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# DAMAGE MECHANICS OF METAL-MATRIX COMPOSITES: VARIOUS LEVELS OF APPROACH

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**Abstract** Constitutive and damage mechanics of composite materials is discussed, with a special attention to unidirectional metal-matrix composites. Macroscopic thermo-elasto-visco-plastic and damage models based on micromechanical approaches are presented that offer various modeling capabilities between a two-phase and a multi-subvolume model. A micromechanics analysis of transverse creep, using periodic homogenization and finite elements demonstrates the role of the fiber-matrix interphase in the creep resistance of the composite. Such a numerical approach can be used to deliver directly the constitutive response at the component level by a multiscale structural analysis using the FE<sup>2</sup> imbricated finite-element method.

## 1. INTRODUCTION

Composite materials and structures differ from metallic ones in the following aspects:

- their low weight, and the corresponding design improvement perspective offered by their high specific mechanical properties;
- their versatility—in many applications, the possibility to optimize the material (volume fraction, sizes, positions, orientations of reinforcements) during the same design stage as the structural component itself;
- their ability to support such inserted systems as optical fibers, piezo-electric sensors, and actuators, leading to many future “smart structure” capabilities, for instance in active or passive in situ health monitoring systems;

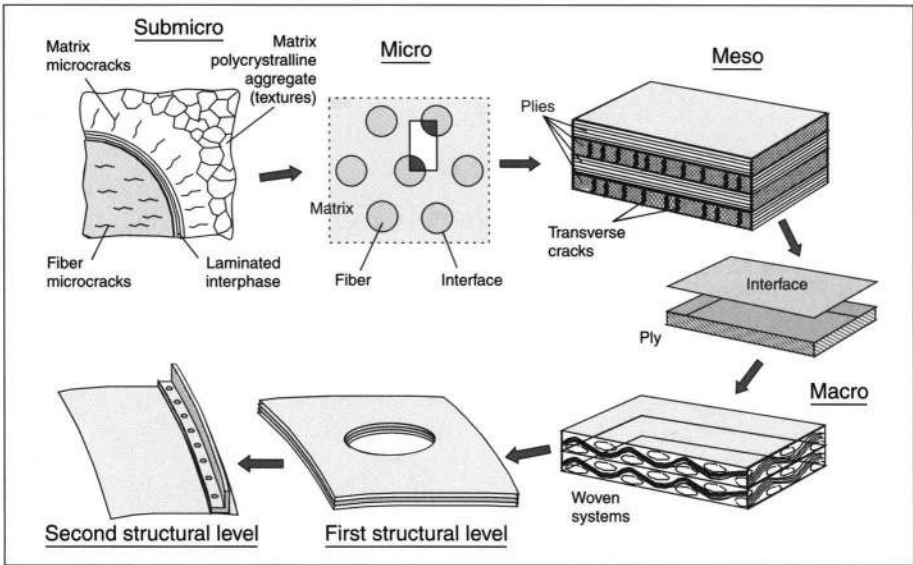


Figure 1 Schematic of the various scales in composite materials and structures.

- their cost, which is often a serious obstacle to their extensive use, as well as the relatively unstable material sources and material manufacturers;
- their multiscale structure, showing many different successive levels, as illustrated schematically in Fig. 1 for structural composites used in aeronautical applications—this is one of the difficulties in the mechanical analysis associated with the use of composites; metals also have several levels of microstructures, but at lower scales, and with a much more random organization;
- their damaging processes, during manufacturing and during operation: in metals initiation of any microcrack of some size could have a significant impact on the remaining lifetime, leading often to an accelerated process under load; contrarily, in composite structures the microcracks can appear very early, leading to a certain deterioration of mechanical properties, but without really affecting the component lifetime—damage in that case is one of the mechanisms by which the heterogeneous material accommodates the applied strain.

In this paper the two last aspects mentioned above will be illustrated and discussed in terms of modeling methodologies, as applied to unidirectional metal-matrix composites (MMCs). The specific structural

applications underlying the developed methods are the bling components (bladed rings) that should replace in some aerojet engines the classical technologies for compressor and turbine discs. The material in the corresponding experimental studies is a SiC/Ti MMC, but many aspects of the methods are still applicable to other classes of metallic composites (at least long fibers). In §2 we present a micromechanics-based set of constitutive and damage equations, exploiting Dvorak's transformation field analysis (TFA) method (Dvorak 1992, Dvorak and Benveniste 1992), which offers several variants for the damage-growth modeling—in the matrix, in the fiber, and at the fiber–matrix interface. The models are set in such a way that they can be considered and exploited as macroscopic constitutive laws. Section 3 summarizes a specific study of creep behavior of the composite (essentially transverse creep), in which is shown the duality between the matrix elasto–visco–plasticity and the interfacial damage, in order to explain experimental results of creep tests realized under vacuum.

Other aspects are treated elsewhere, for instance the component inelastic and damage analysis, based on a true “multiscale analysis”, the FE<sup>2</sup> method, using imbricated finite-element models in order to replace the macrolevel constitutive equations directly (Feyel 1999).

## **2. DEVELOPMENT OF A SET OF MICROMECHANICS-BASED CONSTITUTIVE EQUATIONS**

For the inelastic and damage modeling of composite materials, within a micromechanics based approach, we have various possibilities:

- the tools using averaged quantities in each constituent, similar to those employed for random microstructures, with Eshelby-based operators: in this context localization rules are often based on a secant or tangent writing of the local constitutive equations;
- the more-or-less analytical models based on a mixture of tangent and initial elastic operators, such as the ones developed by Voyiadjis and co-workers (Voyiadjis and Kattan 1993) or by Baptiste (Guo et al. 1997);
- the class of model developed by Aboudi and co-workers (Aboudi 1985, Pindera and Aboudi 1988), which defines a geometrically more-or-less sophisticated unit cell, with local fields approximated through a weak compatibility of constituent elasticity equations;
- the transformation field analysis (TFA) developed by Dvorak (Dvorak 1992, Dvorak and Benveniste 1992), which introduces a

systematic way of defining concentration and influence tensors, but writing a purely elastic rule, treating plastic strain and thermal expansion as eigenstrains of the same nature.

We decide to concentrate on the last type of approach, because it continues to treat elasto–plasticity (or elasto–visco–plasticity) of constituents through the correct strain partitioning. Moreover, within a single and consistent methodology, it offers various refinement possibilities, from the two-subvolume model (two-phase model) to the multi-subvolume model, which eventually degenerates to the finite-element periodic unit cell.

## 2.1. Thermo–elasto–visco–plastic model

The model is built up by combining two kinds of equation. The first is the constitutive equation of each constituent:

$$\boldsymbol{\sigma}_s = \mathbf{L}_s(\boldsymbol{\varepsilon}_s - \boldsymbol{\varepsilon}_s^p - \boldsymbol{\varepsilon}_s^{\text{th}}), \quad (1)$$

$$\dot{\boldsymbol{\varepsilon}}_s^p = \mathcal{F}_s(\boldsymbol{\sigma}_s, \dot{\boldsymbol{\sigma}}_s, \text{ISV}, \dots, T), \quad \boldsymbol{\varepsilon}_s^{\text{th}} = \boldsymbol{\alpha}(T - T_0), \quad (2)$$

in which  $s$  denotes the subvolume numbering (there are possibly several subvolumes for each phase),  $\mathbf{L}_s$  is the initial elastic stiffness operator, and the functional  $\mathcal{F}_s$  will not be defined more precisely—all kinds of local constitutive equations can be used, with internal state variables symbolized here as “ISV”. The thermal expansion  $\boldsymbol{\varepsilon}_s^{\text{th}}$  can be isotropic or not.

The second kind of equation is the localization rule that relates overall to local quantities, taking into account field interactions with eigenstrains. Using TFA, we have for the local strain in the subvolume  $s$ :

$$\boldsymbol{\varepsilon}_s = \mathbf{A}_s \mathbf{E} + \sum_r \mathbf{D}_{sr}(\boldsymbol{\varepsilon}_r^p + \boldsymbol{\varepsilon}_r^{\text{th}}), \quad (3)$$

where  $\mathbf{E} = \langle \boldsymbol{\varepsilon} \rangle = \sum_r c_r \boldsymbol{\varepsilon}_r$  is the average strain (in which  $c_r$  denotes the volume fraction of subvolume  $r$ ),  $\mathbf{A}_s$  and  $\mathbf{D}_{sr}$  being respectively the elastic strain concentration tensor and the influence tensor (between subvolume  $r$  and subvolume  $s$ ). Averaging  $\boldsymbol{\sigma}_s$  in (1) leads to the overall elastic constitutive equation

$$\boldsymbol{\Sigma} = \mathbf{L}(\mathbf{E} - \mathbf{E}^p - \mathbf{E}^{\text{th}}), \quad (4)$$

where  $\boldsymbol{\Sigma} = \langle \boldsymbol{\sigma} \rangle$ ,  $\mathbf{E}^p = \langle \mathbf{B}^T \boldsymbol{\varepsilon}^p \rangle$ ,  $\mathbf{E}^{\text{th}} = \langle \mathbf{B}^T \boldsymbol{\varepsilon}^{\text{th}} \rangle$ , with  $\mathbf{B}$  the stress concentration tensor  $\mathbf{B}_s = \mathbf{L}_s \mathbf{A}_s \mathbf{L}^{-1}$  and  $\mathbf{L}$  the overall elastic stiffness defined by  $\mathbf{L} = \langle \mathbf{L} \mathbf{A} \rangle = \langle \mathbf{A}^T \mathbf{L} \mathbf{A} \rangle$ . The reciprocal formulation can also be defined

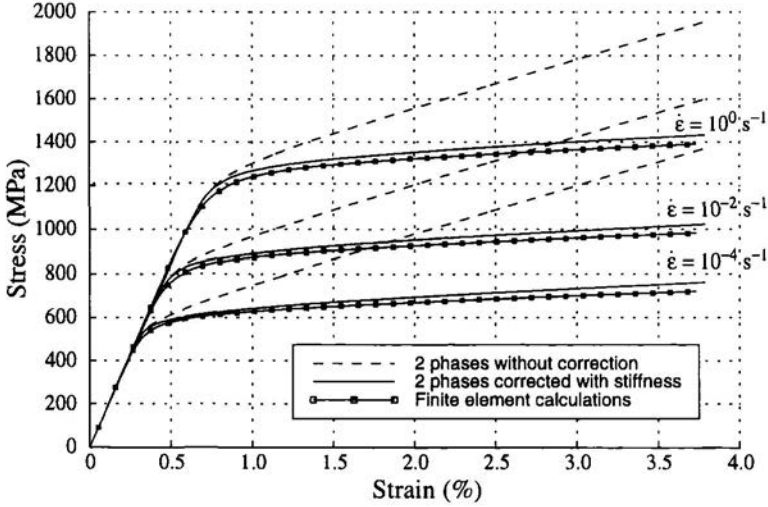


Figure 2 Correction with the asymptotic tangent stiffness for various macroscopic strain rates (volume fraction 35%).

consistently for the stress localization (not needed here). In the case of a two-phase composite system, with one subvolume each, the operators  $\mathbf{A}_s$ ,  $\mathbf{B}_s$  can be obtained from the Eshelby tensor, either through the Mori–Tanaka method (Mori and Tanaka 1973) or through the self-consistent method. The influence tensors are also easily determined for the two-subvolume model, with exact closed-form relations (Dvorak 1992).

When using a more sophisticated model, with several subvolumes per phase, the initial elastic concentration tensors  $\mathbf{A}_s$ ,  $\mathbf{B}_s$  and the influence tensors  $\mathbf{D}_{sr}$  are determined once, by a prior set of numerical analyses (using for instance periodic homogenization and finite-element method, as indicated in Dvorak et al. 1994 or Pottier 1998).

Considering the overly stiff results generally obtained for the two-subvolume model (see Suquet 1997, Zaoui and Masson 1998), we have proposed an asymptotic correction method (Pottier 1998, Chaboche et al. 2000) that modifies the localization rule into

$$\boldsymbol{\varepsilon}_s = \mathbf{A}_s \mathbf{E} + \sum_r \mathbf{D}_{rs} (\mathbf{K}_r \boldsymbol{\varepsilon}_s^p + \boldsymbol{\varepsilon}_s^{\text{th}}), \quad (5)$$

where the 4th-rank tensors  $\mathbf{K}_r$  are determined by identification of the rate form of (5) with its “tangent” format  $\dot{\boldsymbol{\varepsilon}}_s = \mathcal{A}_s \mathbf{E}$  in which the tangent concentration tensor  $\mathcal{A}_s$  is determined from the knowledge of the asymptotic tangent stiffness  $\mathcal{L}_s$  of each subvolume, directly related with

the linear kinematic hardening model used in the matrix. More details are given in (Chaboche et al. 2000).

The improvement of using this correction factor instead of the original TFA is illustrated in Fig. 2 for the transverse traction at three overall strain rates (the matrix is visco-plastic) by comparison with a periodic unit cell finite-element calculation similar to the ones that will be reported in §3. Other comparisons are reported elsewhere (Chaboche et al. 2000).

## 2.2. Various damage modeling approaches

In terms of delivering a micromechanics-based overall constitutive equation of composite materials, three approaches could be used, depending on the kind of deterioration that develops:

- If microcracks are large enough compared with the heterogeneous microstructure (fiber size) we could proceed in two steps: define first an effective homogeneous behavior of the undamaged composite, then input microcracks in it and deliver the overall behavior of the damaged material via a second micro-to-macro analysis. This is the method implicitly used in laminate composites when introducing transverse cracking as a meso-damage in the ply volume elements (Allix et al. 1990).
- If microcracks are small enough, in the matrix or in the fiber, compared with the fiber size, we can use a continuum damage mechanics (CDM) methodology in each of these phases, then first modify the initial local constitutive equations through a micromechanics-based effective stress concept, and second, make the micro-macro averaging of the composite with its damaged constituents. This is for example the method used by Voyiadjis (Voyiadjis and Kattan 1993) for modeling damage in MMCs.
- If microcracks and fiber are of the same size, especially when the crack develops at the fiber-matrix interface, none of the above approaches is acceptable (i.e. there is no scale separability). One simplification consists in approximating the presence of an interface crack by a transverse stiffness reduction in the fiber (Guo et al. 1997). It is also the solution adopted in the direct simplified model developed below. A better approximation is sought by generalizing the TFA method, with a large number of subvolumes, including interphase subvolumes. We present below the corresponding model, called the generalized eigenstrain model.

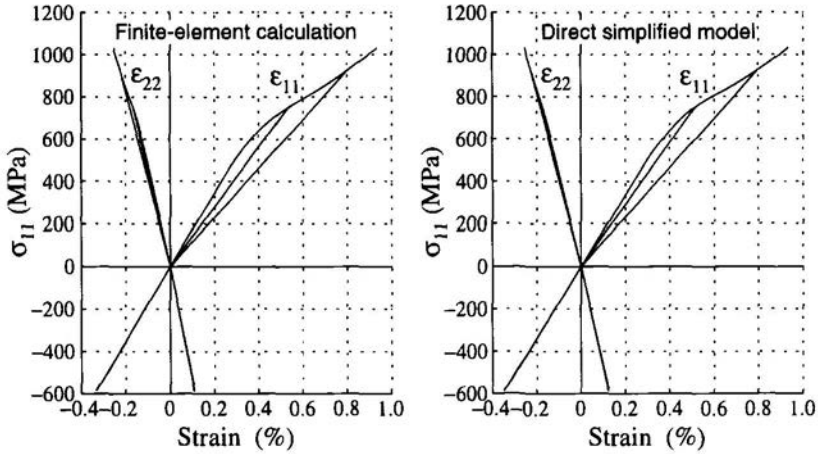


Figure 3 Direct simplified model and finite-element computation for transverse tensile loading–unloading: case of an elastic matrix.

**Direct simplified model (DSM).** We consider only two phases—the matrix and the fiber—and assume a CDM approach to introduce a damage  $d_s$  in each phase (either a scalar or a tensorial variable). Now the constitutive equation is modified as

$$\boldsymbol{\sigma}_s = \tilde{\mathbf{L}}_s(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_s^p - \boldsymbol{\varepsilon}_s^{\text{th}}). \quad (6)$$

The visco–plastic strain rate depends on damage, as well as the elastic stiffness  $\tilde{\mathbf{L}}_s$ . Figure 4 explains the principle of the direct simplified model. Knowing damage  $d_s$  after integration (*i*), we define (*e*) the effective damaged elastic stiffness  $\tilde{\mathbf{L}}_s(d_s)$  (not expressed here). Using closed-form Eshelby solutions and the Mori–Tanaka scheme, we can define the effective strain concentration tensor  $\tilde{\mathbf{A}}_s$ . For a two-phase system, we know explicitly the corresponding  $\tilde{\mathbf{D}}_{sr}$  from  $\tilde{\mathbf{L}}_s$  and  $\tilde{\mathbf{A}}_s$  (Dvorak 1992). The localization rule (*l*), Eqn. (3) (with  $\tilde{\mathbf{A}}_s$  and  $\tilde{\mathbf{D}}_{sr}$  instead of  $\mathbf{A}_s$  and  $\mathbf{D}_{sr}$ ), delivers  $\boldsymbol{\varepsilon}_s$  from the controlled overall strain  $\mathbf{E}$ . Then, we obtain successively  $\boldsymbol{\sigma}_s$  by the elastic constitutive equation (*ce*) and  $\boldsymbol{\Sigma}$  by averaging (*h*).

In this model, the localization tensors have to be defined at each time step of a given loading. It is the reason that the model cannot be generalized to a refined discretization (more than two subvolumes). Figure 3 gives an example of comparison between this direct simplified model and a complete analysis of a unit cell by finite-element and periodic homogenization. This is a theoretical example of transverse tension–compression of a unidirectional elastic metal–matrix composite. The matrix is elastic, the fiber is elastic damageable, and damage deactivation is taken



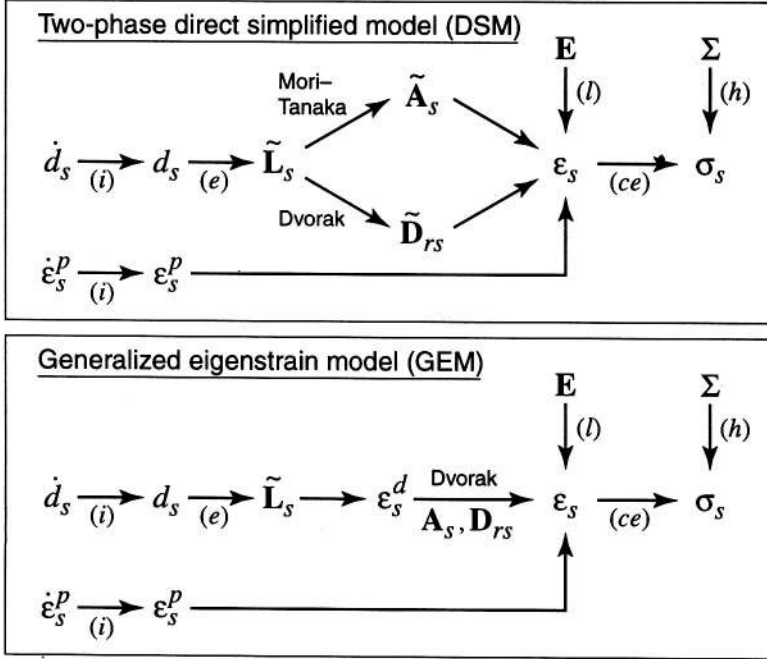


Figure 4. Flowchart of the two damage models.

into account (closure of microcracks under compression). The comparison appears as excellent in that case, as well as in the case where the visco-plasticity of the matrix is taken into account.

**Generalized eigenstrain model (GEM).** This model is developed in order to refine the discretization. Subvolumes are used in order to refine the discretization, to describe the interphase behavior (with damage) and the stress redistributions in the matrix. The model uses only the initial (undamaged) concentration and influence tensors of TFA, obtained initially by a number of elastic finite-element calculations. Its key specificity is to transform the elastic changes due to damage into an additional eigenstrain. We replace (6) by

$$\sigma_s = \mathbf{L}_s(\epsilon_s - \epsilon_s^p - \epsilon_s^{\text{th}} - \epsilon_s^d), \quad (7)$$

where the damage strain  $\epsilon_s^d$  is given as

$$\epsilon_s^d = (\mathbf{I} - \mathbf{L}_s^{-1}\tilde{\mathbf{L}}_s)(\epsilon_s - \epsilon_s^p - \epsilon_s^{\text{th}}). \quad (8)$$

Figure 4 indicates the corresponding flowchart of the model. It uses the following localization equation in place of (5):

$$\boldsymbol{\varepsilon}_s = \mathbf{A}_s \mathbf{E} + \sum_r \mathbf{D}_{sr} \left( \mathbf{K}_r (\boldsymbol{\varepsilon}_s^p + \boldsymbol{\varepsilon}_s^d) + \boldsymbol{\varepsilon}_s^{\text{th}} \right). \quad (9)$$

We remark that the approach is also able to take into account the variations of the elastic stiffness with temperature (without modifying  $\mathbf{A}_s$ ,  $\mathbf{D}_{sr}$ ). Its flexibility and its comparison with experiments appear to be very satisfactory. When applied in the particular case of two subvolumes, the GEM and DSM models deliver exactly the same stress–strain response.

### 3. MICROMECHANICAL NUMERICAL MODELING OF TRANSVERSE CREEP BEHAVIOR

#### 3.1. Experimental study

Silicon carbide–titanium MMCs will be used in the future bling technology as the reinforcement to sustain the high hoop stresses generated by centrifugal effects. The essential characteristics (tensile strength, fatigue, creep) have been studied mainly in the axial direction of unidirectional composites (Ohno et al. 1994). However, transverse creep is a condition that may be of some significance, associated with secondary centrifugal radial stresses generated by external parts and the blades. An experimental study has been performed at ONERA, in order to characterize the creep resistance of the composite under transverse loads (Carrère et al. 2000).

The material is a titanium (Ti6242) matrix reinforced with long unidirectional SiC (SCS6) fibers with a volume fraction of 35%. The specimens used for the experiments are taken from an 8-ply plate manufactured by Snecma, using the fiber-foil technique. The behavior of the titanium matrix is elasto–visco–plastic, the fibers are supposed to remain elastic, and the protecting coat around the fibers (to prevent a reaction between the titanium and the SiC) is a brittle zone where the decohesion and damage in the transverse direction of the composite will take place.

Creep tests were performed at 500°C, under vacuum to avoid environmental problems, for several loads (Carrère et al. 2000). Results obtained are shown in Fig. 5 for the secondary creep rate, and can be summarized as follows:

- Below a critical stress, estimated here around 225 MPa, the creep rate increases gradually and the lifetime is long—there is no rupture even for long creep time (> 1500 h).

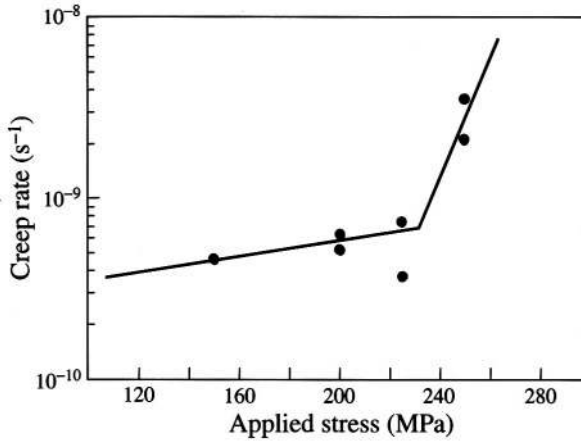


Figure 5 Measured secondary creep rates of SCS6/Ti6242 at 500°C.

- For applied stresses over 225 MPa, the creep rate increases drastically, leading to short failure times.

### 3.2. Micromechanical analysis

A micromechanical analysis based on the periodic homogenization assumption and the finite-element method is performed in order to explain the experimentally observed behavior. The regular position of the fibers

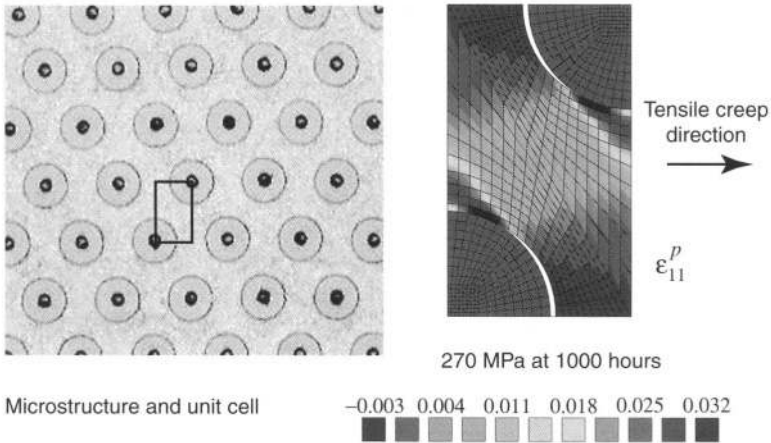


Figure 6 Composite microstructure, definition of a unit cell, and axial visco-plastic strain field after 1000 hours creep under 270 MPa.

inside the matrix, as shown in Fig. 6, allows us to select a unit cell that defines the representative volume element of the composite.

The constitutive behaviors considered for each constituent, fiber, matrix and interphase, are as follows:

- The fiber is elastic and transversely isotropic. Its characteristics are given by literature data and independent measurements.
- The matrix is elasto–visco–plastic, obeying a classical isotropic cyclic constitutive equation that incorporates two back stresses, one of which obeys the nonlinear kinematic hardening rule, the second being linear. This constitutive equation has been determined previously from tension–compression tests performed on pure Ti6242 over the whole temperature domain (20–870°C) (Baroumes and Vincon 1995). The effects of viscosity observed at high temperature are described with a power-law equation determined from relaxation (Malon 2000) and creep tests (Carrère et al. 2000).
- The carbon interphase is treated using interface elements that obey Tvergaard’s progressive debonding model (Tvergaard 1990). In fact we use a modified model (Chaboche et al. 1997), which introduces friction effects during the debonding phase, in order to eliminate the complete shear unloading when debonding is complete. The model combines mode I and mode II decohesion, with a unique damage parameter  $\lambda$  ( $0 \leq \lambda \leq 1$ ), which is the maximum value of the quadratic norm of relative mode I and mode II openings (opening displacement divided by its value at complete debonding). It uses 4 parameters (for each mode a maximum stress and a fracture energy), in addition to the Coulomb friction parameter  $\mu$ . The mode II and friction parameters are determined by push-out tests (Guichet 1998) and the complete numerical analysis of this test. The mode I is adjusted from the transverse tensile test. The corresponding simulations allow us to describe fairly well all the important features: onset of decohesion before plasticity takes place, progressive evolution of the unloading stiffness, then appearance of plasticity and irreversible strains (with hysteretic effects under cyclic loads), interfacial closure effects when reverse loadings are performed, etc.

The numerical simulation performed on the unit cell presented in Fig. 6(b) follows as closely as possible the experimental procedure. The main steps of the calculation are summarized as follows: (i) The first step consists in determining the residual stresses induced by the manufacturing process, and applying the load/temperature history undergone by the composite (referred to as “manufacturing” in the following curves).

(ii) The creep load is imposed, giving the response of the primary state (referred to as “loading”), (iii) Finally, the creep test itself is simulated (secondary stage). The third stage, directly linked to the final failure of the specimen, is not modeled because, in this analysis, the crack initiation and propagation in the matrix are not taken into account.

Figure 7 presents simulated results for two loadings: the first one, 200 MPa, below the critical stress, and the second one, 270 MPa, above this stress. The figure shows the evolution of the damage parameter around the interface between the fiber and the matrix, at different out-

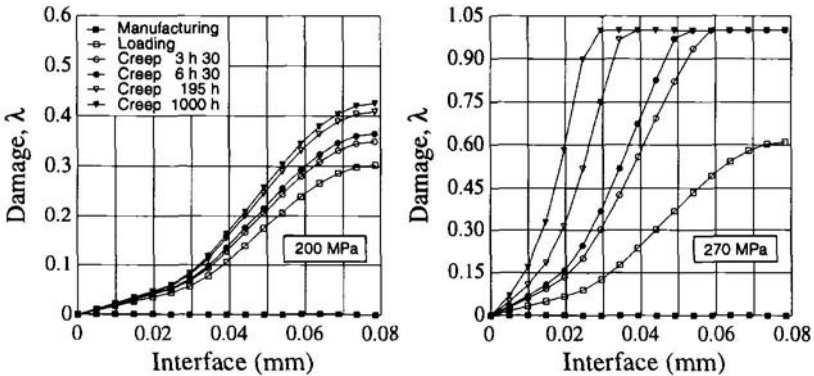


Figure 7 Local interfacial damage evolution for creep under 200 and 270 MPa.

put times corresponding to the steps described before. It can be seen that the manufacturing process does not induce significant damage. Once the load is applied, damage starts and increases progressively. The comparison between the damage evolution induced by the two loads shows that for low levels, damage increases slowly (and sometimes saturates) during the creep without breaking the interface, though for higher loads the interface begins to break and this process continues until the complete interface debonds. Additional results are discussed in (Kruch et al. 2000).

Therefore, the micromechanical analysis confirms that the matrix controls the global deformation of the composite, but the load carried by the matrix is controlled by the strength of the interface. For high stress levels (near the critical stress), the load is carried totally by the matrix (the interface being completely broken), leading to the failure of the composite when a crack initiates and propagates in the matrix. For lower stress levels the interfacial damage is not very important and the load is carried partly by the fibers and the matrix, leading to long lifetimes. The calculated secondary creep rates underestimate the experimental ones by a significant factor. There are two main explanations: (1) the

interface debonding model was determined at room temperature and applied without change for high temperatures (taking into account only the different thermally induced residual stresses), and (2) the titanium matrix was identified from tests on a standard monolithic material whose microstructure could be different from the one of the foil material used to manufacture the composite. The anisotropic texture of the matrix is right now analyzed at ONERA using the electron back-scattering diffraction (EBSD) technique. Numerical computations are under way, taking into account the measured textures in the matrix visco-plastic behavior, by using a polycrystalline aggregate model (Pilvin and Cailletaud 1990).

## 4. CONCLUSIONS

The considered damage modeling approaches have shown the following capabilities for MMCs or similar composite situations:

- Micromechanics-based constitutive equations using the TFA approach are able to describe both thermo-elasto-visco-plasticity of the matrix and the damage evolution in each constituent.
- Two damage approaches—the direct simplified model and the generalized eigenstrain model—with different capabilities in terms of degrees of freedom, are shown to be consistent with periodic homogenization results. Their application in true components as overall constitutive and damage equations is underway.
- The finite-element micromechanics analysis of transverse creep loading conditions is able to predict quite well the experimental results. It assumes periodicity and uses a thermo-elasto-visco-plastic constitutive equation for the matrix and an interface debonding model for the fiber-matrix interphase (determined independently by push-out and transverse tensile tests).
- Improvements in the quantitative prediction of the secondary creep rate is expected by taking into account the true matrix visco-plastic behavior, including texture effects. Such an analysis is underway, using a polycrystalline aggregate model.
- A higher-level multiscale analysis of components (not presented) has also been developed (Feyel 1999). The  $FE^2$  method uses imbricated finite elements in order to replace the overall constitutive equations in a finite-element structural analysis. This method has the objective to deliver all the stress-strain redistributions at the microscale, especially in regions where the two scales do interact, very often the most critical regions of the component.

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