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A new realistic characteristics of real energy conversion processes: A contribution of finite size thermodynamics

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Abstract

The perfection of energy conversion process is currently gauged through out different kind of quality indicator that compare the real performance of the process to that of the ideal reversible Carnot process. The criteria resulting from this commonly used approach give a false idea of the real quality of the energy conversion process. Indeed, the real energy conversion process that generates true energetic power levels is compared to ideal associated Carnot process that generates zero-output power level. The real conversion process implementing finite heat exchanger areas is then compared to the ideal process that needs infinite heat exchanger area to fulfill the same power requirements. This paper presents a new thermo-economic approach, based on Finite Size Thermodynamics, more suitable for qualifying real energy conversion processes. This approach takes into account the external thermodynamic irreversibilities relative to the heat transfer rate through a finite size heat exchanger surface between the external heat sources or sinks and an ideal process without internal thermodynamic irreversibilities. This new approach enables to evaluate more realistically ideal performances of real energy conversion processes. It makes possible the definition of new criteria that characterize more reasonably the quality of a real thermal process compared with the corresponding endoreversible process either for the same power duty (performance criterion) or either for the same involved total heat exchanger surface (technical criterion).

1. Introduction

Thermodynamic optimization of thermal processes is generally carried out in order to minimize thermodynamic irreversibilities created during the energy conversion. In this approach, the perfection of the system is measured through out different kind of quality indicator, that have been formulated by several authors such as Grassman [1] who defined the exergetic efficiency, Kostenko [2] who defined the intrinsic efficiency or such as Le Goff [3] who proposed the exergetic effectiveness. All these quality indicators tend to compare the real performance of the process to that of the ideal process constituted by one or several interconnected Carnot reversible processes.

In the case of thermal energy conversion process, the coefficient of performance (COP) is compared to the maximal performance of the ideal process given by the Carnot-COP. Then, the exergetic efficiency η_{ex} is defined as:

$$\eta_{ex} = \frac{COP_{real}}{COP_{ideal}} \quad (1)$$

However, ideal Carnot processes operate on the reversible and equilibrium principles. Consequently, these ideal processes need heat sinks and heat sources to be in thermal contact with the working fluid and the elementary transformations to take place at an infinitely slow rate. Therefore, an ideal process produces the maximum useful energy for a given set of thermal reservoirs but generates zero output power. The Carnot efficiency that is considered as the standard against all real energy processes is compared, is inappropriate to gauge the quality of real energy conversion from an engineering point of view. Indeed, this long used criterion tends to compare processes with finite power levels to ideal processes with zero power levels. That is equivalent to compare a real process with a finite total exchanger area to an ideal process with an infinite total exchanger area that to fulfill the same thermal power requirements as the real one. This statement leads to unrealistic conclusions concerning the quality of the energy conversion. In order to remove this ambiguity, the real energy conversion process has to compare to a new model of ideal process operating with the same power level or with the same total exchanger area. In this way, one has to consider the energy transfer rates occurring between the reservoirs through finite exchanger areas and the ideal process, called endoreversible process.

Endoreversible thermodynamics (or finite time thermodynamics and finite size thermodynamics) is a subset of irreversible thermodynamics that has been developed for the purpose of engineering science in order to optimize conventional energy conversion processes such as heat engines or heat pumps. Finite Size Thermodynamics involves in its formulation the external thermodynamic irreversibilities relative to the heat transfer rate through a finite size heat exchanger surface between the external heat sources or sinks and an ideal process without internal thermodynamic irreversibilities.

Chambadal [4] and Novikov [5] introduced in 1957 this new thermodynamic concept in the objective of maximization of the power output of power plant. They described independently an endoreversible model (fig. 1) of a heat engine that takes into account the external thermodynamic irreversibilities generated by the heat transfer rate at the high temperature heat source. Curzon-Ahlborn [6] popularized in 1975 this approach by giving a simple expression of the efficiency (called Chambadal-Novikov-Curzon-Ahlborn efficiency) of a heat engine operating at the maximum output power and taking into account the heat transfer resistance both at the cold and the hot reservoirs:

$$\eta_{CNCA} = 1 - \sqrt{\frac{T_o}{T_h}} \quad (2)$$

While the maximum efficiency (at zero power output) is given by the classical expression of the Carnot efficiency:

$$\eta_c = 1 - \frac{T_o}{T_h} \quad (3)$$

Since then, many researchers (Wu [7], De Vos [8], Sieniutycz [9], Bejan [10]...) have investigated and extended the effect of irreversibilities generated by heat and mass transfer rates on thermodynamic processes and cycles.

This new approach proposes new models for ideal thermal processes, so-called endoreversible process that enables to evaluate more realistically ideal performances of real energy conversion processes. The endoreversible cycle concept makes possible the definition of new criteria that characterize more reasonably the quality of a real thermal process compared with the corresponding endoreversible process either for the same power duty (performance criterion) or either for the same involved total heat exchanger surface (technical criterion). In this paper, this approach is applied to thermal energy conversion processes with four heat

sinks or sources that represent the major part of conversion processes (heat engine, refrigerators, heat pumps, heat transformers, co-generation, bi-generation and tri-generation).

2. Endoreversible model for a four-temperature process

The model of the four-temperature endoreversible process is described in Fig. 1. The process operates between a driving high heat source at T_h , a low temperature heat source at the cooling production temperature T_f and two heat sinks at T_u and T_o corresponding respectively to the useful heat production temperature and the environment temperature.

In order to overcome heat transfer resistances, thermal pinches between reservoirs and the reversible internal process are necessary. Internal temperatures T_{hi} , T_{fi} , T_{ui} , and T_{oi} are associated to each reservoir.

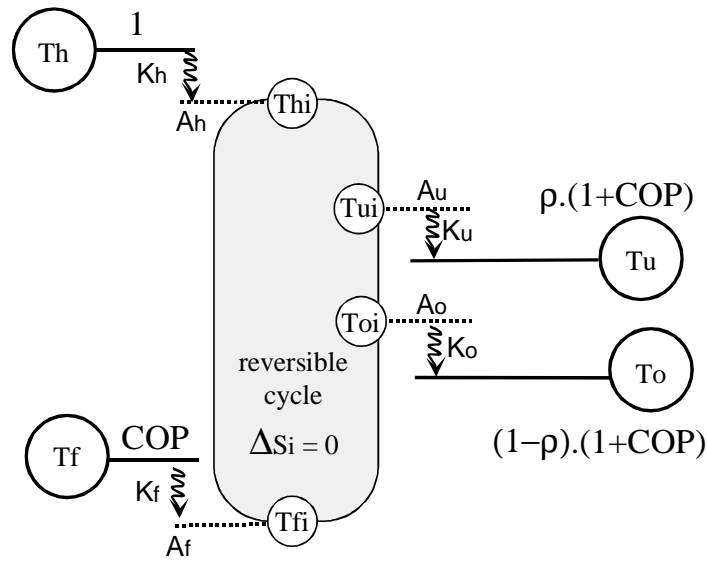


Fig. 1: Endoreversible model of the four-temperature energy conversion process

The rates of heat transfer flow of the endoreversible process are characterized through out two ratio, the coefficient of performance COP and ρ . The COP is defined as the ratio of cooling production relatively to driving heat input rate at the high temperature and ρ is the useful amount of heat production at T_u relatively to overall heat production:

$$COP = \frac{\dot{Q}_f}{\dot{Q}_h} \quad \text{and} \quad \rho = \frac{\dot{Q}_u}{\dot{Q}_o + \dot{Q}_u} \quad (3)$$

The rate of heat transfer from a reservoir and the internal reversible process depends on the thermal pinch, the heat transfer area A of the heat exchanger and the overall heat transfer coefficient K associated to each reservoir. In these conditions, the heat transfer rates are defined relatively to the driving heat input rate at the high temperature as a function of the COP and ρ (Fig.1):

$$\begin{aligned} \dot{Q}_h &= K_h \cdot A_h \cdot (T_h - T_{hi}) = 1 & \dot{Q}_u &= K_u \cdot A_u \cdot (T_{ui} - T_u) = \rho \cdot (1 + COP) \\ \dot{Q}_f &= K_f \cdot A_f \cdot (T_f - T_{fi}) = COP & \dot{Q}_o &= K_o \cdot A_o \cdot (T_{oi} - T_o) = (1 - \rho) \cdot (1 + COP) \end{aligned} \quad (4)$$

The first and second laws of thermodynamics applied to the internal reversible process are expressed as:

$$\dot{Q}_h + \dot{Q}_f = \dot{Q}_o + \dot{Q}_u \quad \text{and} \quad \frac{\dot{Q}_h}{T_{hi}} + \frac{\dot{Q}_f}{T_{fi}} = \frac{\dot{Q}_o}{T_{oi}} + \frac{\dot{Q}_u}{T_{ui}} \quad (5)$$

Using the relations (3) and (4), the coefficient of the endoreversible process yields:

$$COP_{endo} = \frac{\frac{1}{T_{oi}} - \frac{1}{T_{hi}} - \rho \cdot \left(\frac{1}{T_{oi}} - \frac{1}{T_{ui}} \right)}{\frac{1}{T_{fi}} - \frac{1}{T_{oi}} + \rho \cdot \left(\frac{1}{T_{oi}} - \frac{1}{T_{ui}} \right)} \quad (6)$$

Similarly, the same expression can be derived for the four-temperature Carnot process operating with no thermal pinches in contact with the reservoirs:

$$COP_{ideal} = \frac{\frac{1}{T_o} - \frac{1}{T_h} - \rho \cdot \left(\frac{1}{T_o} - \frac{1}{T_u} \right)}{\frac{1}{T_f} - \frac{1}{T_o} + \rho \cdot \left(\frac{1}{T_o} - \frac{1}{T_u} \right)} \quad (7)$$

Because of the necessary thermal pinches to put in power operation the endoreversible process, the ideal Carnot COP is greater than the endoreversible COP:

$$COP_{endo} < COP_{ideal} \quad (8)$$

3. Performance optimization

The objective of this study is to define the optimal distribution of the thermal pinches of an endoreversible process operating between four external temperature levels that are fixed in general by the application. This endoreversible process is characterized by a given coefficient of performance COP and enables the energy conversion with a minimal total heat exchanger area A relatively to a given amount of useful produced power. In definitive, it comes back to determine the optimal distribution of given heat exchanger areas that maximizes the power duty of the endoreversible process operating with fixed external temperature values.

This optimization problem can be handled by the method of the Lagrange multipliers and by solving the associated set of Euler equations in order to determine the optimal internal temperatures of the endoreversible process. The objective function is the total heat exchanger that has to be minimized under the performance constraint of $COP = COP_{endo}$. The related Lagrange function has the form:

$$L = A + \lambda \cdot (COP - COP_{endo})$$

In this formulation, the total heat exchanger area is expressed by:

$$A = A_h + A_u + A_o + A_f \quad (9)$$

Using the set of relations (4), it becomes:

$$A = \frac{1}{K_h \cdot (T_h - T_{hi})} + \frac{\rho \cdot (1 + COP)}{K_u \cdot (T_{ui} - T_u)} + \frac{(1 - \rho) \cdot (1 + COP)}{K_o \cdot (T_{oi} - T_o)} + \frac{COP}{K_f \cdot (T_f - T_{fi})} \quad (10)$$

In addition, the adjoined constraint function is expressed as:

$$COP \cdot \left[\frac{1}{T_{fi}} - \frac{1}{T_{oi}} + \rho \cdot \left(\frac{1}{T_{oi}} - \frac{1}{T_{ui}} \right) \right] - \left[\frac{1}{T_{oi}} - \frac{1}{T_{hi}} - \rho \cdot \left(\frac{1}{T_{oi}} - \frac{1}{T_{ui}} \right) \right] = 0 \quad (11)$$

The resolution of the following related set of Euler equations enables to determine the internal temperatures of the endoreversible process that minimize the total heat exchanger area:

$$\frac{\partial L}{\partial T_{hi}} = 0, \quad \frac{\partial L}{\partial T_{fi}} = 0, \quad \frac{\partial L}{\partial T_{ui}} = 0, \quad \frac{\partial L}{\partial T_{oi}} = 0 \quad \text{and} \quad \frac{\partial L}{\partial \lambda} = 0 \quad (12)$$

The resolution of this set of partial derivative of the Lagrange function yields to a remarkable relation between the optimal internal temperatures and thermal pinches:

$$\lambda = \frac{1}{K_h} \left(\frac{T_{hi}}{\Delta T_h} \right)^2 = \frac{1}{K_f} \left(\frac{T_{fi}}{\Delta T_f} \right)^2 = \frac{1}{K_u} \left(\frac{T_{ui}}{\Delta T_u} \right)^2 = \frac{1}{K_o} \left(\frac{T_{oi}}{\Delta T_o} \right)^2 \quad (13)$$

$$\text{with} \quad \Delta T_h = (T_h - T_{hi}), \quad \Delta T_u = (T_{ui} - T_u), \quad \Delta T_o = (T_{oi} - T_o), \quad \Delta T_f = (T_f - T_{fi})$$

Written in an equivalent form and by using the expressions of heat transfer rates (4), the relation (13) becomes:

$$\frac{\dot{Q}_h}{A_h} \left[\frac{1}{T_{hi}} - \left(\frac{T_h}{T_{hi}} \right)^2 \frac{1}{T_h} \right] = \frac{\dot{Q}_f}{A_f} \left[\frac{1}{T_{fi}} - \left(\frac{T_f}{T_{fi}} \right)^2 \frac{1}{T_f} \right] = \frac{\dot{Q}_u}{A_u} \left[\left(\frac{T_u}{T_{ui}} \right)^2 \frac{1}{T_u} - \frac{1}{T_{ui}} \right] = \frac{\dot{Q}_o}{A_o} \left[\left(\frac{T_o}{T_{oi}} \right)^2 \frac{1}{T_o} - \frac{1}{T_{oi}} \right] \quad (14)$$

This result depicts an equipartition of density rate of entropy production $\dot{\sigma}$ relatively to each heat exchange area for small temperature differences :

$$\dot{\sigma} = \frac{\dot{S}_h^{gen}}{A_h} = \frac{\dot{S}_u^{gen}}{A_u} = \frac{\dot{S}_o^{gen}}{A_o} = \frac{\dot{S}_f^{gen}}{A_f} \quad (15)$$

$$\text{with} \quad \dot{S}_j^{gen} = \dot{Q}_j \cdot \left| \frac{1}{T_j} - \frac{1}{T_i} \right|$$

and where the subscripts j and i is relative respectively to the external temperatures and the internal temperatures.

Furthermore, the relation (13) enables to express the driving temperature differences ΔT_f , ΔT_u and ΔT_o as a function of ΔT_h and external temperatures of reservoirs. Substituting these expressions for ΔT_f , ΔT_u and ΔT_o in equation (11), one solves the obtained equation where the

unknown variable is ΔT_h . Hence, the optimal thermal pinches associated to each heat transfer between the reservoirs and the endoreversible process, can be expressed as:

$$\Delta T_h = \frac{T_h}{\omega \sqrt{K_h} + 1}, \quad \Delta T_f = \frac{T_f}{\omega \sqrt{K_f} + 1}, \quad \Delta T_u = \frac{T_u}{\omega \sqrt{K_u} - 1}, \quad \Delta T_o = \frac{T_o}{\omega \sqrt{K_o} - 1} \quad (16)$$

$$\text{with } \omega = \frac{\frac{1}{T_h \sqrt{K_h}} + \frac{\rho(1+COP)}{T_u \sqrt{K_u}} + \frac{(1-\rho)(1+COP)}{T_o \sqrt{K_o}} + \frac{COP}{T_f \sqrt{K_f}}}{\left(1 - \frac{COP}{COP_{ideal}}\right) \left[\frac{1}{T_o} - \frac{1}{T_h} - \rho \left(\frac{1}{T_o} - \frac{1}{T_u}\right)\right]} \quad (17)$$

These expressions of the optimal thermal pinches may be expressed in a general form:

$$\Delta T_j = \frac{T_j}{\omega \sqrt{K_j} \pm 1} \quad \text{with } \omega = \frac{T_o \cdot \sum \frac{\dot{Q}_j}{T_j \sqrt{K_j}}}{\left(1 - \frac{COP}{COP_{ideal}}\right) \cdot \left[\left(1 - \frac{T_o}{T_h}\right) - \rho \left(1 - \frac{T_o}{T_u}\right)\right]} \quad (18)$$

The sign in the denominator of the generalized expression of the thermal pinch depends on the heat transfer direction: the sign is plus if the heat transfer corresponds to an input of energy and minus if the heat transfer corresponds to an output of energy. The heat transfer rates \dot{Q}_j are normalized by the energy input at the highest temperature level.

These optimal thermal pinches enable then to determine the optimal distribution of the minimal required total area of heat exchangers of the endoreversible process as a function of the different overall heat transfer coefficients, the temperatures of the reservoirs and for given performance characteristics (COP and ρ) of the endoreversible process. Optimal Heat exchanger areas can be expressed relatively to the amount of the useful power duty, the cooling capacity \dot{Q}_f that is equal in the present case to the COP:

$$A_j = \frac{\dot{Q}_j}{K_j \cdot \Delta T_j} \frac{1}{\dot{Q}_f} \quad (19)$$

The optimal and minimal total heat exchanger required for the endoreversible to achieve its power duty is obtained by summing all heat the exchange areas:

$$A = \sum_j A_j = \frac{1}{\dot{Q}_f} \sum_j \frac{\dot{Q}_j}{K_j \cdot \Delta T_j} \quad (20)$$

4. Application to a four-temperature heat conversion process

These optimization results are illustrated and discussed on a four-temperature heat conversion process (Fig.2) that provides for example both heat production at $T_u=70^\circ\text{C}$ and refrigeration effect at $T_f=0^\circ\text{C}$, using a driving heat source available at $T_h=140^\circ\text{C}$ and a heat sink (environment) available at $T_o=20^\circ\text{C}$. The heat exchange coefficients are supposed to be equal

for the purpose to a realistic value of $0.5\text{kW}\cdot\text{m}^{-2}\cdot\text{K}$. This kind of process can be easily implemented using sorption processes such as absorption cycles ($\text{NH}_3/\text{H}_2\text{O}$), adsorption cycles (activated carbon/ NH_3) or thermochemical cycles using ammoniates. Sorption processes need only thermal energy to operate.

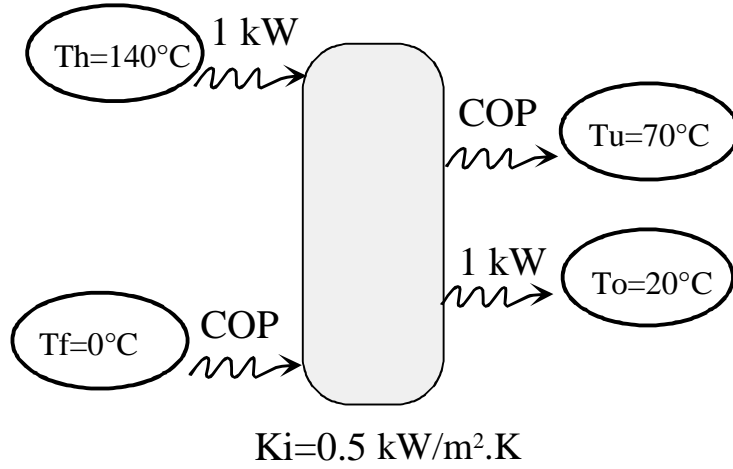


Fig. 2: Power characteristics of the studied four-temperature thermal process

In this application, the process has to provide a heat power production \dot{Q}_u at T_u in the same amount as the cooling capacity \dot{Q}_f at T_f . This condition imposes then an output energy ratio ρ between the useful output energy and the total output energy equal to:

$$\rho = \frac{COP}{1 + COP} \quad (21)$$

The ideal Carnot COP and the exergetic efficiency η_{ex} defined by the relation (1) of the endoreversible process are a function of this output energy ratio and therefore of the COP in this particular case study. Figure 3 shows the dependence of the ideal COP, the exergetic efficiency η_{ex} and the output energy ratio ρ with the COP of the endoreversible process.

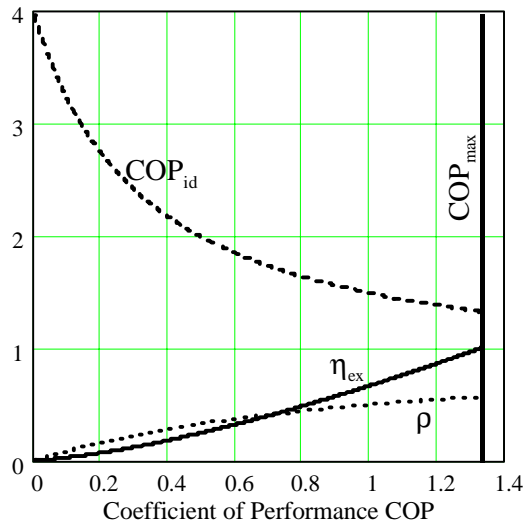


Fig 3: Evolution of the ideal COP, the energy output ratio ρ and the exergetic efficiency η_{ex}

The maximal value COP_{\max} of the COP is obtained when the exergetic efficiency η_{ex} is equal to one, that is to say when $COP=COP_{id}$, and therefore its expression is:

$$COP_{\max} = \frac{\frac{1}{T_o} - \frac{1}{T_h}}{\frac{1}{T_f} - \frac{1}{T_u}} \quad (22)$$

For these operating conditions, the area and the temperature difference associated to each heat exchange of the endoreversible process can be determined. Figure 4 shows the evolution of these areas per kW of cooling capacity (or heating capacity) relatively to the coefficient of performance. The temperature differences decrease toward a zero-value when the COP become close to the ideal COP.

The curve of the total area of heat exchangers has minimum value. The left branch of this curve is uninteresting from an engineering point of view: the total area of heat exchanger increases drastically while the COP decreases to a non-zero value. The more interesting part of this curve is the right side: the COP increases with increasing value of the total heat exchange area and reach an infinite value when the COP tends toward the ideal Carnot COP.

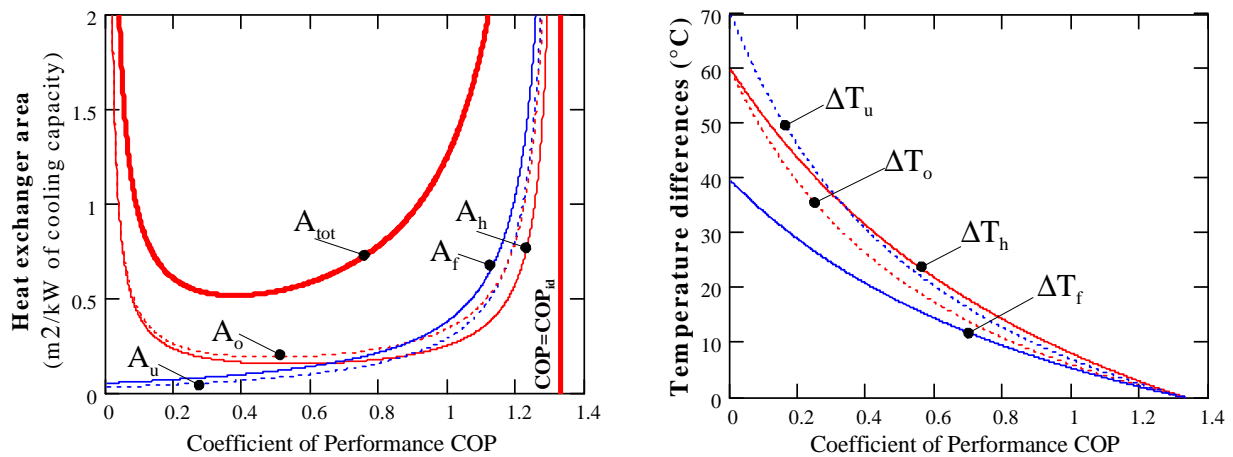


Figure 4: Evolutions of heat exchanger areas and temperature differences associated to each thermal reservoir

These graphs show the necessary operating conditions of the ideal process to achieve the same power duty as an endoreversible process: infinite heat exchanger areas and zero temperature differences.

4. Results and discussion

An endoreversible process is a more realistic model than Carnot process. It only takes into account the external irreversibilities due to energy transfer through finite areas. Therefore, all real process that really generates internal irreversibilities needs a greater total heat exchanger area than the associated endoreversible cycle in order to achieve the same performance. The curve of total area represents then the minimum required heat exchanger area for any real process in order to achieve the same duty as the endoreversible process. Finally, all real processes that operate with these four external temperatures are enclosed by this endoreversible curve (Fig. 6).

This statement enables to provide new criteria for a realistic characterization of real energy conversion processes. Indeed, let us consider a real process (point X) with a given performance COP_{real} and operating with a finite total heat exchanger area A_{real} . Up to now, the quality of this real process is measured through the classical exergetic efficiency η_{ex} , defined in (1), that compares the real performance COP_{real} to that of the ideal process COP_{id} . Now, to come back to what we developed previously, this criterion does not reflect the quality of real processes operating with true power levels.

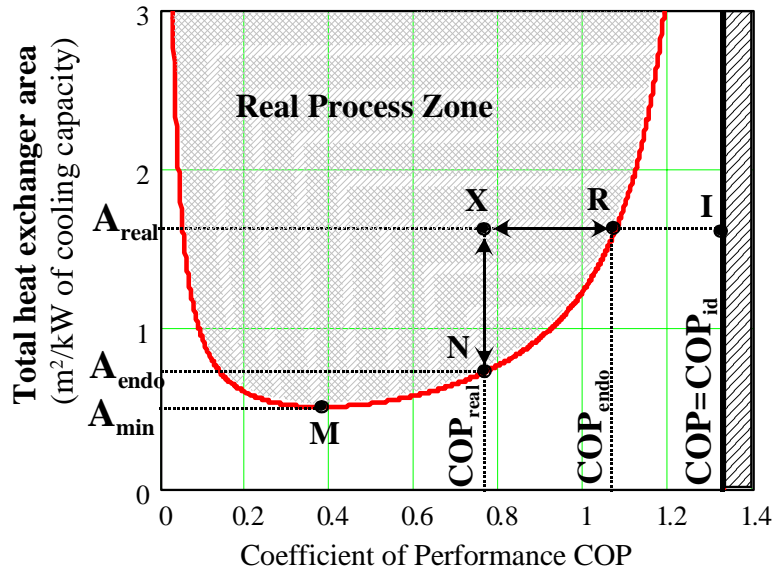


Figure 6: Points of comparison that make possible a realistic characterization of real energy conversion process

The study carried out here enables to characterize more practically a real process from an engineering point of view. The first proposed criterion is an endoreversible exergetic efficiency η_{endo} in contrast of the classical exergetic efficiency η_{ex} . This new efficiency tends to compare (Fig. 6) the real performance COP_{real} (point X) to that of the associated endoreversible process COP_{endo} (point R) that makes use of the same total heat exchanger A_{real} of the real process instead of the ideal performance COP_{id} (point I) that needs infinite heat exchanger area:

$$\eta_{endo} = \frac{COP_{real}}{COP_{endo}} \quad (23)$$

The second proposed criterion characterizes technical aspects of the real process, and more precisely the optimal use of the total area of heat exchangers. The technical efficiency η_s compares the total heat exchangers A_{real} implemented in the real process to the minimum required total heat exchanger A_{endo} implemented in the associated endoreversible process achieving the same performance COP_{real} as the real one.

$$\eta_s = \frac{A_{endo}}{A_{real}} \quad (24)$$

This last technical criterion is of great importance from an engineering point of view. It characterizes the technical perfection of a real process.

The study performed here for a heat conversion process can be easily generalized to heat engine by assimilating mechanical energy to heat energy available to an infinite temperature

level. Therefore, same conclusions can be drawn. Figure 7 illustrates this generalization on an endoreversible heat engine operating between a heat source at 600°C and a heat sink at 20°C. The point M corresponds to the minimum total area of heat exchanger of the endoreversible heat engine and is obtained for the Curzon-Albhorn efficiency (the so-called *nice radical*). The Curzon-Albhorn efficiency ρ_{CA} is the particular efficiency at the maximum output power for an endoreversible heat engine with a given total heat exchanger area. This is equivalent to say that the Curzon-Albhorn efficiency is obtained for the minimum total heat exchanger area of an endoreversible process with a given output power.

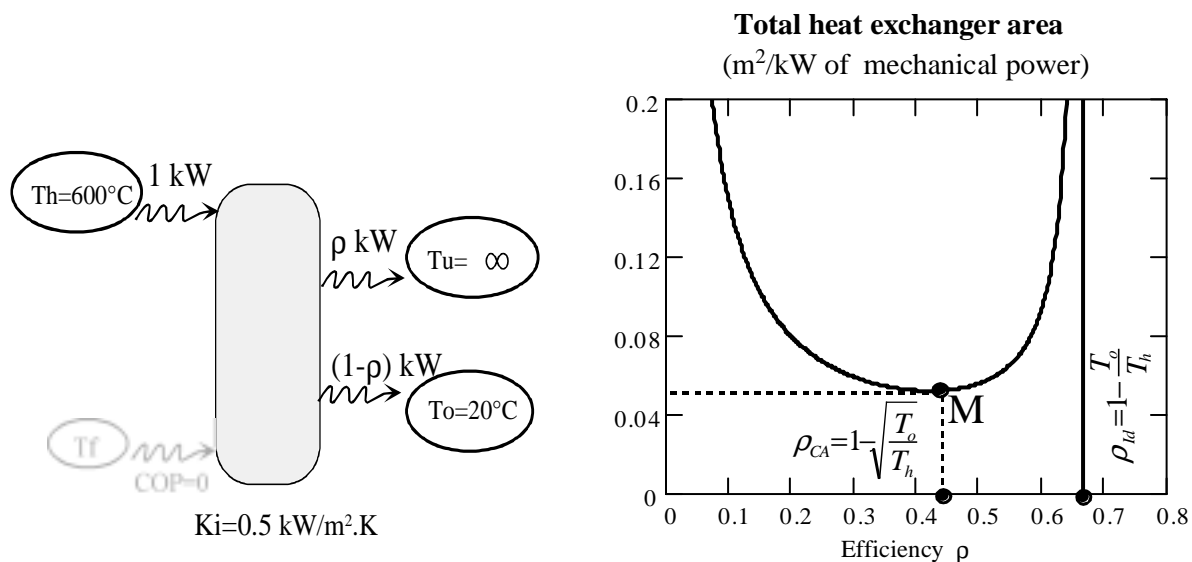


Figure 7: Endoreversible curve of heat engine

Conclusions

A new kind of characterization of any real energy conversion processes using finite size thermodynamics has been conducted. It leads to new criteria that characterize more realistically the energy conversion quality as well as the technical quality of real process from an engineering point of view. This new approach can be applied to other energy conversion such as cogeneration or tri-generation processes.

Acknowledgements

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