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Grey area mitigation in Equivalent-DES using commutation error estimators

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1 Introduction

In seamless hybrid RANS/LES (sHRL) methods, the filtering operator is a complex blending between the ensemble-average (associated with RANS) and convolution filtering (associated with LES) of the Navier-Stokes equation. Unfortunately, differential operators do not commute with the sHRL filtering operation: the derivative of a filtered quantity is not equal to the filtering of the derivative of that quantity. Formally, a commutation error E_c then arises:

$$E_{ci} \equiv \frac{\widetilde{Du_i}^H}{Dt} - \frac{D\widetilde{u_i}^H}{Dt} . \quad (1)$$

where the operator $\widetilde{\cdot}^H$ is sHRL filtering. This error is non-negligible in areas with steep transition in resolution level (see e.g. [1]). Although it may not be the only source, this commutation error is undeniably one cause of the modeled scale depletion / log-layer mismatch issues which plague sHRL methods. A simple and comprehensive correction to this issue is presented hereafter.

2 Estimation of the commutation error

Following the additive filter idea of Germano [2], the hybrid filter H of a sHRL method may be expressed as:

$$\widetilde{\cdot}^H = b \widetilde{\cdot}^F + (1 - b) \widetilde{\cdot}^E , \quad (2)$$

where $\widetilde{\cdot}^F$ represents a filter which is arbitrarily well-resolved and $\widetilde{\cdot}^E$ the ensemble-average. The parameter b is a blending factor, which may vary in space and time.

The above equation allows to express the commutation error E_c between the H -filtering and any differential operator \mathcal{D}_i :

$$\widetilde{\mathcal{D}_i \cdot}^H = \mathcal{D}_i \widetilde{\cdot}^H - \frac{\partial b}{\partial x_i} (\widetilde{\cdot}^F - \widetilde{\cdot}^E) . \quad (3)$$

In the present work, an estimation of the commutation error E_c is presented, by (i) linking the blending parameter b in Eq. (2) to the energy ratio parameter r_k defined by the fraction of modeled turbulent kinetic energy within the total (modeled+resolved) turbulent kinetic energy, and (ii) choosing $F \equiv I$, i.e. no filtering at all.

To establish a relationship between the blending factor b and the energy ratio r_k , we use the expression of the turbulent stress at the H level, since the cutoff only appears explicitly within the turbulence equations. Recalling the turbulent stresses $\tau_{ij}^{\mathcal{O}}$ associated with an arbitrary filter \mathcal{O} , i.e. the part of the velocity fluctuations filtered by \mathcal{O} :

$$\tau_{ij}^{\mathcal{O}} = \widetilde{u_i u_j}^{\mathcal{O}} - \widetilde{u_i}^{\mathcal{O}} \widetilde{u_j}^{\mathcal{O}} , \quad (4)$$

and substituting the definition of the blended filter Eq. (2), gives the ensemble-averaged turbulent stress at the H level, keeping in mind that $\tau_{ij}^F = 0$:

$$\widetilde{\tau_{ij}^E}^H = (1 - b^2) \tau_{ij}^E . \quad (5)$$

Keeping in mind that $\widetilde{\tau_{ii}^H}^E = r_k \tau_{ii}^E$ and assuming isotropy of b and r_k , one establishes a relationship between b and r_k :

$$b = \sqrt{1 - r_k} . \quad (6)$$

With b known, Eq. (3) can now be applied to the flow equations, to estimate commutation correction terms. For the momentum equations, assuming b constant in time, the correction E_{ci} reads:

$$\begin{aligned} E_{ci} = & \frac{1}{\rho} \frac{\partial b}{\partial x_i} (\widetilde{p}^F - \widetilde{p}^E) + \frac{\partial b}{\partial x_j} \left(\widetilde{u}_i^F \widetilde{u}_j^F - \widetilde{u}_i^E \widetilde{u}_j^E \right) \\ & - \frac{\partial b}{\partial x_j} (\tau_{ij}^E) - 2\nu \frac{\partial b}{\partial x_j} \frac{\partial}{\partial x_j} \left(\widetilde{u}_i^F - \widetilde{u}_i^E \right) - \nu \frac{\partial^2 b}{\partial x_j^2} \left(\widetilde{u}_i^F - \widetilde{u}_i^E \right) . \end{aligned} \quad (7)$$

The F -filtered quantities in (7) are unknown, but were conveniently evaluated in [1]. For instance, the F -filtering operation over any quantity ϕ yields (as long as $b \neq 0$):

$$\widetilde{\phi}^F = \widetilde{\phi}^H + \frac{1-b}{b} \left(\widetilde{\phi}^H - \widetilde{\phi}^E \right) \Leftrightarrow \widetilde{\phi}^F = \widetilde{\phi}^E + \frac{\widetilde{\phi}^H - \widetilde{\phi}^E}{b} . \quad (8)$$

The above formulation is invalid for the case $b = 0$, which corresponds to the RANS limit. Within the RANS area, this is no problem, since E_c vanishes anyway. But at the limit between $b = 0$ and $b > 0$, Eq. (8) cannot provide the F -filtered variables anymore. In this case, there are two possibilities:

1. preventing $b = 0$, by setting a minimum value b_{min} . In [1], $b_{min} = 0.15 \Leftrightarrow r_{k,max} \approx 0.98$.
2. reconstructing the F -field from the E -field and the τ_{ij}^E stresses, instead of from the E - and H -fields. A similar strategy has been successfully used by [3].

The second solution above, is presented hereafter:

$$\widetilde{u}_i^F = \widetilde{u}_i^E + \underbrace{\sqrt{\tau_{ii}^E} \mathcal{N}(0,1)}_{\approx u_i'^E} , \quad \widetilde{p}^F = \widetilde{p}^E + \underbrace{\frac{\rho \tau_{ii}^E}{2} \mathcal{N}(0,1)}_{\approx p''^E} , \quad (9)$$

where $\mathcal{N}(0,1)$ is a random number with a normal distribution of average 0 and standard deviation 1. Regarding F velocities in Eq. (9), the Einstein convention does not apply.

Furthermore, the total turbulent kinetic energy (resolved+modeled) must remain unaffected by extra terms accounting for commutation errors. Therefore, as E_c changes the balance of resolved turbulent kinetic energy, this must be compensated at the subfilter level. This implies that transport equation for any second turbulent scale, must also be corrected.

3 Turbulence model

The chosen turbulent closure is the $k - \omega$ SST of [4]. The hybridization is performed on the sink term of the unresolved turbulent kinetic energy k_u , following the ‘‘Equivalent DES’’ method described in [5] :

$$\underbrace{\beta^* \omega k}_{\text{RANS}} \rightarrow \underbrace{\frac{k_u^{3/2}}{L}}_{\text{hybrid}} \quad \text{with} \quad L = \frac{r_k^{3/2} \sqrt{k_{\text{tot}}}}{C_\mu \psi \omega_{\text{tot}}} \quad (10)$$

where ω^* is the transported specific dissipation (second scale of the turbulence model), and ψ is given by [6] :

$$\psi = \frac{\beta}{C_\mu \gamma + r_k (\beta - C_\mu \gamma)} \quad (11)$$

where the parameters β , C_μ and γ are defined as in the RANS version.

Furthermore, the eddy viscosity related to the unresolved motion, is given by [6]:

$$\nu_u = \frac{a_1 k_u}{\max \left[a_1 \psi \omega^* ; \widetilde{S} F_2 \right]} \quad (12)$$

where:

$$\tilde{S} = \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}} \quad , \quad \tilde{S}_{ij} = \frac{1}{2} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) \quad (13)$$

and F_2 is formally identical to its RANS version. Moreover, the target energy ratio is given by:

$$r_k = \frac{1}{\beta_0} \left(\frac{\pi \sqrt{k_{\text{tot}}}}{\Delta C_\mu \omega_{\text{tot}}} \right)^{-2/3} \quad , \quad \beta_0 = 0.44 \quad , \quad \Delta = \Omega_{\text{cell}}^{1/3} \quad (14)$$

4 Numerical methodology

The computations were run using *Code_Saturne*. ([détails code](#))

To assess the effect of accounting for commutation errors, four configurations will be considered:

- (a) Basic Equivalent-DES, initialized with the proper repartition between modeled and resolved turbulent kinetic energy,
- (b) Same as (a) + the energy partition is enforced during the computation
- (c) Same as (a) + commutation error
- (d) Same as (b) + commutation error

The chosen test case is the periodic channel flow at $Re_\tau = 395$, where the resolution varies only in the normalwise direction.

5 Results

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