s,1} = n_1 (v_{y,1} - v_{sh}) \quad (\text{A.1})$$

$$n_0 M_0^2 c_{s,0}^2 + p_0 = n_1 M_1^2 c_{s,1}^2 + p_1, \text{ and} \quad (\text{A.2})$$

$$h_0 + M_0^2 c_{s,0}^2 / 2 = h_1 + M_1^2 c_{s,1}^2 / 2, \quad (\text{A.3})$$

with  $p_{1,2}$  denoting here the pressures,  $h_{0,1} = p_{0,1}/n_{0,1} + \epsilon_{0,1}$  the enthalpies, and  $\epsilon_{0,1}$  the internal energies. These relations express the balance for the continuity, the momentum (Euler equation), and the energy (Bernoulli equation) between each side, respectively, supposing steady state and absence of any potential force on either side.

The shock discontinuity in general includes ion heating with entropy increase over the shock front<sup>32</sup>, yielding a jump in density and in temperature,

$$n_1/n_0 = \frac{\gamma+1}{2} M_0^2 / [1 + \frac{\gamma-1}{2} M_0^2], \quad (\text{A.4})$$

$$T_1/T_0 = (2\gamma M_0^2 + 1 - \gamma) / [(\gamma - 1)(n_1/n_0)]. \quad (\text{A.5})$$

The case of an isothermal plasma (with  $\gamma = 1$ , degrees of freedom  $\rightarrow \infty$ ) is a particular case, for which Eq. (A.3), relating the internal energies on both sides, can be disregarded.

We consider here the isothermal plasma case for simplicity, also in the simulations, with  $c_{s,0} = c_{s,1} \equiv c_s$ , by disregarding Eq. (A.3), so that the system of relations reduces to

$$n_0 M_0 = n_1 M_1, \quad (\text{A.6})$$

$$\pi_0 := n_0 (M_0^2 + 1) = n_1 (M_1^2 + 1) =: \pi_1. \quad (\text{A.7})$$

The resulting density jump is then essentially a function of the Mach numbers,  $n_1/n_0 = M_0^2 = M_1^{-2}$  with  $M_0 M_1 = 1$ .

From the simulations one can determine the shock speed in the laboratory frame,  $v_{sh}$ , which can be deduced from the space-time slope of the shock front as seen in Figs. 4. One can consequently determine the Mach numbers  $M_0$  and  $M_1$  on each side of the shock, furthermore  $n_1$ , while  $n_0$  and  $M_{in}$  are known from the unperturbed plasma. Knowing these value, the values for  $M_0 = M_{in} - v_{sh}/c_s$ , for  $M_1 = (v_{y,1} - v_{sh})/c_s$ , and the ratio  $n_1/n_0$  can be deduced. It furthermore follows

$$-v_{sh}/c_s = \sqrt{n_1/n_0} - M_{in}, \quad (\text{A.8})$$

which has the consequence that the outbreak of a shock running against the incident flow, namely  $-v_{sh} > 0$ , can only occur for a sufficiently high density jump, namely  $n_1/n_0 > M_{in}^2$ .

$\langle U \rangle / T$	$M_{in}$	$n_1/n_0$	$n_0 M_0$	$n_1 M_1$	$\pi_0$	$\pi_1$	type
0.073	1.1	1.39	1.26	1.27	2.58	2.56	RPP
0.10	1.1	1.51	1.29	1.33	2.66	2.68	RPP
0.11	1.1	1.54	1.29	1.32	2.67	2.66	RPP
0.1	1.2	1.64	1.33	1.39	2.77	2.81	RPP
0.11	1.2	1.67	1.35	1.40	2.82	2.83	RPP
0.13	1.2	1.74	1.36	1.38	2.86	2.85	RPP
0.073	1.1	1.31	1.18	1.20	2.40	2.42	SSD $\delta_m = 0.6$
0.143	1.1	1.86	1.46	1.54	3.15	3.14	SSD $\delta_m = 0.6$
$M_{in}$	$\langle U \rangle / T$	$n_1/n_0$	$n_0 M_0$	$n_1 M_1$	$\pi_0$	$\pi_1$	type
1.1	0.10	1.51	1.29	1.27	2.58	2.56	RPP
1.2	0.10	1.64	1.33	1.39	2.77	2.81	RPP
1.3	0.10	1.71	1.40	1.37	2.95	2.80	RPP
1.1	0.143	1.86	1.46	1.54	3.15	3.14	SSD $\delta_m = 0.6$
1.2	0.143	2.06	1.52	1.59	3.31	3.30	SSD $\delta_m = 0.6$

TABLE I. As a function of the Mach number of the flow,  $M_{in}$  and of the ponderomotive potential  $\langle U \rangle / T$ , the density jump  $n_1/n_0$  of the shock, as well as the continuity and momentum balance according to Eqs. (A.6-A.7) for selected cases, both for RPP and SSD cases (with SSD phase depth  $\delta_m = 0.6$ ).

Table I shows the results for simulations of selected RPP and SSD cases. The balance between the Rankine-Hugoniot relations conserving mass- and momentum flow on each side of the shock, in the proper frame of the shock, is generally quite well reproduced, as expected for well-developed shocks. One can find a generally better balance for the momentum flow,  $\pi_j = n_j(M_j^2 + 1)$ ,  $j=0,1$ . The discrepancy in the mass balance  $n_j M_j$  slightly increases, however, systematically with  $\langle U \rangle / T$ .