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Load identification and damage evaluation of a beam subjected to moving loads by using co-located strain and acceleration measurements

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ABSTRACT

Bridge structural health monitoring and assessment are important subjects nowadays. In this paper, we propose a novel method which allows to evaluate both the bridge structural health and its loads based on strains and accelerations measured at the same location. The bridge is considered as governed by the Euler-Bernoulli beam equation. The strain can be obtained directly from sensors such as fiber optics. This type of sensors has many advantages such as high precision, high sampling frequency, durability... so that the dynamic strain component can be acquired easily. By decomposing the obtained signals into static and dynamic components, the correlation between strain and acceleration for the same type of component is clarified. The proportional coefficients, which reflect the intrinsic state of the beam and its load, can be obtained by optimization. From the variation of these coefficients, information on the damage of the bridge can be obtained. Moreover, in case of multiple loads (multi-axle vehicle), the distance between the axles of the vehicle can also



be determined. Numerical results with several scenarios will be presented to demonstrate the efficiency and the robustness of the method.

Keywords: Damaged Identification, Moving Load, Bridge, Strain, Acceleration.

1 INTRODUCTION

Since most of bridges have been constructed in the 20th century, bridge Structural Health Monitoring (SHM) and assessment is an important subject nowadays. Many studies about vibration-based structural damage identification have been carried out the last decades. The presence of damage decreases the global rigidity of the structure and leads to a reduction of its natural frequencies. Modal properties such as natural frequencies and mode shapes have been used as damage detection criteria [1, 2]. However, the change in natural frequencies is relatively small, and the measure is influenced by environmental conditions such as temperature, humidity...[3]. Allemang *et al.* [4] presented the Modal Assurance Criterion (MAC) that studies the correlation between vibration modes of a structure by two different approaches. Based on MAC, Lieven *et al.* [5] proposed the COMAC (Coordinate Modal Assurance Criterion) as an extension of MAC. In reality, these methods require a large number of sensors.

The advantage of a moving load is that the excitation does not depend only on time, but also on space. From the dynamic response of the beam, we can find out its modal properties. Zhu and Law [6] presented a method that allows to identify the location of the damage from a single deflection time history at the midpoint of a simply supported beam, by using the Continuous Wavelet Transform (CWT). Nie *et al.* [7] used the time series of two distinct measured displacements in order to locate the damage. These methods work well when the velocity of the moving load is very small compared to the first critical velocity (i.e. resonance velocity [8]) of the beam.

Nowadays, thanks to advanced technology, bridges can be monitored continuously. Unlike the traditionally electrical strain gauge, a long-gauge optical strand measures the average strain over its length. Hence, it has a higher probability to cover the damage. Moreover, Optical Strand sensors have many advantages such as high-sample strain measurements over long periods, high precision and durability [9].

In this paper, we present firstly a novel method that allows to evaluate the beam-like bridge structural health and eventually the loads of the vehicle in the absence of damage, by using the proportional coefficients between the measured signals (strain, acceleration). Moreover, this method can be used to determine the distances between the axles of the vehicle. In order to validate the method, we apply the optimization procedure to the passage of a 3-axle vehicle over a simply supported beam and re-evaluate the intrinsic properties of the beam and the vehicle. Then, a parametric study is carried out to study the dependence of the proposed coefficients to the properties of the damage (location, damage level, damage length...).

2 FORMULATION AND IDENTIFICATION METHODS

2.1 Analytical formulation

The study examines a simply supported beam of length L as shown in Fig. 1. We assume that the beam follows the Euler-Bernoulli beam theory, the governing equation of the beam which describes the vertical displacement $u(x, t)$ is [10]:

$$EI \frac{\partial^4 u}{\partial x^4} + EI^* \frac{\partial^5 u}{\partial t \partial x^4} + c \frac{\partial u}{\partial t} + \rho S \frac{\partial^2 u}{\partial t^2} = P \delta(x - vt), \quad 0 \leq x \leq L. \quad (1)$$



where EI is the bending stiffness, ρS is the mass per unit length of the beam. In Eq. 1, the term $c(\partial u/\partial t)$ represents the external damping mechanism whereas the term $EI^*(\partial^5 u/\partial t \partial x^4)$ represents the internal damping mechanism, $\delta(x)$ denotes the Dirac distribution.

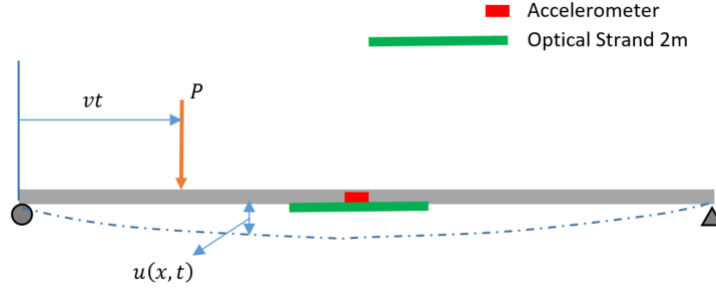


Figure 1: A simply supported beam subjected to a single moving force.

The beam is simply supported at its two ends, which leads to the following boundary conditions:

$$\begin{cases} u(0, t) = u(L, t) = 0 \\ M(0, t) = M(L, t) = 0 \Rightarrow u''(0, t) = u''(L, t) = 0 \end{cases}, \quad 0 \leq t \leq \frac{L}{v}, \quad (2)$$

where $M(x, t) = EIu''(x, t)$ is the bending moment of the beam. The prime operator $(u)' = \partial u/\partial x$ denotes the derivative of the function $u(x, t)$ with respect to x .

Moreover, before the time when the moving load enters the beam, it is considered at rest at all space points. Therefore, the following initial conditions are taken:

$$\begin{cases} u(x, 0) = 0 \\ \dot{u}(x, 0) = 0 \end{cases}, \quad 0 \leq x \leq L. \quad (3)$$

The dot operator $\dot{u} = \partial u/\partial t$ denotes the derivative of the function $u(x, t)$ with respect to t .

2.2 Forced responses of the beam under moving load

By modal decomposition, the solution of Eq. (1) can be expressed as a linear combination of the mode shapes $\phi_n(x)$ and the generalized coordinated $q_n(t)$ of the n^{th} mode:

$$u(x, t) = \sum_n \phi_n(x) q_n(t). \quad (4)$$

From the initial conditions (Eq. 2), and mode shapes which are normalized such that $\int_0^L \phi_m(x) \phi_n(x) dx = \delta_{mn}$, where δ_{mn} is the Kronecker delta, the form of $\phi_n(x)$ for a simply supported beam can be expressed explicitly as:

$$\phi_n(x) = \sqrt{2/L} \sin\left(\frac{n\pi x}{L}\right) = \mathcal{A} \sin(\lambda_n x), \quad (5)$$

in which $\lambda_n = n\pi x/L$ and $\mathcal{A} = \sqrt{2/L}$.

Since the mode shapes are orthonormal, the generalized coordinates $q_n(t)$ are solutions of the following differential equation:

$$\ddot{q}_n(t) + 2\omega_n \zeta_n \dot{q}_n(t) + \omega_n^2 q_n(t) = \frac{P}{\rho S} \phi_n(vt), \quad (6)$$

where $\omega_n = \lambda_n^2 \sqrt{EI/\rho S}$ are the angular frequencies of the undamped system and ζ_n the damping ratio. The initial conditions that are described in Eq. (3) can be written as:



$$\begin{cases} q_n(0) = 0, \\ \dot{q}_n(0) = 0. \end{cases} \quad (7)$$

The time-history of the strain at a given point located at a distance z_0 below the neutral axis of the beam is given by:

$$\varepsilon(x, t) = z_0 u''(x, t) = -z_0 \sum_n \lambda_n^2 \phi_n(x) q_n(t). \quad (8)$$

As defined, the acceleration at this point is the second derivative of the displacement with respect to time:

$$a(x, t) = \ddot{u}(x, t) = \sum_n \phi_n(x) \ddot{q}_n(t). \quad (9)$$

Particular and homogeneous solutions. The general solution of Eq. (6) is the sum of a particular solution and the homogeneous solution:

$$q_n(t) = q_n^p(t) + q_n^h(t). \quad (10)$$

Substitute Eq. (10) into the Eq. (4), we have:

$$u(x, t) = \sum_n \phi_n(x) (q_n^p(t) + q_n^h(t)) = u^p(x, t) + u^h(x, t). \quad (11)$$

By the same concept as the displacement, the strain and the acceleration are also decomposed into two components: the particular component and the homogeneous one, in which only the time-dependent party $q_n(t)$ is decomposed:

$$\varepsilon(x, t) = -z_0 \sum_n \lambda_n^2 \phi_n(x) (q_n^p(t) + q_n^h(t)) = \varepsilon^p(x, t) + \varepsilon^h(x, t), \quad (12)$$

$$a(x, t) = \sum_n \phi_n(x) (\ddot{q}_n^p(t) + \ddot{q}_n^h(t)) = a^p(x, t) + a^h(x, t). \quad (13)$$

Particular component. Hereafter, for simplification, we neglect the damping effect. By using the method of undetermined coefficients, a particular solution of Eq. (6) is given by the following expression [8]:

$$q_n^p(t) = \frac{\mathcal{A}P}{\rho S \omega_n^2 f(\gamma_n, \zeta_n)} \sin(\Omega_n t - \varphi_n), \quad (14)$$

where $\Omega_n = \lambda_n v = n\pi v/L$ and $\gamma_n = \Omega_n/\omega_n = v/v_{cn}$. We denote $v_{cn} = \lambda_n \sqrt{EI/\rho S}$ the critical velocity which corresponds to the n^{th} vibration mode of the beam. $f(\gamma_n, \zeta_n) = \sqrt{(1 - \gamma_n^2)^2 + (2\zeta_n \gamma_n)^2} \approx 1 - \gamma_n^2$ for some small value of γ_n, ζ_n ($\leq 10\%$) and $\tan \varphi_n = 2\zeta_n \gamma_n / (1 - \gamma_n^2)$.

The particular component of strain and acceleration are then calculated:

$$\varepsilon^p(x, t) = -z_0 \sum_n \lambda_n^2 \phi_n(x) q_n^p(t), \quad (15)$$

$$a^p(x, t) = \sum_n \phi_n(x) \ddot{q}_n^p(t) = - \sum_n \lambda_n^2 v^2 \phi_n(x) q_n^p(t). \quad (16)$$

From Eqs. (15), (16), we deduce the linear relation between the particular acceleration and the particular strain:



$$a^p(t) = \frac{v^2}{z_0} \varepsilon^p(t). \quad (17)$$

Homogeneous components. The right hand side of the Eq. (6) is set to be zero. Its homogeneous solution can be written as:

$$q_n^h(t) = [C_n \sin(\omega_n^d t) + D_n \cos(\omega_n^d t)] e^{-\omega_n \zeta_n t}, \quad (18)$$

with $\omega_n^d = \omega_n \sqrt{1 - \zeta_n^2}$ the damped angular frequency of the system for the n^{th} natural mode. The coefficients C_n, D_n are determined from the initial conditions in Eq. (7). We deduce also:

$$\begin{cases} q_n^h(0) = -q_n^p(0), \\ \dot{q}_n^h(0) = -\dot{q}_n^p(0). \end{cases} \quad (19)$$

When the load passes the beam at small velocity ($\gamma_1 \leq 10\%$) and the damping ratio is also small ($\zeta_n \leq 5\%$), we can assume that: $\ddot{q}_n^h(t) \approx -\omega_n^2 q_n^h(t)$. We have:

$$\varepsilon^h(x, t) = -z_0 \sum_n \lambda_n^2 \phi_n(x) q_n^h(t), \quad (20)$$

$$a^h(x, t) = \sum_n \phi_n(x) \ddot{q}_n^h(t) \approx - \sum_n \omega_n^2 \phi_n(x) q_n^h(t). \quad (21)$$

Note that $v_{cn} = \lambda_n \sqrt{EI/\rho S} = \omega_n/\lambda_n$, we deduce the following relation between the homogeneous acceleration and the homogeneous strain for the n^{th} mode:

$$a_n^h(t) = \frac{v_{cn}^2}{z_0} \varepsilon_n^h(t). \quad (22)$$

2.3 Load and damage identification by optimization

Load and damage identification. Now, by suitable sensors placement, we study only the first mode of the homogeneous components. The dynamic strain and acceleration can be written as:

$$\varepsilon(t) \approx \varepsilon^p(t) + \varepsilon_1^h(t), \quad (23)$$

$$a(t) \approx a^p(t) + a_1^h(t). \quad (24)$$

From the Eqs. (23), (24) and the proportional relations that are described in Eqs. (17), (22), we have:

$$\varepsilon(t) = \frac{z_0}{v_{c1}^2} a(t) + \left(1 - \frac{v^2}{v_{c1}^2}\right) \varepsilon^p(t). \quad (25)$$

We introduce the quasi-static strain, which can be defined as the limit of the particular strain when the velocity of the load approaches zero. As only the first mode is retained, for a small speed of the load ($\gamma_1 = v/v_{c1} \leq 10\%$), from Eq. (14), we have:

Essayer de clarifier comment on obtient cette relation à partir de (14)

$$\varepsilon^s(t) \approx (1 - \gamma_1^2) \varepsilon^p(t) = \left(1 - \frac{v^2}{v_{c1}^2}\right) \varepsilon^p(t). \quad (26)$$

On the other hand, the quasi-static strain can be expressed as:



$$\varepsilon^s(t) = z_0 \frac{M(t)}{EI} = z_0 \frac{P}{EI} \mathcal{M}_{se}(t). \quad (27)$$

where $\mathcal{M}_{se}(t)$ is the bending moment influence line ordinate at strain sensor's location in time t .

Hence, the Eq. (25) becomes:

$$\varepsilon(t) = \frac{z_0}{v_{c1}^2} a(t) + z_0 \frac{|P|}{EI} \mathcal{M}_{se}(t) = c_d a(t) + c_s \mathcal{M}_{se}(t), \quad (28)$$

where $c_d = z_0/v_{c1}^2 = z_0 \rho S L^2 / \pi^2 EI$ and $c_s = z_0 P / EI$.

For the measured signal ε_m, a_m , we can define the objective function J_ε as the sum of squared difference between the measured and the approximated strain signal:

$$J_\varepsilon = \|\varepsilon_m(t) - c_d a_m(t) - c_s \mathcal{M}_{se}(t)\|_2^2. \quad (29)$$

We search the coefficients c_d, c_s that minimize J_ε . In case of an intact beam, the coefficient c_s is proportional to the moving load. Otherwise, when the beam is damaged, both c_s and c_d is inversely proportional to the bending stiffness EI . When the load of the vehicle is unknown, we can use c_d to evaluate the damage of the beam.

Axle spacing identification. In case of the passage of a multiple-axle vehicle, the quasi-static strain is determined by superposition of the quasi-static strain which is caused by each individual force. Assume that a vehicle of M axles passes over the beam with a constant velocity v . Let's denote d_i the distance between the i^{th} and $(i+1)^{\text{th}}$ axle and $D_{i-1} = \sum_{j=1}^{i-1} d_j$ is the distance between the first axle and the i^{th} axle ($i \geq 2$). By convention, $D_0 = 0$. The quasi-static strain is given by:

$$\varepsilon^s(t) = \frac{z_0}{EI} \left[P_1 \mathcal{M}_{se}(t) + P_2 \mathcal{M}_{se}\left(t - \frac{D_1}{v}\right) + P_3 \mathcal{M}_{se}\left(t - \frac{D_2}{v}\right) + \dots \right]. \quad (30)$$

The objective function J_ε in the Eq. (23) now becomes:

$$J_\varepsilon = \left\| \varepsilon_m(t) - c_d a_m(t) - \sum_{i=1}^M c_{si} \mathcal{M}_{se}\left(t - \frac{D_{i-1}}{v}\right) \right\|_2^2 \quad (31)$$

where $c_{si} = z_0 P_i / EI$.

For a set of given distances $\mathbf{d} = \{d_1, d_2, \dots, d_M\}$, we search firstly a set of values $\mathbf{c}_{\min} = \{c_d^{\min}, c_{s1}^{\min}, c_{s2}^{\min}, \dots, c_{sM}^{\min}\}$ which minimize Eq. (22). Then, we optimize the following objective function J_d :

$$J_d = \left\| \varepsilon_m(t) - c_d^{\min} a_m(t) - \sum_{i=1}^M c_{si}^{\min} \mathcal{M}_{se}\left(t - \frac{D_{i-1}}{v}\right) \right\|_2^2 \quad (32)$$

Once the distances \mathbf{d}^* that minimize the function J_d are determined, we have the associated coefficients $\mathbf{c}^* = \{c_d^*, c_{s1}^*, c_{s2}^*, \dots, c_{sM}^*\}$ that have the same meaning as mentioned before.

3 NUMERICAL RESULTS AND DISCUSSION

3.1 Intact beam: Loads and axle distances identification

In this section, a simply supported Euler-Bernoulli beam is studied. The length of the beam is $L = 10 \text{ m}$. The cross section is square of dimensions $b = h = 10 \text{ cm}$. The beam is made of steel, which has the modulus of elasticity $E = 210 \text{ GPa}$ and the density $\rho = 7800 \text{ kg/m}^3$. The damping ratio of steel is equal to 2%. The fundamental frequency of the beam is $f_0 = (\pi/2L)\sqrt{EI/\rho S} =$



2.35 Hz. The beam is subjected to a 3-axle vehicle whose loads are $P_1 = 1 \text{ kN}$ and $P_2 = P_3 = 2 \text{ kN}$. The distances between the loads P_1 - P_2 and P_2 - P_3 are respectively $d_1 = 3.5 \text{ m}$ and $d_2 = 1.4 \text{ m}$ (Fig. 2). At $t = 0$, the vehicle enters the beam with constant speed $v = 5 \text{ m/s}$, which is about 11% v_{c1} . Two sensors are placed at the midpoint of the beam, namely an accelerometer and an optical strand (long fiber optic strain sensor) of length 2 m.

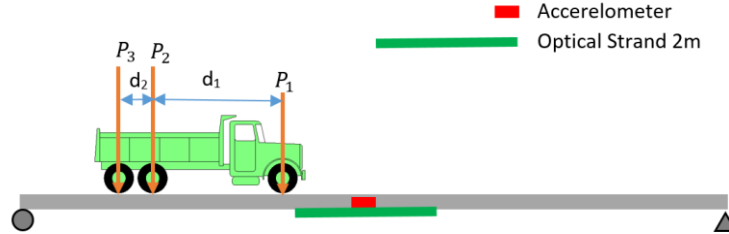


Figure 2: Three-axle vehicle passing a simply supported beam at constant speed.

The responses are calculated analytically and the 12 first frequencies are taken into account, so that the cumulative effective mass factor is 90,4%. Then, these responses are filtered by a low-pass filter with a cut-off frequency that is equal to $1.7f_0$. Then, they are served as input of optimization process.

Table 1 presents the results of the optimization process in comparison with the reference for some parameters. The errors of the estimated values are relatively small.

Table 1: Results of optimization

Parameter	Notation	Unit	Reference values	Estimated values	Error (%)
1 st critical velocity	v_{c1}	m/s	47.06	47.29	0.50
Distance P_1 - P_2	d_1	m	3.50	3.50	0.02
Distance P_1 - P_2	d_2	m	1.40	1.42	1.39
First axle load	P_1	N	1000	993.15	0.68
Second axle load	P_2	N	2000	1975.55	1.22
Third axle load	P_3	N	2000	2032.04	1.60

3.2 Damage beam: Parametric study

For the parametric study, we use the same steel beam as the previous example. A two-axle vehicle $P_1 = 1 \text{ kN}$ and $P_2 = 2 \text{ kN}$ passes over the beam with constant speed $v = 5 \text{ m/s}$, which is equal to 11 % the first critical velocity of the beam. The distance between these loads is $d = 3 \text{ m}$ (Fig. 3). The damage is modelled as a uniform decrease of bending stiffness over the length of the damaged zone [11]. In this study, we take a damaged zone of length l_d , with various **locations** and various reductions of bending stiffness ΔEI (the orange segment). The location of the damage is characterized by the position of its centre (x_c). Moreover, we denote l_0 the overlaying length of the damaged and the strain sensor (the green segment). An accelerometer and an optical strand of length 2 m are placed at the midpoint of the beam, like the previous example.

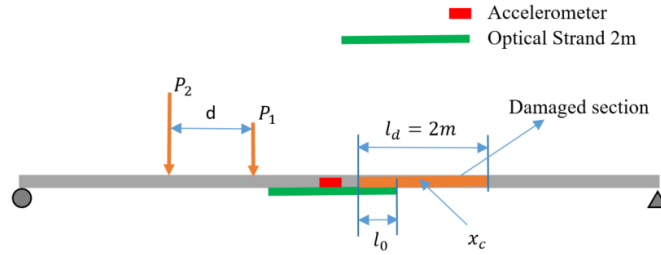


Figure 3: Damaged beam subjected to a 2-axle moving load.

The beam is modelled in *code_Aster*, an Open Source FEM software. The FEM model has 40 linear beam elements, which follows the Euler-Bernoulli beam theory. The transient response is calculated on a generalized basis, using Newmark's scheme for temporal integration with $\beta = 1/4$ and $\gamma = 1/2$. The maximum frequency taken into account is up to 100 Hz. The sampling frequency of the responses is set to be 100 Hz, which is common to most optical strands in service. In this parametric study, we investigate 2 cases:

1. The damaged zone is located at the center of the beam: $x_c = L/2$. Its length l_d and the reduction of stiffness ΔEI are of various values.
2. The length of the damage is fixed at $l_d = 2\text{ m}$, its position x_c and the reduction of stiffness ΔEI change (Fig. 3).

Aligner à droite le paragraphe ci-dessous

Figure 4 shows the dependence of relative change on c_d as a function of the length of the damage and the reduction of the stiffness ΔEI in case 1. When the length of the damage exceeds the length of the optical strand, the coefficient c_d remains constant (Fig. 4a). We can also see this on Fig 4b, the curves that correspond to $l_d = 2, 3, 4\text{ m}$ are coincident. When the stiffness reduction is small ($\leq 20\%$), the relative change of c_d varies linearly in terms of ΔEI . Over this value, the change on c_d is more sensitive to the level of the damage.

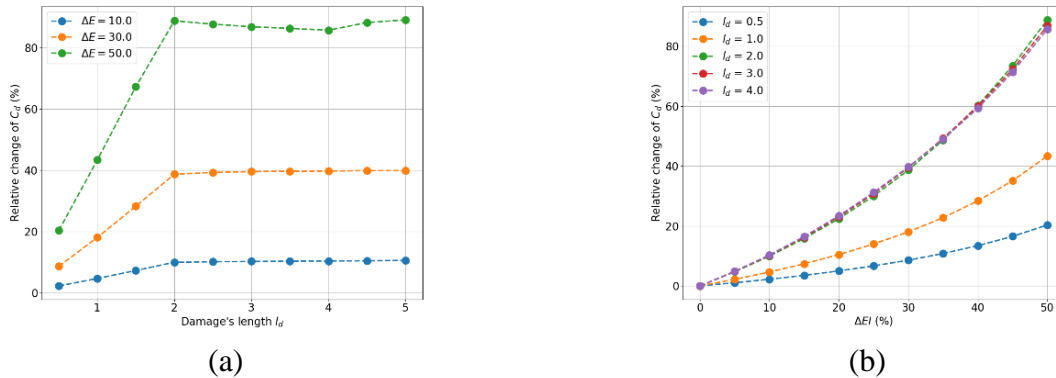


Figure 4: Case 1: Relative change of the coefficient c_d as a function of (a): the length of the damage l_c and (b): reduction of stiffness ΔEI .

Figure 5 shows the results of case 2. When the damage is located at midspan of the beam, the change on c_d is maximal (Fig 5b.). On Fig 5b., we see that the curves corresponding to $x_c = 4\text{ m}$ or $x_c = 6\text{ m}$ are very close, because they have the same overlaying length ($l_0 = 1$). When the damage is not covered by the optical strand, the coefficient c_d does not change.

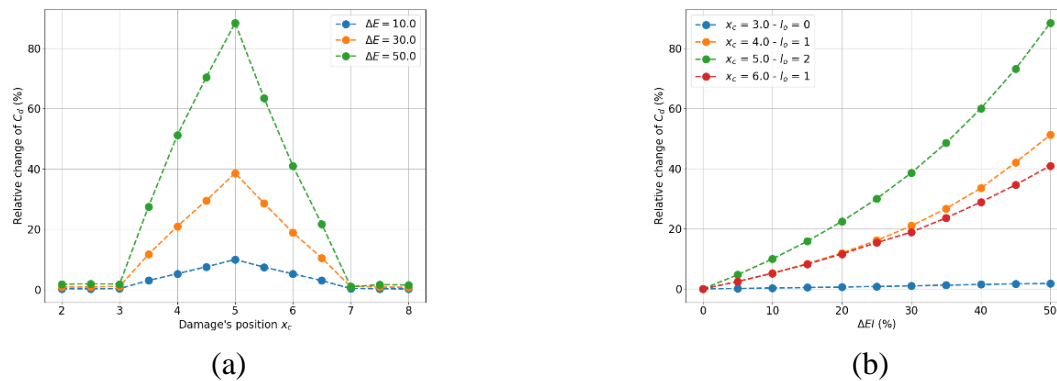


Figure 5: Case 2: Relative change of the coefficient c_d as a function of (a): the position of the damage x_c and (b): reduction of stiffness ΔEI .

In both cases, the estimated distance obtained from the optimization procedure is $d = 3 \pm 0.05\text{m}$, The accuracy is equal to 1.67 %.

CONCLUSION

In this paper, we have presented a novel method that allows to detect the presence of a damage on a simply supported beam and the distance between axles of a vehicle. Numerical results demonstrate the sensitivity of the proposed method to damage. In the case of an intact structure, the method can be used to determine the vehicle's load.

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