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Adaptive quaternion control for a quadcopter vehicle: real-time validation in presence of wind gusts

J. M. Arizaga¹, J. Cariño², H. Castañeda¹, D. Mercado³ and P. Castillo²

Abstract—An adaptive control for a quadcopter aerial vehicle exposed to aggressive wind gusts is presented in this paper. The control scheme is composed by two parts; firstly, the attitude dynamics is robustly stabilized using a controller based on the quaternion formulation. Then, a translational flight control law with adaptive properties is designed using the sliding mode approach. The stability analysis of the whole system is proved using the Lyapunov theory. The performance of the closed-loop system is validated in real-time experiments and for validating the adaptive and robust properties of the controller, strong wind gusts are applied during flight tests. A video and some graphs, obtained from these experiments, illustrate the effectiveness and robustness of the proposed control strategy.

I. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) come in a wide variety of forms and shapes, but they are often modeled as rigid bodies moving in a 3D environment. They can be classified as either fully-actuated or underactuated, depending on their degrees of freedom (DoF) with respect to their control inputs. The quadcopter and other types of multirotor vehicles are underactuated platforms that have what is known as coupled dynamics that force the attitude of the vehicle to be linked to its position dynamics.

In general, the control design of an underactuated system with coupled dynamics is a challenge, which is the main reason why it is still an active research subject. A common solution for this problem is to use an internal attitude control loop to guide the actuator's force with an outer control loop. Some examples of this approach include the works presented in [10], [11], [12], [15], and [22]. The main idea behind this is to allow the design of the position control to be independent of the attitude one, but in reality, the attitude references, that the position algorithm gives, are highly nonlinear and thus require to be conservative to preserve stability due to unmodeled dynamic effects.

Many works regarding quadcopter models, like the ones presented in [9], [2], [3] and [20], use Euler angles to represent the orientation of the vehicle. This implementation becomes highly nonlinear in an automated system due to the

use of trigonometric functions, numerical rounding errors, and the presence of the gimbal lock problem [18]. Alternative attitude representations can be found in [23], of which the main ones used for the control of UAVs include rotation matrices, unit quaternions, and axis-angle.

Rotation matrices use nine numbers to represent the orientation of an object instead of the three used in Euler angles, which gives them redundancy to mitigate the effects of numerical errors. Nonetheless, rotation matrices still require computational resources to be used for attitude calculations, including rotating a vector from one reference frame to the other and numerically stabilizing them. Unit quaternions are numerically more stable because, much like rotation matrices, they have redundant information available that helps to mitigate any possible rounding errors. The main advantage that unit quaternions have over Euler angles is that they avoid the gimbal lock problem. The main disadvantage of using unit quaternions for the control of UAVs is due to their nonlinear nature. Some results that use this representation are [1], [4], [16] and [19].

The angle-axis representation, much like the Euler angles, uses three values to represent attitude, and sometimes four can be used if the quantity of rotation is separated from the axis of rotation. Their relationship with quaternions can be seen in the Euler-Rodrigues formula, which describes the attitude for rigid bodies. The main advantage that they have is that their kinematics can be used to obtain a linear representation of the attitude system of a rigid body. In this work, a combination of both unit quaternions and axis-angle representations is used to mitigate the effects of the coupled dynamics.

Moreover, different algorithms have been studied to manage the trajectory of a quadcopter unmanned aerial vehicle (UAV), ranging from classical PID control, or nonlinear controllers such as backstepping, to intelligent control methods, such as fuzzy logic, as described in [6], [13]. However, the design of these algorithms usually involves precise knowledge of the system and its environment, or a complex calibration and training method, as well as possessing limited capabilities to handle uncertainties and external disturbances [8]. This is a disadvantage when the UAV is deployed in environments with wind gusts, requiring a controller capable of compensating for them while maintaining the desired trajectory [5], [14], [17].

Pure sliding mode control (SMC) is robust to parametric uncertainties and bounded external disturbances. It also has

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convergence in finite time. However, the major drawbacks of sliding modes are the presence of chattering, as well as the overestimation of control gains. To solve this, some variants have been proposed that minimize considerably the undesired effects, and several of them are discussed in [21]. Similarly, in [8] an adaptive sliding mode control (ASMC) is proposed, which takes advantage of pure SMC, but provides a significant reduction in chattering, while the adaptability regulates the control gain without overestimating it.

In this paper, we focus on addressing the problem of coupled dynamics in the quadcopter platform using two strategies: first, a fusion between quaternion and axis-angle attitude representations is used to have a linear representation of the attitude system and its references, second, an adaptive sliding mode control (ASMC) is used to compensate for the unmodeled dynamic effects and any other external disturbances. This approach is evaluated experimentally, demonstrating the ability of the UAV to compensate strong wind gusts and maintain the desired position.

This article is organized as follows: in section II the required mathematical background, particularly for quaternion algebra and system model is presented. In section III the control algorithms are developed. Here, the stability analysis of the closed-loop system using the Lyapunov theory is demonstrated. The experimental results are presented in some graphs in section IV, and the conclusions are summarized in section V.

II. PRELIMINARIES AND SYSTEM MODEL

In this work, two types of representations will be used to present the attitude of the vehicle: unit quaternions and an axis-angle vector. Quaternions are considered to be hyper-complex numbers that belong to the group \mathbb{H} . A quaternion $\mathbf{q} = q_0 + \vec{q} \in \mathbb{H}$ can be represented as a number with a scalar part $q_0 \in \mathbb{R}$ and an imaginary 3D vector $\vec{q} = [q_1 \ q_2 \ q_3]^T \in \mathbb{R}^3$.

Unit quaternions have the property $\|\mathbf{q}\| = 1$ for some $\mathbf{q} \in \mathbb{H}$. They can be used to rotate a 3D vector from one reference frame $\vec{v} \in \mathbb{R}^3$ into another $\vec{v}' \in \mathbb{R}^3$. The operation describing the rotation of \vec{v} into the new reference frame is given using the Euler-Rodrigues formula:

$$\vec{v}' = \mathbf{q}^* \otimes \vec{v} \otimes \mathbf{q} \quad (1)$$

where $\mathbf{q}^* := q_0 - \vec{q}$ denotes the quaternion conjugate and \mathbf{q} the unit quaternion defined as:

$$\mathbf{q} := \cos \frac{\gamma}{2} + \hat{n} \sin \frac{\gamma}{2} \quad (2)$$

where $\hat{n} \in \mathbb{R}^3$ serves as the 3D unitary axis of rotation and $\gamma \in \mathbb{R}$ is the amount of rotation that the vector \vec{v} is rotated around the axis.

Notice that a 3D vector, as \vec{v} can be seen as a quaternion with a null scalar part and is often referred as a pure quaternion. Therefore, the previous product in (1) represents

the product of two quaternions $\mathbf{q}, \mathbf{r} \in \mathbb{H}$ that is defined using the vector inner \cdot and outer \times products as:

$$\mathbf{q} \otimes \mathbf{r} := (q_0 r_0 - \vec{q} \cdot \vec{r}) + (q_0 \vec{r} + r_0 \vec{q} + \vec{q} \times \vec{r}) \quad (3)$$

The square of the norm of a quaternion $\mathbf{q} \in \mathbb{H}$ is denoted using the product and conjugated as:

$$\|\mathbf{q}\|^2 := \mathbf{q}^* \otimes \mathbf{q} = \mathbf{q} \otimes \mathbf{q}^* = q_0^2 + q_1^2 + q_2^2 + q_3^2 \quad (4)$$

Thus, if one considers a rigid body, a single unit quaternion is sufficient to describe the rotation of any vector from the body frame towards an inertial frame and vice versa. Thus, it is sufficient to successfully describe the body's attitude.

The axis-angle representation uses a single 3D vector $\vec{\gamma} \in \mathbb{R}^3$. Equation (1) can be also used to describe the attitude of a rigid body if one takes into account that $\vec{\gamma} := \hat{n} \gamma$.

The explicit mathematical relationship between the axis-angle formula and the unit quaternion representation can be described using the following quaternion natural logarithm [18]

$$\ln \mathbf{q} := \begin{cases} \frac{\vec{q}}{\|\vec{q}\|} \arccos q_0, & \vec{q} \neq 0 \\ 0, & \vec{q} = 0 \end{cases} \quad (5)$$

This relationship is $\vec{\gamma} := 2 \ln \mathbf{q}$. Note that (5) only applies to unit quaternions.

System model

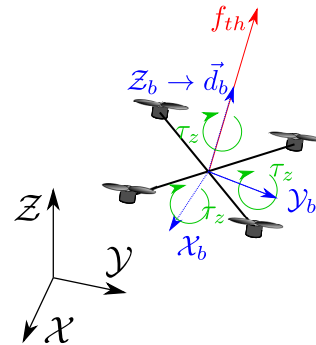


Fig. 1. The quadcopter model with its four control inputs

The dynamic model of the quadcopter, illustrated in Fig. 1, using the quaternion representation can be described as:

$$\begin{aligned} \ddot{\vec{p}} &= \mathbf{q} \otimes \begin{bmatrix} 0 \\ 0 \\ \frac{f_{th}}{m} \end{bmatrix} \otimes \mathbf{q}^* + \vec{g} \\ \dot{\mathbf{q}} &= \frac{1}{2} \mathbf{q} \otimes \vec{\Omega} \\ \dot{\vec{\Omega}} &= J^{-1} (\vec{\tau} - \vec{\Omega} \times J \vec{\Omega}) \end{aligned} \quad (6)$$

where $\vec{p}, \dot{\vec{p}}, \ddot{\vec{p}} \in \mathbb{R}^3$ are the 3D position, speed and acceleration of the aerial vehicle, respectively, in the inertial frame. The unit quaternion $\mathbf{q} \in \mathbb{H}$ denotes its attitude. The mass of the vehicle is $m \in \mathbb{R}_+$, the effect of the gravity in the inertial frame is denoted by $\vec{g} \in \mathbb{R}^3$, and the system's

inertia matrix $J \in \mathbb{R}^{3 \times 3}$, with $J > 0$ is considered to be constant. The terms $\vec{\Omega}, \dot{\vec{\Omega}} \in \mathbb{R}^3$ represent the angular velocity and acceleration, respectively, w.r.t. the body frame. The variables $f_{th} \in \mathbb{R}$ and $\vec{\tau} \in \mathbb{R}^3$ are the vehicle control inputs.

III. CONTROLLER DESIGN

Two controllers are proposed, one based on the adaptive sliding mode approach and the second one based on the quaternion formulation for assuring the attitude stabilization. Both controllers allow to the quadcopter to be robust with respect to external and unknown perturbations.

A. Adaptive position control

For an easy designing of the control laws, we rewrite the translational part (\mathcal{H}_{pos}) of system (6) as follows

$$\ddot{\vec{p}} = \vec{u} + \vec{g} \quad (7)$$

$$\text{with } \vec{u} := \mathbf{q}^* \otimes \begin{bmatrix} 0 & 0 & \frac{f_{th}}{m} \end{bmatrix}^T \otimes \mathbf{q}.$$

Notice that $\vec{p}, \dot{\vec{p}} \in \mathbb{R}^3$ are the 3D position and translational velocity of the vehicle, respectively, thus, the following sliding surface can be proposed as

$$\vec{\sigma} = \dot{\vec{p}} - \dot{\vec{p}}_d + \vec{\lambda}(\vec{p} - \vec{p}_d), \quad (8)$$

where \vec{p}_d denotes the reference vector, while $\vec{\lambda} \in \mathbb{R}^{3 \times 3}$ is a diagonal matrix gain with $\lambda_i > 0$, for $i = 1, 2, 3$.

In addition, by differentiating $\vec{\sigma}$ w.r.t. time, and taking into account the dynamics of system (7), the following result is obtained:

$$\dot{\vec{\sigma}} = \vec{u} + \vec{g} - \ddot{\vec{p}}_d + \vec{\lambda}(\dot{\vec{p}} - \dot{\vec{p}}_d) + \vec{\Delta}(t). \quad (9)$$

the term $\vec{\Delta}(t)$ is included to consider the unmodeled nonlinear dynamics that could be present as external disturbances and/or uncertainties in the dynamic system, and that could affect the quadcopter position. Moreover, \vec{u} is considered a vector containing the input force exerted by the quadcopter in the three axis, and proposed as

$$\vec{u} = -\vec{g} + \ddot{\vec{p}}_d - \vec{\lambda}(\dot{\vec{p}} - \dot{\vec{p}}_d) + \vec{u}_\sigma, \quad (10)$$

where \vec{u}_σ is given by the following adaptive sliding mode controller:

$$\vec{u}_\sigma = -\vec{K}(t) |\vec{\sigma}|^{\frac{1}{2}} \text{sign}(\vec{\sigma}) - \vec{k}_2 \vec{\sigma}, \quad (11)$$

where $\vec{K}(t)$ dynamics is described by

$$\dot{\vec{K}}(t) = \begin{cases} \vec{k}_1 \text{sign}(|\vec{\sigma}| - \vec{\mu}), & \text{if } \vec{K} > \vec{K}_{min}, \\ \vec{K}_{min}, & \text{if } \vec{K} \leq \vec{K}_{min}. \end{cases} \quad (12)$$

Notice that this controller tunes its gains to ensure minimal control effort, which is described by a diagonal matrix $\vec{K}_{min} \in \mathbb{R}^{3 \times 3}$, ensuring no zero control. Diagonal matrix gain $\vec{k}_1 \in \mathbb{R}^{3 \times 3}$ provides the adaptation rate, while $\vec{\mu} \in \mathbb{R}^3$ is a factor used to detect the loss of the sliding mode, and thus increase or decrease the gains if it is required. Furthermore,

robustness to bounded perturbations/uncertainties properties of standard sliding mode control is maintained.

Then, without loss of generality, by considering the controller (10) in closed loop with the sliding surface dynamics (9), one gets

$$\dot{\vec{\sigma}} = \vec{u}_\sigma + \vec{\Delta}(t). \quad (13)$$

Observe that quadcopters are limited by physical characteristics and they can not compensate infinity perturbations. Therefore, they are considered globally bounded and can be written in the following form

$$|\vec{\Delta}(t)| \leq B_1 \in \mathbb{R}_+^3 \quad (14)$$

Lyapunov's direct method is used to determine the stability of the adaptive sliding mode controller - ASMC. Hence, $V(t)$ is proposed as a candidate function:

$$V(t) = \frac{1}{2} \vec{\sigma}^T(t) \vec{\sigma}(t), \quad (15)$$

with $V(0) = 0$ and $V(t) > 0$ for $\vec{\sigma} \neq 0$. The time differentiation of the candidate function must be negative defined. Thus,

$$\dot{V}(t) = \vec{\sigma}^T(t) \dot{\vec{\sigma}}(t) < 0, \quad \sigma \neq 0. \quad (16)$$

Introducing (13) into (16) it leads to:

$$\begin{aligned} \dot{V}(t) &= \vec{\sigma}^T \left(\vec{u}_\sigma + \vec{\Delta}(t) \right) \\ &= \vec{\sigma}^T \left(-\vec{K}(t) |\vec{\sigma}|^{\frac{1}{2}} \text{sign}(\vec{\sigma}) - \vec{k}_2 \vec{\sigma} + \vec{\Delta}(t) \right) \\ &= -|\vec{\sigma}^T| \vec{K}(t) |\vec{\sigma}|^{\frac{1}{2}} - \vec{k}_2 \vec{\sigma}^T \vec{\sigma} + \vec{\sigma}^T \vec{\Delta}(t) \\ &\leq -|\vec{\sigma}^T| \vec{K}(t) |\vec{\sigma}|^{\frac{1}{2}} - \vec{k}_2 \vec{\sigma}^T \vec{\sigma} + |\vec{\sigma}^T| B_1 \end{aligned} \quad (17)$$

Notice that $\dot{V}(t) < 0$, therefore, the following sufficient condition must be met:

$$K(t) |\vec{\sigma}|^{\frac{1}{2}} + \vec{k}_2 |\vec{\sigma}| > B_1 \quad (18)$$

This leads to a constrained adaptive gain:

$$K(t) > |\vec{\sigma}|^{-\frac{1}{2}} (B_1 - \vec{k}_2 |\vec{\sigma}|). \quad (19)$$

In this analysis we consider that the disturbance is bounded. Note that to ensure stability, the magnitude of the control signal must be greater than the magnitude of the disturbance.

B. Attitude controller

From (6), the rotational part (\mathcal{H}_{att}) of the system using axis-angle representation can be represented as

$$\dot{\vec{\eta}} := \begin{bmatrix} \vec{\gamma} & \vec{\Omega} \end{bmatrix}^T \in \mathbb{R}^6 \quad (20)$$

where the vector $\vec{\gamma} \in \mathbb{R}^3$ is the new orientation representation using the angle-axis description.

Taking into account the relationship between the axis-angle representation and the quaternion attitude, an attitude error state can be defined as:

$$\vec{\eta}_e := \begin{bmatrix} \vec{\gamma}_e \\ \vec{\Omega} \end{bmatrix} \quad (21)$$

where the axis angle error is defined as $\vec{\gamma}_e := 2 \ln \mathbf{q}_e$. The quaternion attitude error $\mathbf{q}_e \in \mathbb{H}$ is defined using the quadcopter's attitude \mathbf{q} and the reference attitude $\mathbf{q}_d \in \mathbb{H}$ as

$$\mathbf{q}_e := \mathbf{q}_d^* \otimes \mathbf{q} \quad (22)$$

It is assumed that \mathbf{q}_d is constant.

The differentiation of the state $\vec{\eta}_e$ with respect to time results in:

$$\dot{\vec{\eta}}_e = \begin{bmatrix} \vec{\Omega} \\ J^{-1} (\vec{\tau} + \vec{\Omega} \times J \vec{\Omega}) \end{bmatrix} \quad (23)$$

It is assumed that the control algorithm is robust enough to ensure that $\mathbf{q}_e \rightarrow 0$, then it is obvious that the terms $\vec{\Omega} \times J \vec{\Omega}$ will be small. Therefore considering these values small enough, equation (23) can be simplified as:

$$\dot{\vec{\eta}}_e = A_\eta \vec{\eta}_e + B_\eta \vec{\tau} \quad (24)$$

The constant matrices A_η and B_η are defined as:

$$A_\eta := \begin{bmatrix} \vec{0} & I_3 \\ \vec{0} & \vec{0} \end{bmatrix}, \quad B_\eta := \begin{bmatrix} \vec{0} \\ J^{-1} \end{bmatrix} \quad (25)$$

Then, the control law can be proposed as

$$\vec{\tau} := -\vec{k}_{\eta_p} \vec{\gamma}_e - \vec{k}_{\eta_d} \vec{\Omega} \quad (26)$$

with \vec{k}_{η_p} and \vec{k}_{η_d} are positive gains and need to be chosen appropriately for making the system Hurwitz stable.

Substituting (26) into (24), it yields

$$\dot{\vec{\eta}}_e = \begin{bmatrix} \vec{0} & I_3 \\ -J^{-1} \vec{k}_{\eta_p} & -J^{-1} \vec{k}_{\eta_d} \end{bmatrix} \vec{\eta}_e \quad (27)$$

Rewriting the above using (5) it yields

$$\vec{\tau} := -2 \vec{k}_{\eta_p} \ln \mathbf{q}_e - \vec{k}_{\eta_d} \vec{\Omega} \quad (28)$$

Notice that $\vec{\Omega} \times J \vec{\Omega}$ are unmodeled parameters not considered in the attitude control, and they can produce uncertainties in the dynamic performance. Nevertheless, observe that these unmodeled parameters are included in $\vec{\Delta}(t)$ and compensated in the position controller. Therefore, without loss of generality, we can consider that the control law (28) stabilizes the attitude system in (6) towards the state $\mathbf{q} \rightarrow \mathbf{q}_d$ and $\vec{\Omega} \rightarrow 0$ as $t \rightarrow \infty$.

C. Quadcopter control law

Notice that the attitude controller stabilizes the orientation with respect to a desired quaternion. This desired quaternion, $\mathbf{q}_d \in \mathbb{H}$, is designed using the translation controller as

$$\mathbf{q}'_d := \left(1 + \hat{\vec{u}} \cdot \hat{d}_b \right) + \left(\hat{d}_b \times \hat{\vec{u}} \right) \quad (29)$$

$$\mathbf{q}_d := \frac{\mathbf{q}'_d}{\|\mathbf{q}'_d\|}$$

where $\hat{\vec{u}}$ is the direction of the translational controller, $\hat{d}_b \in \mathbb{R}^3$ is the normalized direction of the thrust force $f_{th} = \|\vec{u}\|$ fixed in the body frame as shown in Fig. 1.

From the previous, observe that both controllers are linked by the desired quaternion. On one hand, the desired quaternion is used in the attitude controller for assuring the system stability, see (28). On the other hand, this desired quaternion is the direction of the control vector for the translational part.

Remark from (29) that the goal of the quaternion reference attitude is to make the quadrotor thrust force $\mathbf{q} \otimes [0 \ 0 \ f_{th}]^T \otimes \mathbf{q}^*$ direction coincides with the direction of the position force \vec{u} . Notice also that the magnitude of the thrust force f_{th} is the real control input that can be obtained from the magnitude of \vec{u} as $f_{th} := \|\vec{u}\|$.

From subsection III-B, note that the attitude controller $\vec{\tau}$ has been proven stable for constant attitude references and small angles. However, any difference in the thrust direction, as explained before, can be seen as a perturbation of the position system, but the position controller was designed to be robust even in presence of undesired inner performances of the system. Therefore, without any loss of generality, the asymptotic stability of the quadcopter system in (6) can be guaranteed even in these conditions, and we can conclude that $\mathbf{q} \rightarrow \mathbf{q}_d$, $\vec{\Omega} \rightarrow 0$, $\vec{p} \rightarrow \vec{p}_d$ and $\dot{\vec{p}} \rightarrow \dot{\vec{p}}_d$ as $t \rightarrow \infty$.

IV. EXPERIMENTAL RESULTS

This section presents the results of the experimental validation of our proposal. The objective is to evaluate the robustness of the quadcopter being handled by the controller developed in section III, flying in the presence of aggressive wind gusts. A block diagram of the full control scheme is illustrated in Fig. 2.

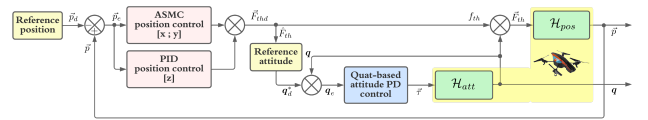


Fig. 2. Flowchart of the control scheme on the experimental quadcopter. H_{pos} and H_{att} represent the dynamic equations for the translational and rotational model.

The experiment involves performing an indoor hover flight at a given position. Once the quadcopter has stabilized, wind gusts of varying magnitude and duration are induced using a leaf blower, see Fig. 3. Such gusts are completely unknown for the controller. The quadcopter used in this test is a Parrot AR.Drone. The experiment takes 60 seconds. The measurement of the UAV rotation is obtained by its internal sensors, while its position and velocity are measured with an Optitrack motion capture system, which runs at a frequency of 100 Hz. The controller parameters used to perform this experiment are shown in the Table I.

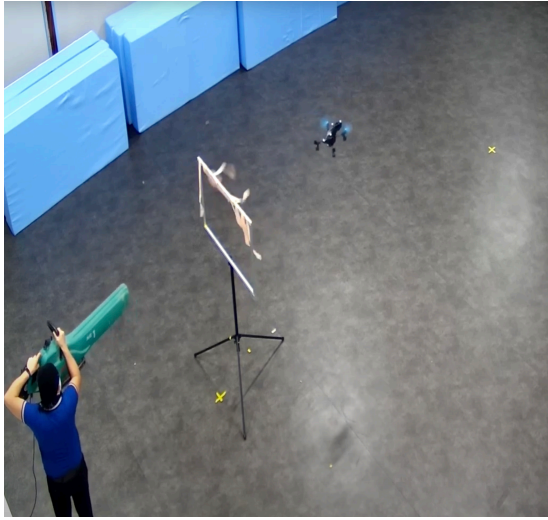


Fig. 3. Picture of the real time experiment when the quadcopter is exposed to wind gusts.

TABLE I
POSITION AND ATTITUDE CONTROLLERS PARAMETERS

Position adaptive controller (x, y)		Attitude controller	
Parameter	Value	Parameter	Value
$\bar{\lambda}_{x,y}$	diag(2.9, 2.9)	$\bar{k}_{\eta p}$	diag(12, 12, 0.6)
$\bar{k}_{1x,y}$	diag(0.1; 0.1)	$\bar{k}_{\eta d}$	diag(0.2, 0.2, 0.2)
$\bar{K}_{min_{x,y}}$	diag(0.1; 0.1)	z system	
$\bar{\mu}_{x,y}$	diag(0.3; 0.3)	\bar{k}_z	[0.6, 0.1, 0.4]
$\bar{k}_{2x,y}$	diag(0.07; 0.07)		

The flight performance of the quadcopter is shown in Fig. 4. From $t = 0s$ to $t = 12.20s$, the UAV is manually controlled to move it to the desired hovering point. At $t = 12.21s$ the adaptive controller (ASMC) takes control of the quadcopter, setting as reference values, the position of the vehicle at that particular moment. The red highlight in the figure indicates the period where the aircraft is disturbed by the wind gusts. The effectiveness of the ASMC controller in keeping the UAV stable in spite of the observed wind.

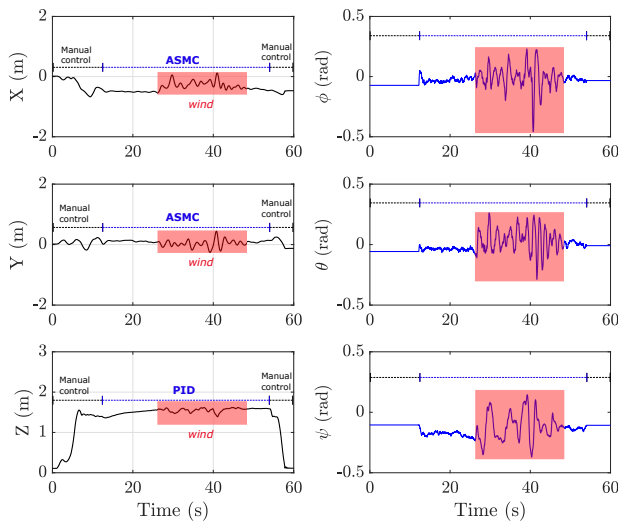


Fig. 4. Position and orientation performances of the quadcopter aerial vehicle in presence of aggressive wind gusts.

In Fig. 5, the norm of the orientation error is shown on a logarithmic scale. Variations in the angles are required for position control, and thus are associated in this scenario with wind gusts. In spite of this, the orientation error remains bounded.

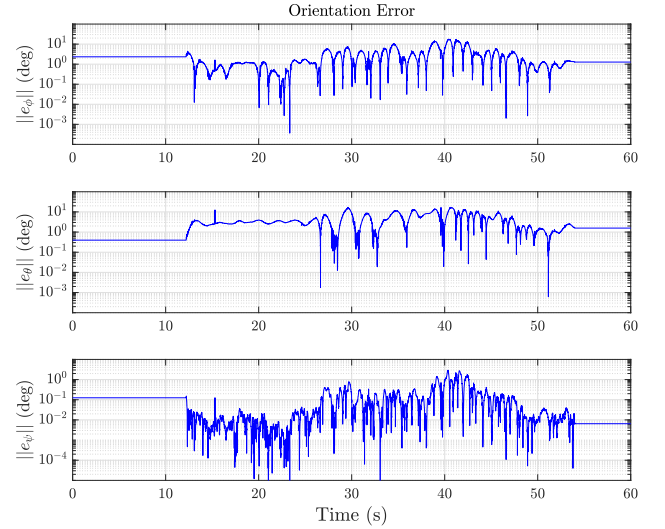


Fig. 5. The quaternion values were converted to their equivalent values in Euler angles for showing the quadcopter orientation error.

In Fig. 6, the horizontal position control signals generated by the ASMC scheme are shown. These signals are in the range of $[-1 : 1]$ normalized to the maximum thrust capacity of the quadcopter prototype. These control signals are analyzed in combination with the data of Fig. 7, corresponding to the adaptive gains of the ASMC. Incremental peaks of adaptation are observed once the first gusts are present. From $t = 30s$ and up to $t = 43s$ further adaptation is required to maintain vehicle position. Once the gusts disappear, the adaptation decreases to its minimum value. This demonstrates the ability of the proposed scheme to maintain the position and stability of the quadcopter while compensating for the wind disturbances.

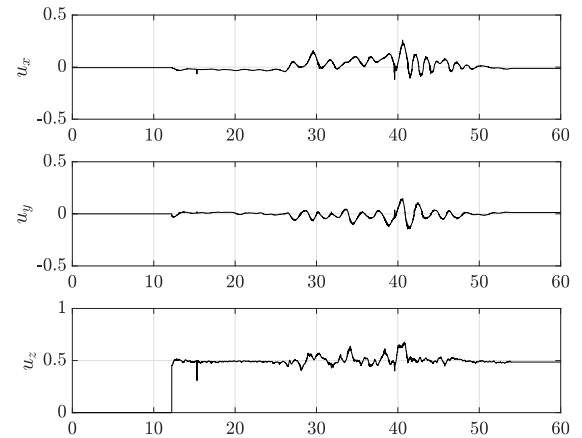


Fig. 6. Adaptive position control performance of \vec{u} . Notice that the control action changes when the perturbation is applied for compensating it.

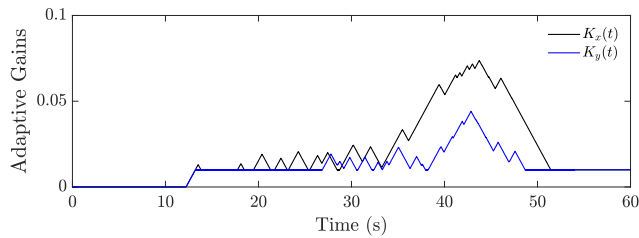


Fig. 7. Adaptive control gains performances obtained during experiments.

Finally, in <https://youtu.be/C6fUAKi0Ywg> a video of such experimental test is shown.

V. CONCLUSIONS

In this work, a robust control scheme for a quadcopter aerial vehicle in the presence of wind gusts has been presented. The vehicle orientation was modeled through the quaternion and axis-angle approach, allowing a linear representation of the rotational subsystem, and decoupled from position dynamics. Based on this scheme, an adaptive sliding mode controller for vehicle position was proposed. In addition, using Lyapunov's analysis, the stability of the system was guaranteed. Then, our proposal has been evaluated through an experimental implementation, deploying the aerial vehicle in a scenario with aggressive wind gusts. Experimental results demonstrate the effectiveness of our proposal by ensuring the stability of the quadcopter despite environmental disturbances.

Future work will include validate the controller for aggressive trajectories and delays in the data position.

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