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# Managing flow problems defined on time-expanded networks through a project/lift decomposition

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## 1 The Time-Expanded 2 Flow Problem (TE2FP)

When trying to solve problems with arcs whose status depend on the time, a straightforward way is to search for the optimal solution in a Time-Expanded Network (TEN) as did Krumke and al.[1]. A connection between two nodes in this network represents the crossing of an arc at a given time. Those kind of networks are used, among other applications, for evacuation routing problems as did Park and al.[2].

Using a TEN has a lot of advantages because it does not contain cycles and Mixed Integer Linear Programs are simpler. However, TENs depend on a time discretization. Unfortunately, the number of arcs of a TEN grows very quickly depending on the time discretization used. Of course, one can have an idea of the solution of their problem by using a coarse discretization, but the rounding errors are multiplied with every edge used in the solution.

We are working here on a *Dial A Ride Problem* defined on a network  $G$  with a time horizon  $T_{max}$ . We are required to transport multiple commodities. For every commodity  $k$ ,  $q^k$  units of commodity must travel from their origin  $o^k$  to their destination  $d^k$ . They can be pickup after the date  $t^k$  and must be delivered at most  $\delta^k$  time units after. Also, the number of vehicles is not fixed beforehand and the length of an arc is time dependant (i.e.  $L_e$  is a function of the time  $L_e : t \mapsto L_e(t)$ ). Lastly, we add an important hypothesis: preemption is allowed. It means that one commodity may be handled by several vehicles.

A time-expanded network  $G^T$  is constructed by copying every node of  $G$  for every possible date of the time discretization used. Therefore, a node in this network corresponds to a node of  $G$  at a certain date. Then, let  $X$  and  $Y = (Y^k, k = 1 \dots, K)$  be two flows that satisfy Kirchhoff laws on every node of the time-expanded network  $G^T$ . Their meaning is that  $X$  is integer and represents the number of vehicles on each edge and that  $Y^k$  is real and represents the quantity of commodity  $k$  travelling on each edge. Those two vectors are linked by the fact that if commodities are transported through an arc, they must be transported by enough vehicles.

### Coupling constraints

On any edge  $e = ((v_1, t_1), (v_2, t_2))$  with  $v_1 \neq v_2$  and  $t_2 = t_1 + L_{(v_1, v_2)}(t_1)$ :

$$C.X_e \geq \sum_k Y_e^k \quad (1)$$

But the number of variables is way too high and this model cannot give a precise solution in a reasonable amount of time. Therefore, we want to project the TE2FP on the graph  $G$ , solve this projected problem and lift its solution back into a TE2FP solution.

## 2 The projected problem

In the projected problem, we search for two flows  $\bar{X}$  (integer) and  $\bar{Y} = (\bar{Y}^k, k = 1 \dots, K)$  (real) defined on the network  $G$  that satisfy Kirchhoff laws and the Coupling Constraints. But they now need to satisfy sub-tour constraints.

### Extended no sub-tour constraints

The time dimension may be implicitly reintroduced in the projected model by noticing that if the vehicles spend more than  $T_{max} \cdot Q$  time in a given area  $S$ , then at least  $Q$  vehicles must enter into  $S$ . Therefore, no sub-tour constraints can be extended as:

$$T_{max} \cdot \sum_{e \in \delta^-(S)} x_e \geq \sum_{e \in \bar{\delta}(S)} l_e \cdot x_e, \quad S \subset N \setminus \{Depot\} \quad (2)$$

where  $N$  is the set of nodes of  $G$ ,  $\delta^-(S)$  is the set of arcs starting from outside  $S$  and ending inside  $S$  and  $\bar{\delta}(S)$  is the set of arcs which either start or end (or both) in  $S$ .

### Commodities must follow acceptable paths

There is a risk that the solution  $(\bar{X}, \bar{Y} = (\bar{Y}^k, k = 1 \dots, K))$  of the projected problem does not correspond to a feasible solution  $(X, Y = (Y^k, k = 1 \dots, K))$  of the original problem. We ensure that  $\bar{Y}$  can be lifted as a feasible vector  $Y$  by requiring from  $\bar{Y}$  to be decomposable into a collection of acceptable paths:

**Définition 1** *A path  $\gamma$  from  $o^k$  to  $d^k$  is acceptable if and only if there is  $\gamma_{o^k}$  (resp.  $\gamma_{d^k}$ ) a path from the depot to  $o^k$  (resp. from  $d^k$  to the depot) such that:*

$$L(\gamma_{o^k}) + L(\gamma) + L(\gamma_{d^k}) \leq T_{max} \quad (3)$$

To verify if all those constraints are respected, the following problem is written, with  $\Gamma$  the set of acceptable paths  $\gamma$  and  $y_\gamma = \begin{cases} 1 & \text{if } e \in \gamma \\ 0 & \text{otherwise} \end{cases}, e \in E$ :

$$(P) \quad \begin{aligned} & \text{Does it exist } (\lambda_\gamma)_{\gamma \in \Gamma}, \\ & \text{s.t. } \sum_k \bar{Y}^k = \sum_{\gamma \in \Gamma} \lambda_\gamma \cdot y_\gamma \\ & \lambda_\gamma \geq 0, \quad \forall \gamma \in \Gamma \end{aligned}$$

However,  $\Gamma$  is the set of all possible paths and not just the set of shortest paths. If  $T_{max}$  is large enough,  $\Gamma$  contains all possible paths. Therefore, this decomposition problem will be solved by column generation or, in its dual version, by cuts generation (i.e. path generation).

## 3 The lift issue

It consists in turning the projected solution  $\bar{X}, \bar{Y} = (\bar{Y}^k, k = 1 \dots, K)$  into a solution  $X, Y = (Y^k, k = 1 \dots, K)$  of the original problem. We deal with it in a heuristic way by solving a sequence of min cost flow problems defined on specific small-size sub-networks of the time-expanded network  $G^T$ .

## References

- [1] Sven O Krumke, Alain Quilliot, Annegret K Wagler, and Jan-Thierry Wegener. *Relocation in carsharing systems using flows in time-expanded networks*. Springer, 2014.
- [2] Inhye Park, Gun Up Jang, Seho Park, and Jiyeong Lee. *Time-Dependent Optimal Routing in Micro-scale Emergency Situation*. IEEE, 2009.