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Applications of single photons to quantum communication and computing

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1 Supplementary Information

1.1 How to define a single photon and to characterize a single photon source?

Due to wave-particle duality, a quantum state of light possesses two complementary descriptions. The first one that we shall denote the Fock representation expresses the quantum state of a quantized mode of light in terms of photon numbers, a typical corpuscular approach (1.1.1). Thereafter we shall discuss the wave description which is a spatio-temporal description (1.1.2).

1.1.1 Corpuscular approach

1.1.1.1 A single-photon in the Fock representation: $N=1$ Fock state of a given mode

A mode is a solution of the classical Maxwell's equations. In many applications, it is convenient to consider monochromatic modes of the plane wave type, with well-defined frequency, direction of propagation and polarisation, having in mind that plane waves constitute convenient mathematical idealisations, but do not exist in nature because they are not localized, contrary to light pulses produced in a laboratory. However, their properties are very close to those of quasi-monochromatic wave packets which are superpositions of plane waves with very close frequencies/wavelengths. The spread of the wavelength of a wave packet $\delta\lambda$ is typically quite smaller than its central wavelength so that:

$$\delta(\lambda)/\lambda \ll 1 \text{ thus } \lambda/\delta(\lambda) \gg 1$$

by making use of the Heisenberg uncertainties, one finds that:

$$\delta(x) \cdot \delta(1/\lambda) = \left(\frac{\delta(x)}{\lambda}\right) \cdot \left(\frac{\delta(\lambda)}{\lambda}\right) \geq 1/4\pi$$

Finally, one gets

$$\delta(x)/\lambda \geq \lambda/4\pi\delta(\lambda) \gg 1 \text{ so that } \delta(x) \gg \lambda$$

The central wavelength λ of a wave packet is thus quite smaller than the spatial size $\delta(x)$ and a wave packet can be seen as a monochromatic plane wave modulated by an envelope of spatial extent quite larger than its wavelength.

It can be shown that, in a quantum approach, the physical properties of a quasi-monochromatic mode are equivalent, in good approximation, to those of a quantised one-dimensional harmonic oscillator whose Hamiltonian can be expressed in terms of the photon number operator \hat{N} through $\hat{H} = \hbar\omega(\hat{N} + 1/2)$ with $\omega = \sqrt{\frac{k}{m}}$ being the classical pulsation of the oscillator. The eigenstates of \hat{H} (and thus \hat{N}) are the so-called Fock states and, in this approach, a single photon is nothing else than an elementary excitation or $N = 1$ Fock state $|N = 1\rangle$ meaning that it is an eigenstate of \hat{N} for the eigenvalue $N = 1$ (and thus an eigenstate of \hat{H} for the eigenvalue $3\hbar\omega/2$). Such a state can be obtained after letting act the creation (or raising) operator $\hat{a}^\dagger = \frac{1}{\sqrt{2}}(\hat{q} - i\hat{p}_q)$ (where $\hat{q} = \hat{x}/\sqrt{\hbar/m\omega}$ and $\hat{p}_q = \frac{\partial}{i\partial q}$) on the vacuum state (which is itself a $N = 0$ Fock state $|N = 0\rangle$), of energy eigenvalue $\hbar\omega/2$. Note that the Hamiltonian of a one-dimensional harmonic oscillator ($\hat{H} = \hat{p}_x^2/2m + k\hat{x}^2/2$) can be expressed in terms of the creation and annihilation operators (resp. $\hat{a}^\dagger = \frac{1}{\sqrt{2}}(\hat{q} - i\hat{p}_q)$ and $\hat{a} = \frac{1}{\sqrt{2}}(\hat{q} + i\hat{p}_q)$) through the relation:

$$\hat{H} = \hbar\omega \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) = \hbar\omega \left(\hat{N} + \frac{1}{2} \right) = \hbar\omega \left(\sum_{N=0,1,2,\dots}^{\infty} N |N\rangle \langle N| + 1/2 \right)$$

The N Fock state can be found using the following relation:

$$|N\rangle = \frac{(\hat{a}^\dagger)^N}{\sqrt{N!}} |N=0\rangle$$

The Fock state representation emphasises the corpuscular nature of light, but in virtue of wave-particle duality, quantum light pulses are also waves localised in space and travelling throughout time. These waves can be described thanks to Glauber's formalism as explained in the next paragraph.

1.1.1.2 How to characterize a single photon source (1): measurement of $g^{(2)}$

Glauber's theory describes what to measure experimentally in order to ensure one's source does provide single photons. For that, one must measure the intensity autocorrelation function of the light electromagnetic field under consideration. The so-called $g^{(2)}(\tau)$ function is the second-order correlation function of a signal with itself, in this case as a function of time τ . The $g^{(2)}(\tau)$ function reads as follows:

$$g^{(2)}(\tau) = \frac{\langle \hat{a}^\dagger(t) \hat{a}^\dagger(t+\tau) \hat{a}(t+\tau) \hat{a}(t) \rangle}{\langle \hat{a}^\dagger(t) \hat{a}(t) \rangle^2}$$

with \hat{a}^\dagger and \hat{a} being respectively the photon creation and photon annihilation operators (see previous and following sections). As these operators are directly linked to the electric field and as such to the light intensity, this definition and the one given in the main text are equivalent. This one is simply the quantized version of the 'simpler' one given in the main text of the part 1.

In practice, it does mean one should be able to tell how many photons are detected within a given short-enough time window (on the order of a typical lifetime emitter). This is in fact very difficult to do experimentally as detectors are usually not fast enough to recover from one detection to the next one and they are usually not very good at discriminating the number of photons detected within such short time windows. This is where the famous Hanbury Brown and Twiss (HBT) experiment comes where a 50/50 beamsplitter is used to measure the $g^{(2)}$ function (see Figure 1 main text). The idea is very simple. Take a single photon source and analyse it with two single photon detectors at each output port of a beamsplitter, a truly single photon will not be able to trigger both detectors at the same time. Because there is a single quantum of energy, the photon, the particle behaviour shows up and only one detector at a time can fire, but not both. This is quite convenient as one can then circumvent the detector issues by using two of them because when one fires and thus becomes unusable for a certain time, the second one is ready to receive a potential second photon. Thus, the observed photon antibunching behaviour tells you that if you were to obtain coincident detections by the two detectors after the beamsplitter, no coincidence should occur at zero delay ($\tau=0$) between the two detectors and $g^{(2)}(0)=0$. The term antibunching is used in order to emphasise the fact that we have one and only one photon at a time¹. We say we have photon antibunching of an emitter. Glauber showed that the quantum formalism could be applied to this experiment in the same way [Gla63a]. Since then, the interaction of matter with photons has been described and studied at length but the experimental proof that single photons actually exist was 'only' obtained in 1977 by H. Kimble, M. Dagenais and L. Mandel [Kim77]. They used a single atomic transition from a beam of excited hot atoms. This first result on the statistics of light showed that single photons do exist for sure and that they are not only some kind of convenient theoretical tool.

1.1.1.3 $N=1$ Fock state versus weak coherent state

Coherent states are eigenstates of the operator $\hat{a} = \frac{1}{\sqrt{2}}(\hat{q} + i\hat{p}_q)$ for the (complex) eigenvalue $\tilde{\alpha}$. One can show that coherent states $|\alpha\rangle$ are superpositions of Fock states (for all values of N , a positive integer) with a Poisson distribution of average $\mu = |\tilde{\alpha}|^2$. A coherent state is given by:

¹ As a side note, the original experiment by Hanbury Brown and Twiss was devised in the 1950's for astronomy purposes as they wanted to study the coherence of light emitted from stars in space using two spatially separated detectors to measure light-intensity correlation [Han56].

$$|\alpha\rangle = e^{-\frac{|\tilde{\alpha}|^2}{2}} \sum_{N=0}^{+\infty} \frac{\tilde{\alpha}^N}{\sqrt{N!}} |N\rangle$$

Weak coherent states are coherent states with $\mu \ll 1$. Weak coherent states are empty ($N=0$) with probability close to $1-\mu$, they possess one single photon with probability μ , two photons with probability $\mu^2/2 \ll \mu$, three photons with probability $\mu^3/6 \ll \mu^2/2 \ll \mu$ and so on.

Within a good approximation, a weak coherent state is thus a superposition of nothing with probability $1-\mu-\mu^2/2$, of a single photon with probability μ and of a pair of photons with probability $\mu^2/2$ (here we neglect higher orders).

In a HBT device, the probability of coincident clicks at zero delay is therefore not equal to zero as for the single photon state: the probability of a double click is on the order of $\mu^2/2$ which makes it possible to discriminate single photons from weak coherent pulses.

In the case of weak coherent pulses, one can show that $g^{(2)}(0)=1$ while $g^{(2)}(0)>1$ for classical light (for example for thermal states).

When the probability of a double click is zero ($g^{(2)}(0)=0$) we know that the state of the incoming photon must be a single photon state. For two photons arriving at the same time, we can show that $g^{(2)}(0)=1/2$, hence the criteria of $g^{(2)}(0)<1/2$ for a single photon source. “ $1-g^{(2)}(0)$ ” can therefore be considered as a measure of the Fock purity or photon number purity (also called photon antibunching), which is maximal and equal to 1 in the case of single photon states, with coherent states of light delimitating the border line between antibunched pulses and bunched (negative purity) classical pulses.

1.1.2 Wave approach

1.1.2.1 Spatio-temporal wave function of a single-photon: Glauber's first order correlation function.

If we wish to properly describe single photons propagating in space and time, let us firstly introduce a one-photon state (in Schrödinger representation) as follows:

$$|\psi(t)\rangle = \sum_{\lambda=\pm} \int d^3k \cdot c_{\lambda}(\mathbf{k}, t=0) e^{-ic\|\mathbf{k}\|t} |1_{\lambda,\mathbf{k}}\rangle$$

Here $c_{\lambda}(\mathbf{k}, t=0)$ is a complex amplitude while $\hat{a}_{\lambda,\mathbf{k}}^{\dagger}|0\rangle = |1_{\lambda,\mathbf{k}}\rangle$ represents the quantum state of a single-mode single-photon for a (plane wave) transverse mode of a wave vector \mathbf{k} and polarisation λ . It is obtained after letting act the raising (creation) operator $\hat{a}_{\lambda,\mathbf{k}}^{\dagger}$ associated to this mode on the vacuum state here denoted $|0\rangle$. The vacuum state itself is the tensor-product of the vacuum states associated to each transverse mode ($|0\rangle = \otimes |0_{\lambda,\mathbf{k}}\rangle, \forall \lambda, \mathbf{k}$). The integral on all $|c_{\lambda}|^2$ is equal to one by normalisation of the probability distribution of the single photon state over various modes. The associated Glauber first-order correlation function reads [Gla63b, Scu97]:

$$\psi^E(x, t) = ic \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^3} \cdot \sqrt{\|\mathbf{k}\|} e^{i(kx-c\|\mathbf{k}\|t)} c_{\lambda}(\mathbf{k}, t=0) \varepsilon_{(\lambda)}(\mathbf{k})$$

where $\varepsilon_{(\lambda)}(\mathbf{k})$ represents the direction of polarisation of the corresponding mode.

As it is well-established in the standard theory of photodetection [Scu97], the modulus squared of $\psi^E(x, t)$ is proportional to the probability of detecting a photon at time t in a detector located at position x and this can be interpreted as a kind of single photon (spatiotemporal) wave function. In analogy with Maxwell's classical theory, the photon wavefunction $\psi^E(x, t)$ can also be interpreted as the single photon electric field generated by a quantum dipole. The single photon wavefunction is also useful to describe light which is prepared at time $t=0$ in the single-mode (polychromatic) Fock state [Tit66, Smi07]:

$$|N\rangle = \frac{(\hat{a}^{\dagger})^N}{\sqrt{N!}} |N=0\rangle \text{ (with } \hat{a}^{\dagger} = \sum_{\lambda=\pm} \int d^3k \cdot \bar{c}_{\lambda}(\mathbf{k}) \varepsilon_{(\lambda)}(\mathbf{k}) \hat{a}_{\lambda,\mathbf{k}}^{\dagger} \text{)}$$

The N -photon wave function then factorises into the product of similar single-photon wave functions:

$$\psi^E(x_1, x_2, \dots, x_N, t) = \psi^E(x_1, t) \cdot \psi^E(x_2, t) \dots \psi^E(x_N, t)$$

where each individual (single photon) wavefunction obeys Maxwell equations.

We note that a wavefunction-related definition using Fock states of a single photon source could be the following where:

$$|\psi\rangle = a_1|0\rangle + a_2|1\rangle + a_3|2\rangle$$

with the efficiency of the single photon source defined as $|a_2|^2 + |a_3|^2$ and the purity is defined as $1 - |a_3|^2$.

1.1.2.2 How to characterize a single photon source (2): Indistinguishability and the HOM effect

The notion of spatio-temporal wavefunction of a single photon leads to other specifications that must be given for a single photon and in particular its so-called degree of coherence, namely the photon distinguishability. As we show below, when two photons, in the same quantum optical state, impinge on each entrance port of a beamsplitter, their bosonic behaviour kicks in and thus the two photons always come out of the beamsplitter together (see Figure 1 in the main text). Destructive interferences eliminate the case where they exit separately. This is only 'perfect' if the photons are indistinguishable.

The effect was first shown experimentally in 1987, when Leonard Mandel and his team observed this so-called Hong-Ou-Mandel (HOM) effect using twin photons created by the optical non-linear process known as spontaneous parametric down-conversion (SPDC) [Hon87] (see main text).

To explain this indistinguishability effect between two photons, let us represent each photon by a wave packet (the aforementioned Glauber first order correlation function) and let us denote the corresponding wave functions (see Figure S1):

$$\Psi_{in}^{HOR} \text{ and } \Psi_{in}^{VERT}$$

Then, the 'in' state, prior to the passage through the mirror is described by the symmetrized two-photon wave function:

$$\frac{1}{\sqrt{2}} \left(\Psi_{in}^{HOR}(x_1) \Psi_{in}^{VERT}(y_2) + \Psi_{in}^{HOR}(x_2) \Psi_{in}^{VERT}(y_1) \right)$$

After passing through the mirror, each reflected wave gets dephased by a factor $\pi/2$, while the transmitted waves do not undergo any dephasing. At the output of the beamsplitter we get thus:

$$\begin{aligned} & \frac{1}{\sqrt{2}} \left[\left(\frac{1}{\sqrt{2}} \right) i \Psi_{out}^{HOR-ref}(y_1) + \Psi_{out}^{HOR-trans}(x_1) \right] \left(\frac{1}{\sqrt{2}} \right) \left(\Psi_{out}^{VERT-trans}(y_2) + i \Psi_{out}^{VERT-ref}(x_2) \right) \\ & + \frac{1}{\sqrt{2}} \left[\left(\frac{1}{\sqrt{2}} \right) i \Psi_{out}^{HOR-ref}(y_2) + \Psi_{out}^{HOR-trans}(x_2) \right] \left(\frac{1}{\sqrt{2}} \right) \left(\Psi_{out}^{VERT-trans}(y_1) \right. \\ & \left. + i \Psi_{out}^{VERT-ref}(x_1) \right) \end{aligned}$$

If the two incoming photons share a same wave function, that is to say if :

$$\Psi_{out}^{HOR-ref}(y) = \Psi_{out}^{VERT-trans}(y) = \Psi_{out}(y) \text{ and } \Psi_{out}^{HOR-trans}(x) = \Psi_{out}^{VERT-ref}(x) = \Psi_{out}(x)$$

then the expression above simplifies to :

$$\begin{aligned} & \left(\frac{i}{\sqrt{2}} \right) \left(\Psi_{out}^{HOR-trans}(x_1) \Psi_{out}^{VERT-ref}(x_2) + \Psi_{out}^{HOR-ref}(y_1) \Psi_{out}^{VERT-trans}(y_2) \right) \\ & = \left(\frac{i}{\sqrt{2}} \right) \left(\Psi_{out}(x_1) \Psi_{out}(x_2) + \Psi_{out}(y_1) \Psi_{out}(y_2) \right) \end{aligned}$$

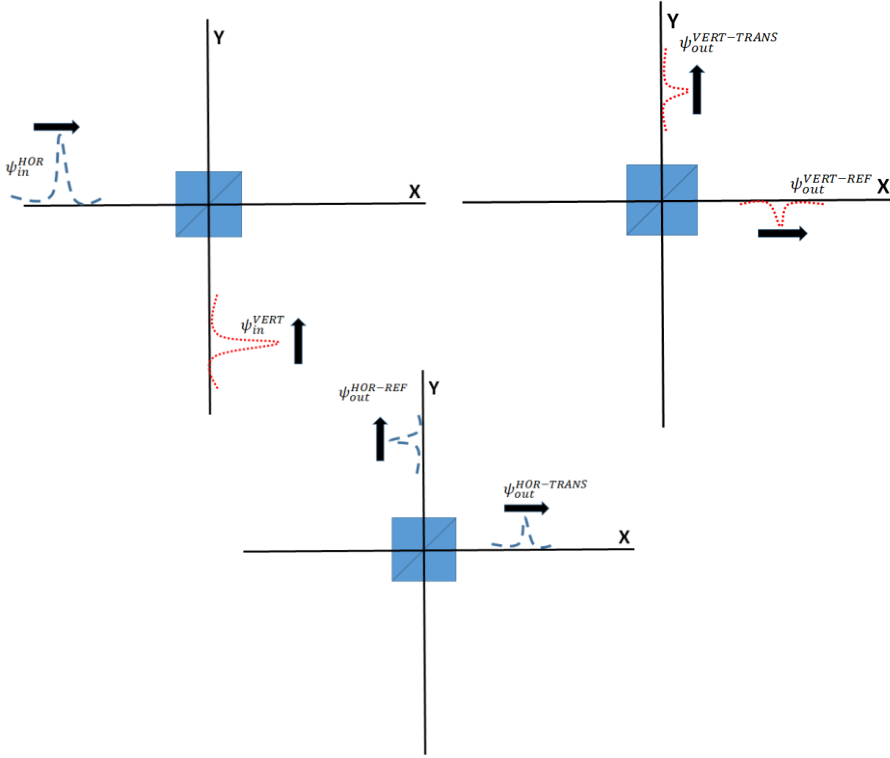


Figure S1: Hong-Ou-Mandel (HOM) effect: photon impinging on a beamsplitter.

which means that either both photons are located along the horizontal axis (X_{out}) or they are located along the vertical axis (Y_{out}). The absence of simultaneous clicks in the horizontal and vertical detectors at the output is thus a signature of the equivalence of the wave functions of the photons sent along the incoming horizontal and vertical axes.

An alternative derivation can be done in terms of creation operators and is the one often found in the literature. For this derivation, let us associate the incident photon in the horizontal (vertical) arm to the creation operator $(\hat{a}^\dagger)_{in}^{HOR}$ ($(\hat{b}^\dagger)_{in}^{VERT}$), then the 'IN' state reads $(\hat{a}^\dagger)_{in}^{HOR} (\hat{b}^\dagger)_{in}^{VERT} |Vacuum\rangle$ and the 'OUT' will read as:

$$(1/2) \left(i(\hat{a}^\dagger)_{out}^{HOR-ref} + (\hat{a}^\dagger)_{out}^{HOR-trans} \right) \left(i(\hat{b}^\dagger)_{out}^{VERT-ref} + (\hat{b}^\dagger)_{out}^{VERT-trans} \right) |Vacuum\rangle$$

which is a superposition of a state with one photon in each arm of the form:

$$\left(-(\hat{a}^\dagger)_{out}^{HOR-ref} (\hat{b}^\dagger)_{out}^{VERT-ref} + (\hat{a}^\dagger)_{out}^{HOR-trans} (\hat{b}^\dagger)_{out}^{VERT-trans} \right) |Vacuum\rangle$$

and a state with two photons in one arm of the form:

$$\left(i(\hat{a}^\dagger)_{out}^{HOR-ref} (\hat{b}^\dagger)_{out}^{VERT-trans} + i(\hat{a}^\dagger)_{out}^{HOR-trans} (\hat{b}^\dagger)_{out}^{VERT-ref} \right) |Vacuum\rangle$$

By simplifying the notation we get $(-\hat{a}_V^\dagger \hat{b}_H^\dagger + \hat{a}_H^\dagger \hat{b}_V^\dagger) |Vacuum\rangle$ when the photons are emitted along two distinct arms, and $i\hat{a}_{V(H)}^\dagger \hat{b}_{V(H)}^\dagger |Vacuum\rangle$ when they are in the same arm. Obviously when $\hat{a} = \hat{b}$, both photons are emitted in the same arm.

To conclude, if the wave packets are perfectly identical, they do overlap at the outputs of the beamsplitter and this bunching effect will be at maximum. This property is very important for quantum computation for instance. On the other hand, if the two photons are slightly different, the two wave packets are not quite the same, thus the overlap is not maximum and thus the bunching is not perfect. The HOM experiment is very similar to the one previously described for the HBT experiment. In both cases we measure the coincidence count rate between the two outcome ports of the beamsplitter using two single photon detectors but in the case of HOM we send two pulses, one on each entrance port of a beamsplitter and only one pulse in the case of HBT. Finally, we must stress that information can be encoded in different degrees of freedom of the photon such as path or polarisation. Like standard electromagnetic waves, which have two possible orthogonal polarisation states, a single photon possesses two orthogonal polarisation states. This property is particularly interesting when using entangled photons for quantum communications for instance, but that is beyond the scope of this article [Cou18].

1.1.3 Some comments in relation with entanglement

It is sometimes difficult to completely disconnect single photons and photon entanglement, especially when the ‘single’ photons are created by pairs like in the case of SPDC (heralded single photon source in Fig. 2 in main text).

Note that if we send two identical photons through the same input port of a beamsplitter, for instance if at the input, the wave function is $\Psi_{in}^{HOR}(x_1)\Psi_{in}^{HOR}(x_2)$, then at the output, we get the wavefunction:

$$\frac{1}{2} \left(i\Psi_{out}^{HOR-ref}(y_1) + \Psi_{out}^{HOR-trans}(x_1) \right) \left(i\Psi_{out}^{HOR-ref}(y_2) + \Psi_{out}^{HOR-trans}(x_2) \right)$$

so that the probability to find two photons in the same output port is $\frac{1}{4}$ for each of them, and $\frac{1}{2}$ in different ports. The states of the two photons factorise and the statistical distributions of clicks is the product of the statistical distributions associated to each photon.

States which do not factorise are said to be entangled, for instance the state

$$\frac{i}{\sqrt{2}} \left(\Psi_{out}(x_1)\Psi_{out}(x_2) + \Psi_{out}(y_1)\Psi_{out}(y_2) \right)$$

that we get at the output of the beamsplitter when two identical photons are injected in a HOM device is an entangled state and the statistical distributions of clicks is then not equal to the product of the statistical distributions associated to each photon. The statistics of both photons are actually strongly correlated; if a photon is found in one arm, then the other one must be found in the same arm. There is no simple classical explanation of the correlations of entangled systems. In Maxwell’s theory, there is even no room for such a concept because multiparticle quantum systems are described in the configuration space, while Maxwell’s field belongs to the usual 3D physical space.

Entanglement does not require the presence of two photons as one can also entangle two degrees of freedom of a single photon. For instance, a single photon prepared in the state

$$\frac{1}{\sqrt{2}} \left(\Psi_{out}(x_1)|HOR.pol\rangle + \Psi_{out}(y_1)|VERT.pol\rangle \right)$$

is an entangled state with equal weight superposition of a state horizontally polarised located in the horizontal output and a state vertically polarised located in the vertical output arm.

Somewhat abusively, one may formally consider that a state $\frac{1}{\sqrt{2}} \left(\Psi_{out}(x_1) + \Psi_{out}(y_1) \right)$ obtained by injecting a single photon at the input of a 50/50 beamsplitter is entangled because it can be interpreted as the 50/50 superposition of a state which is the product of “one photon in the horizontal arm” with “nothing in the vertical arm” with a state which is the product of “one photon in the vertical arm” and “nothing in the horizontal arm”:

$$\frac{1}{\sqrt{2}}(\Psi_{out}(x_1) + \Psi_{out}(y_1)) =$$

$$\frac{1}{\sqrt{2}}(|OnePhotonInHORarm\rangle|VacVERTarm\rangle + |VacHORarm\rangle|OnePhotonInVERTarm\rangle)$$

But it is not clear how to exploit this kind of entanglement which is therefore sometimes called weak entanglement.

1.2 How do you ‘make’ single photons?

1.2.1 Single photons from two-level systems

As discussed in the main text, the natural method of producing single photons is to isolate an emitter with a well-defined radiative two-level system and this can be implemented using various systems (see [Figure 2a in the main text](#)). There are two main categories which we could call the ‘natural’ category and the ‘engineered’ category. The ‘natural’ category is the most obvious one theoretically although challenging experimentally. It consists in using single atoms or single molecules given to us by nature. For single atoms, Kimble *et al.* used them to show for the first time the effect of photon antibunching in 1977. Since then, people do trap them in optical or microwaves cavities or using a Paul trap with dynamical electric fields to trap single ions. Single molecules on the other hand can be ‘trapped’ in a solid state matrix and that was the case for the first detection of single photons coming from a single molecule in 2000 [Mic00]. Single molecules are still used these days although they do tend to bleach in time plus they do possess very simple energy schemes which is not suitable for most applications. The second category is the ‘engineered’ one and that usually involves using semiconductor or high-band gap materials. Semiconductors and in particular electron-hole complexes called excitons can behave as single emitters providing we have a confinement to spatially restrain these excitons. Quantum dots (QDs) and nanocrystals are the prime example of such systems. For nanocrystals, it is a matter of synthesizing them chemically in solution and end up with nanosize particles of semiconductors. Excitons can be created optically in these nanocrystals and single photons can be observed upon emission. In this case, the two-level is given by: lower-state = no exciton created in the nanocrystal and upper-state = one exciton created optically or electrically. Nanocrystals can also suffer from bleaching as well as blinking because the exciton created can be trapped at the surface of the nanocrystal due to surface states. This problem hardly occurs in semiconductor QDs, grown physically by epitaxy for instance. In this case, a QD can benefit from the complex energy structure of its original bulk semiconductor and becomes interesting for quantum computation applications or for quantum cryptography when the polarisation of the photons is controlled [Wan19b]. Nanocrystals do tend to work mostly at room temperature but hardly any coherence and indistinguishability is expected from them, as opposed to the QDs which can really behave like ‘artificial atoms’ but with the down-side of only working at low temperatures. The reason for that is that these single photon sources are within a matrix and thus coupling to phonons is quite strong which can then destroy the coherence and even the single ‘photon-ness’.

There is a third category which in a way is a mix between the two and that is the use of defect centres occurring in large band-gap materials such as diamond or SiC. This is a mix in a sense that these defects are atomic-like but they can also be engineered by ion implantation or other techniques such as annealing the material. The most famous example is the nitrogen-vacancy (NV) defect centre in diamond. This defect is the replacement of one carbon atom by a nitrogen atom next to a carbon vacancy. This system is interesting for many reasons and that is because it does have a rather complex energy level system that can be manipulated optically, using microwaves and using magnetic fields. The NV centres are the most known ones but there are plenty of defects that are under study such as silicon-vacancy (SiV), germanium-vacancy (GeV), nickel-related (NE8) etc. [Jel06].

From the wide range of new emerging photon source technologies, it is worth mentioning that devices based on carbon nanotubes [Xe18] are highly promising for triggered single photon generation with emission in all relevant communication bands with the potential for electrical operation at GHz speeds and at room temperature. Defects in 2D materials [Liu19] such as transition-metal dichalcogenides can be electrically excited and manipulated as well [Pal16], but the potentially greatest advantage of this very young field is the prospect of massively simplified device fabrication compared to epitaxially grown QDs described previously [Sen17].

1.2.2 Case study of an “engineered” single photon source: the epitaxial quantum dot

To date, for quantum communications and quantum computation, sources have been the leading solid state candidate as single photon sources starting from the early experiments [San01] with random Stranski–Krastanov grown QD samples. Addressing the efficiency and indistinguishability criteria for quantum computation has taken a little longer. The problem of making QD single photons indistinguishable was first addressed by Santori *et al.* [San02] using sequentially created single photons and using a pump laser close to the QD resonance (p-shell pumping). The visibility in these early experiments were primarily limited by Stark induced line diffusion due to the excess electrons created by off-resonant pumping. Once resonant pumping schemes became viable [He13] and then coherent π pulse pumping, near-unity interference between sequential emissions [Wei14] and good interference visibility between separate QDs [Gao13] have been achieved. The development of pillar microcavities [Pel02] has led to engineering of high efficiency single photon sources using QDs. The Purcell effect is in this case responsible for the good properties.

Single photon sources are the result of the interaction between light and matter and as such, we must point to the important feature of the so-called Purcell effect [Pur46]. What has been known and famously pointed out by Edward Purcell since 1946 is the fact that the spontaneous emission rate of an emitter is not an unalterable intrinsic property of the emitter. If the environment of the emitter changes, its spontaneous emission rate changes and as such the emitted photons too. Purcell extracted an enhancement factor - now-called Purcell factor - that predicts how this rate will be modified if the photon is put in a different environment than the infinitely extended vacuum. In particular, when a single two-level emitter is placed in a small cavity, resonant at the spontaneous emission wavelength, the emission of the emitter can be enhanced (as energy has to be conserved, its spectral linewidth gets narrower at the same time in the ideal case). This very important result has the effect of increasing the spontaneous emission rate as first observed in 1983 by Serge Haroche’s team [Goy83] and is used now in various systems and experiments either to increase the emission rate or to beat the decoherence of the system (therefore increase the indistinguishability of the photons) or both. The Purcell effect has since been observed for QDs using optical cavities. Initially these sources have been separately engineered for efficiency (>75% into useful modes) and purity (up to 99.5%) [Dus19]. The latest experiments on pillar microcavity sources pumped using resonant π pulses have combined high extraction efficiency and near-unity indistinguishability [Din16,Som16]. This brings these sources close to satisfying the requirements for linear optics quantum computing applications with proof of principle experiments showing boson sampling (see main text) with 4-5 single photons [Lor17,He17c,Wan18a] now superseded by 20 photons in 60 modes [Wan19a]. Good QD sources are typically found by studying many dots in random locations because Stranski–Krastanov growth leads to a wide range of sizes and thus emission wavelengths. Recent advances have led to the engineering of resonant cavities around pre-characterised dots [Dou08,Pre20] although in longer term it would be useful to develop sources with dots placed in pre-seeded locations [Sch09].

QD single photon sources in pillar microcavities show the highest external quantum efficiency (useful for linear optics computing experiments) so far but of course, for the future, integrating a photonic waveguide circuit with a QD single photon source will guarantee emitted single photons are

coupled efficiently to the required waveguide modes. To this end, various groups have worked towards aligning and coupling dots to photonic crystal waveguides. Similarly, the deterministic generation of entangled photon pairs from bi-excitons [Ben00] including 2-photon coherent control pumping schemes [Jay13] are now able to provide the high purity photons required for quantum computational scale up [Wei22] and research into doubly resonant cavities [Dou10] and broadband dielectric antenna structures [Che18] are increasing efficiency towards values useful for scalable quantum computation. However, when extended to 1.3-1.6 μm wavelengths and electrical pumping, these sources are already useful for quantum communications applications as discussed in the main text.

1.2.3 Creating heralded single photon sources from spontaneous non-linear processes

The idea of heralding single photons originated from early experiments proving that spontaneous parametric sources of photon pairs [Bur73] can be configured to emit non-classical light (see also Figure 2 in the main text for a typical heralded source using SPDC). Both electronically post-selected [Hon86] and optically gated single photon states [Rar86,Bri11] were generated in these experiments. The latter optically generated single photon Fock state sources are the ideal source for intensity transmission imaging. In a recent experiment [Sab19], a high efficiency source of optically gated single photons was used to measure the transmission image of a weakly absorbing sample showing an order a factor of 1.8 (2.7dB) improvement in precision over classical methods. This experiment was based on a bulk source designed for high heralding efficiency into a single mode fibre where heralding efficiencies greater than 80% were achieved.

The early experiments in pair photon interference and entanglement were carried out using bulk sources [Hon87,Rar90]. In the mid 1990's various groups realized that scaling beyond 2-photon experiments could be possible using bulk parametric sources pumped by pulsed lasers [Rar95,Wei95] leading to 3-photon experiments showing quantum interference between photons from separate sources [Rar96] and teleportation [Bou97]. These then led on to a series of experiments involving 4 [Wal05] 6, 8, and most recently 10 photon entanglement [Wan16a]. The limits to scaling beyond these small numbers of photons are effectively the heralding efficiency, or lumped efficiency of detection of the photon pairs and the limited visibility (or fidelity of) the interference effects. These original experiments have been carried out using bulk crystals leading to the need to filter collection into single spatial modes using single mode fibre collection optics and requiring high pump laser powers. Guided wave solutions have emerged as a leading contender to replace bulk sources starting from the development of pairs in a single mode within second order non-linear materials [Bon99,Tan01], four wave mixing in fibres [Rar05] and most recently in waveguide quantum circuits [Sil11].

The HOM interference visibility in SPDC is limited by spectro-temporal purity and by higher number photon terms (number purity). In the original proposals [Rar95,Wei95] it was shown that by narrowband filtering the photon pairs arising in pulsed parametric down-conversion such that the effective photon length is longer than the pump pulse forces the photon into a single spectro-temporal mode at the expense of source brightness. Similarly, operating at low average numbers of pairs per mode will also restrict the background from higher order (2-pairs, 3-pairs per pulse) events. However, the counting rate C_N in an $2N$ -fold coincidence measurement evolves as

$$C_N \sim \eta^{2N} \mu^N R$$

Where η is lumped detection efficiency, μ is the average number of pairs per pulse and R is pump pulse repetition rate. Thus, we see immediately that we sacrifice efficiency and brightness to achieve high fidelity interference typically ending up with efficiency $\eta < 0.2$ and pulse occupancy $\mu < 0.025$ Assuming a pump rate of a < 1 GHz this leads quickly to ten-fold coincidence rates less than one per week.

This then has been the driver for recent research looking to find ways to increase or even avoid exponentially falling rate while improving visibility of interference and thus of quantum gates. Dispersion engineering to develop natural spectro-temporal purity heralded single photons

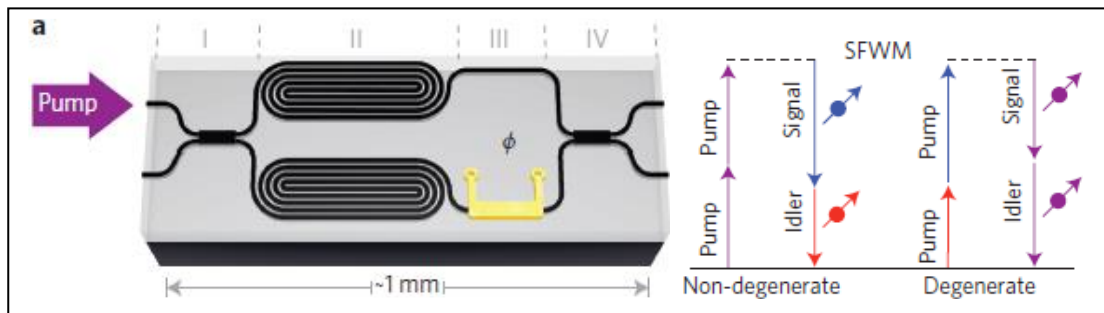


Figure S2: Extending pair photon sources and entanglement to silicon chip based photonic circuits. The pump laser creates photon pairs in the coiled waveguide sections via the process of four wave mixing. The pair of sources are arranged inside a Mach-Zehnder interferometer and varying the phase allows the demonstration of 2-photon interference effects and entanglement generation (figures from [Sil14], and cover-page of Nature photonics).

(avoiding filter losses) was first introduced by groups working with bulk crystals [Gri01] then extended to fibre sources [Hal09,Coh09] with significant interference visibility improvements. This dispersion engineering is now being transferred into the integrated sources being developed on silicon waveguides [Sil14,Far18,Ver17] (see Figure S2) and we expect to see high visibility interference on-chip experiments in the near future [Pae19]. The other area being addressed is how to reduce background coincidences from higher order number terms which occur. This can be addressed by using many sources to herald single photons and actively multiplexing to obtain high efficiency heralded single photons with very low higher order multiphoton terms [Men16].

1.2.4 Spin-photon hybrid systems

Proposals to entangle emitted (and scattered) single photons with the ground state spin of an atom came from atom-cavity QED research in the late 1990's [Dua04] and have been realised with trapped ions [Bli04] and atoms [Wil07]. These schemes have then been adapted in various ways to solid state two-level systems in cavities [Hu08,Bon10] and are in general very relevant for quantum communications (section 2) and quantum computation (section 3) applications in general. Further schemes to generate cluster states in emission suitable for large scale computation have been proposed by Lindner et al. [Lin09]. Various groups have now demonstrated such spin photon interfaces in emission from NV-centres in diamond [Hen17], scattering from dots [And16,And19,Sto17] and recently in diamond waveguide structures containing silicon vacancy (SiV) centres cooled to low temperatures [Ngu19]. Progress in NV-centres where ground state spins can be stored for milliseconds has allowed demonstrations of loophole free Bell inequalities and other quantum communication primitives between spatially separated solid state systems [Pfa14, Hen17] and such local clusters of up to 10-spin qubits [Bra19] have been proposed as processing nodes linked by quantum photonic channels to make a distributed computing architecture. Decoherence in charged QDs happens in a few ns which hinders the progress towards large cluster states and even simple quantum memory and repeater schemes. Some progress has been achieved by applying strong magnetic field to make spins precess at rates of 25-60 GHz using picosecond pulses to coherently write information onto and off the dots [Gre13] however efficient entanglement between dot spin and single photons still eludes us.

1.3 Basic tools for quantum technologies

To understand 2 of the 5 applications of single photons discussed in the main text and the accompanying Review, namely in quantum communications and quantum computing, one needs a

crash-course in quantum technologies in order to acquire the basic tools necessary for understanding how quantum optics and quantum photonics is used and is useful for quantum technologies.

1.3.1 Quantum bit

The first very important notion is the notion of a quantum bit of information, or qubit, which is the quantum counterpart of a classical bit of information in our current computers. The definition of a quantum bit of information $|\psi\rangle$ is given by:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

where α and β are two complex numbers. A quantum bit is basically a quantum superposition state between two states. The more intuitive one in quantum mechanics is the spin $\frac{1}{2}$ of an electron with two states up or down. For photons, one can have various states $|0\rangle$ and $|1\rangle$ with the polarisation state perhaps being the most used in practice, given by:

$$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$$

where $|H\rangle$ and $|V\rangle$ are the horizontal and vertical linear polarisation states (within the laboratory frame or the optical table and so on) respectively. In the more general case, $|\psi\rangle$ is an elliptical polarisation state and depending on the values of α and β , one can have linear states, circular states or elliptical states.

By the same token, we can define a two-qubit system considering for example two spin $\frac{1}{2}$ (state '+' for $+1/2$ and '-' for $-1/2$) A and B that are spatially separated. The most general two-qubit pure state would be:

$$|\psi\rangle = \alpha|1_A\rangle \otimes |1_B\rangle + \beta|1_A\rangle \otimes |0_B\rangle + \gamma|0_A\rangle \otimes |1\rangle + \delta|0_A\rangle \otimes |0_B\rangle$$

which we can write in qubit notation:

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

where \otimes represents the tensor product.

1.3.2 Factorisable states, entanglement & Bell states

To understand the notion of entanglement, we first need to understand the notion of a factorisable state in quantum mechanics. A factorisable state between two systems A and B can be written in qubit notation like:

$$|\psi\rangle = (A|0_A\rangle + B|1_A\rangle) \otimes (C|0_B\rangle + D|1_B\rangle)$$

By definition, an entangled state is a state that cannot be written in a factorisable state like above. For two spin $\frac{1}{2}$ particles or for two qubits, there exist the 4 Bell states in qubit notation which are:

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|0_A\rangle \otimes |1_B\rangle + |1_A\rangle \otimes |0_B\rangle)$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|0_A\rangle \otimes |1_B\rangle - |1_A\rangle \otimes |0_B\rangle)$$

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|0_A\rangle \otimes |0_B\rangle + |1_A\rangle \otimes |1_B\rangle)$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}(|0_A\rangle \otimes |1_B\rangle + |1_A\rangle \otimes |0_B\rangle)$$

These 4 states cannot be written in a factorisable state of system A and system B. It is therefore an entangled state (these are the so-called 4 Bell states). In practise with single photons, the systems A and B are single photons and the coding 0 or 1 can be done onto their polarisation states [Kwi95c],

their angular momentum states [Mai01] but in also in time (also called time-bin entanglement) [Fra89] or phase and momentum [Rar90].

1.3.3 Examples of single qubit and two-qubit gates

There are many single qubit gates (more information can be found for instance here [Bar15]) that take in a given single qubit state and transform it into another single qubit state. One of the most used single qubit gate is the so-called Hadamard gate. This gate can transform a pure given incoming state (say $|0\rangle$) into a superposition state of $|0\rangle$ and $|1\rangle$. As we are dealing with 2 dimensional qubit state, the Hadamard gate is a 2x2 matrix and it reads as follows:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

One can easily verify that the effect of the Hadamard gate onto the state $|0\rangle$ will give the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ which is a superposition state. In the case of the polarisation state of a single photon as qubit, it is very easy to implement a Hadamard gate by simply rotating the polarisation state using an optical half-waveplate for instance.

For a two-qubit gate, again, there are many different types of 2 qubit gates. Perhaps the most used gates are the CNOT (control-not gate) or the CPhase (Controlled-phase gate), where we have commonly a 'control' qubit that will control the state of the 'target' qubit. For the CNOT gate, as we deal with 2 qubits, the CNOT gate can be represented by a 4x4 matrix and it is given by:

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

In plain words, a CNOT gate acts on the target qubit depending on the state of the control qubit. If the control qubit is in the $|0\rangle$ state thus the target state remains unchanged: if the target state was $|0\rangle$ it remains $|0\rangle$ and if it were $|1\rangle$, it remains $|1\rangle$ (this can be seen in the 2x2 upper-left matrix of the CNOT matrix). Now if the control qubit is in the $|1\rangle$ state then the target qubit will have its state flipped from $|0\rangle$ to $|1\rangle$ or from $|1\rangle$ to $|0\rangle$ (this can be seen in the 2x2 lower-right matrix of the CNOT matrix).

For the CPhase gate, it is given by:

$$CPhase = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

This kind of two-qubit gate on two qubits made of single photons is actually quite challenging to produce and requires some not so trivial optical circuitry as demonstrated [O'B03, Gas04, Kie05].

1.4 Useful concepts for quantum communications

1.4.1 Entanglement swapping and quantum repeater

Entanglement swapping describes a scheme where entanglement that is initially created within independent sub-systems (such as entangled photon pairs), is extended to entanglement between the sub-systems and their qubits. In photonic schemes, this is typically achieved by performing a projective entanglement operation (BSM) between two photons from two independent entangled pairs, thereby extending the entanglement to the two partner photons, which have never interacted [Zuk93].

For applications, this enables the spatial extension of entanglement over longer distances which makes it an essential technology for so-called quantum repeaters [Bri98]. In a quantum repeater chain, pair-wise entanglement that is originally created over several short-distance links is successively

extended over longer distances by nested entanglement swapping operations until extending to the two outermost points of a long-distance link. Quantum communication in a quantum repeater would then be established via teleportation of quantum bits from one end to the other of the long-distance link.

1.4.2 Decoy protocol

The decoy protocol [Lo05,Zha06] allows for the detection of an eavesdropping attempt in QKD with WCS sources, by using additional so-called decoy states with varying intensity that are randomly time-multiplexed with the quantum signal. Typical implementations make use of the vacuum state (no photon being transmitted) and a state with lower mean photon number compared to the signal used for transmission of quantum bits. A so-called photon-number-splitting attack would have to introduce a photon-number dependent attenuation which can be detected by measuring changes in the photon number distribution of the decoy states.

1.4.3 Quantum memories

Quantum memories are in general systems that can store and retrieve qubits. Of particular importance are quantum memories that perform these tasks with photonic qubits. With photonic quantum technologies being mainly limited by their intrinsically probabilistic character, quantum memories are required for synchronization in most popular schemes for quantum computation and quantum repeaters. This requires storage of qubits with high efficiency and fidelity and on-demand retrieval, requiring an extremely high degree of control over the interaction of light and matter. A broad overview of current quantum memory technology being investigated can be found for example in [Hes16] and references therein.

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