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CHAPTER 2

PRACTICE AND PROFESSION Everyday mathematics and their transmission in Mesopotamia and Egypt

CÉCILE MICHEL

The mathematics we use today has its roots in Antiquity and, in various aspects, more specifically in Near Eastern antiquity. Their inventors and practitioners have left texts – only a small portion of which have survived to date – that reflect their knowledge and the practice they made of mathematics. However, these texts are unevenly distributed in time and space, partly because of their more or less durable medium.

In Mesopotamia and surrounding areas, more than a million cuneiform texts, for the great majority written on clay tablets, but also on stones or other media, have been discovered. They cover more than three millennia from the end of the 4th millennium BCE to the first century CE and were found in a very large area from the Mediterranean Sea to the Persian Gulf and Iran, from Anatolia to Egypt (El Amarna).¹ Some sites have produced only a few tablets, while others dozens of thousands of them. About a dozen languages have used the cuneiform script, the main ones being Sumerian and Akkadian. Assyriologists consider that about 2,000 of these texts have a mathematical content, most of them were found in an educational context. But dozens of thousands of other clay tablets linked to the administration of palaces and temples or to private economic activities contain quantifications and computations and thus may be linked to the practice of mathematics.

During roughly the same period in Egypt, besides monumental hieroglyphic inscriptions engraved or painted on the walls, surviving texts are written in Hieratic and Demotic, mainly on papyri and ostraca. Ostraca are numerous from the middle of the second millennium and on; there are almost 10,000 ostraca written in Demotic. Papyri hardly survived time compared to clay tablets, and we count about as many of Egyptian Hieratic papyri as cuneiform mathematical texts. For the first millennium, there are a little less than 5,000 papyri written in Demotic.² Taking apart the ostraca, for which there is no evaluation of the number of texts with a mathematical content, the corpus of Egyptian mathematical texts represents about one per cent of the Mesopotamian one. This disproportion in the available sources is reflected in this chapter which focuses on Mesopotamia with some hints about mathematical practices in Egypt. Such a little number of mathematical texts might be explained by the fact that the context of discovery of the Egyptian papyri is limited to graves and temples.³

Most of the authors of these mathematical texts remain unknown. Were they professionals of mathematics? A 'profession' is understood as a paid activity performed by a person who went through a specific training or education. There is no word in Sumerian, Akkadian or Egyptian that we would translate by 'mathematician'. By contrast, there are words to refer to 'astronomers.' In Akkadian, astronomy practitioners are called *tupšar Enuma Anu Enlil*, 'the scribes of the Enuma Anu Enlil' which is the longest Babylonian treaty of astrology written on 70 tablets. In Egyptian, *imy-wnw.t* is 'the one in charge of the hours', generally translated as the priest-astronomer. The absence of a word for 'mathematician' reflects the absence of mathematical professionals despite the presence of mathematical problems whose resolution required specific and advanced training. Mathematics in antiquity was studied and subsequently taught and developed by masters. Those masters, however, who, were equally knowledgeable in various other 'sciences' as astronomy, medicine or divination as we gather from their archives or libraries.

Besides 'mathematical texts', thousands of administrative and economic texts from various cultural environments testify to the use of mathematics outside the scholarly milieu. Quantification and computation are practiced daily by many actors, whether they are employed by institutions, palaces or temples, or act as independent workers, such as merchants and some craftspeople. Since our main sources of information concerning these practitioners of mathematics are textual, their identification is unavoidably linked to their ability to write.

For more than a century, it was generally accepted that literate and numerate people in the Ancient Near East represented a small elite. However, for two decades, in-depth studies on tablets found in the context of 'schools' at Nippur dating to the beginning of the second millennium, on the materiality of texts and on paleographical analyses of cuneiform tablets have highlighted several levels in the practice of writing. School masters or scribes working for the power have been through the complete scribal curriculum reconstructed by Assyriologists. They were the authors of advanced mathematical texts, of official letters and treaties, literary composition, astrological and divinatory treaties. A second group of people only finished the elementary level and can write family and commercial contracts as well as private letters; they were able to solve simple mathematical exercises. Others, have learned to read, write and count at home from a family member. Depending on the periods, regions and milieus considered, the cuneiform syllabary used to write Akkadian had different degrees of complexity, varying from one hundred to more than two hundred and fifty signs, including some Sumerian logograms. Moreover, the technique of cuneiform writing is very simple: each sign for logograms, syllables and numbers consists of a composition of wedges, each wedge being obtained by impressing a stylus in fresh clay. Numbers are written with only two types of wedges (vertical and chevron). For all these reasons, literacy and numeracy, at least during the first half of the second millennium, was quite widespread among both men and women. During the first millennium, the Aramaic language was gradually gaining ground. It is written in ink on papyrus or other perishable media. The Akkadian language written with cuneiform signs on clay tablets remained the scholarly language in cultural and religious context. Because of the increased lack of texts of practice, it is difficult to evaluate the proportion of the literate and numerate population.

In Egypt, Hieroglyphs were reserved to monumental writing. During the third and the second millennia, hieratic was the cursive everyday script, and it was usually written on papyri. Since this media hardly survived time, there are too few texts to answer the question of literacy and numeracy. Thanks to the use of demotic written on ostraca and limestone during the first millennium, many more texts have been found. However, there is still no clear idea about the number of literate and numerate Egyptians for this period.

Despite the abundance and variety of sources available, at least for ancient Mesopotamia, the texts document the type of mathematics involved, but rarely the authors of computations. Most often these texts being not signed, the identity of mathematical practitioners must be guessed only through their activities and the context in which the texts were discovered (Michel and Chemla 2020b). Then it is possible to consider their role and status in society. Naturally, only the individuals who left archives are documented, which induces a bias in our study. This is especially true for third and second millennia Egypt where only members of the upper class have benefitted from decorated tombs in which, occasionally, some papyri were discovered.

The practitioners of mathematics in the Ancient Near East came from a great variety of milieus and the mathematical methods they learned and used varied accordingly. These methods were taught in 'schools', in private context by masters, or passed on from generation to generation. Mathematics usually played an important role in the education.

MATHEMATICS IN PRACTICE AND THEIR PRACTITIONERS

Many fields of daily life activities in Antiquity required some knowledge in mathematics. Their actors, whether officials working for palaces or temples, or independent workers, merchants, and craftspeople, could be numerate enough to proceed themselves to the mathematical operations necessary for their activities. Such a study implies gathering dossiers, outside the corpus of mathematical texts, including administrative and economic texts, as well as letters. It is often difficult to draw the portrait of these mathematics practitioners and to distinguish between scribes or high-level officials with multiple competences and specialists under their authority. Moreover, many of these individuals remain anonymous. However, this section is an attempt to highlight the role and status of some of them in the society. Leaving aside the writers of mathematical texts who are at the center of the next section, a few samples of practitioners of mathematics borrowed both from Mesopotamia and Egypt are presented here; they deal with land surveying, management of estate, trade and architecture.

Mesopotamian Land Surveyors

Agriculture was the main resource of ancient Mesopotamia. The administration of land was centered on the temple or royal estate. Land measurement with rope and rods of different lengths is well-developed during the third millennium. In the South of Mesopotamia, the necessary irrigated agriculture had an impact on the shape of the elongated fields, their smaller side receiving the water from the irrigation channel. Beside a heavy infrastructure for the building and management of dams, dikes and canals reflected in mathematical texts, specific personnel were dedicated to the measurement of fields and the estimation of the cultivating area. The archives found at Girsu around 2350, and belonging to the domain administered by the queen, illustrate the land surveyors at work (Lecompte 2020). To measure fields, land surveyors did not take into account their shape, but proceeded with length measurement values and approximations. They used the 'land surveyor formulae', *i.e.* the product of the average of the irregular quadrilateral sides. C. Lecompte has identified three different professions linked to land surveying and registration: the land recorder, the geometer who proceed to the measures, perhaps together with the field assessor. In this hierarchical organization, it is most probable that the land recorder was the one to proceed to the computations.

At the end of the third millennium, in order to face the management of new territories, the Ur III state implemented extensive administrative and economic reforms. An increased number of scribes worked for the state, recording daily accounts, then summarized in annual balance accounts based on estimated theoretical expenditures. Among the recorded data was a precise quantification of irrigated arable land. S. Rost presents the case of a large domain, Da-Ummaki, located in the in the Umma province which was under the responsibility of two families of land surveyors, in which skills were transmitted from father to son (Rost 2020). Descendants of both families held key administrative positions in the province.

The high social status of land surveyors is still visible in late second millennium Kassite Babylonia thanks to *kudurrus*-stones recording royal grants of land which could be measured in seeds necessary to sow the field. The measurements were done by a land surveyor and supervised by local and royal authorities (Robson 2008: 167-176). A dozen land surveyors attested on these inscriptions are said to be descendant of Arad-Ea, who lived two or three centuries earlier. Arad-Ea, himself an expert in accountancy, was the son of the governor of Nippur. Thus, these land surveyors belonged to the lineage of

a very powerful and rich family which included royal scribes and high officials.

As an echo of the tasks performed by land surveyors, field plans from the late third millennium as well as mathematical texts from the Old Babylonian and perhaps also the Kassite periods concern the computation of the areas of field of different shapes (Nemet-Nejat 1993: 72-78).

Egyptian Land Surveyor

The land surveyor profession is presumably as ancient in Egypt and might be attested by autobiographic hieroglyphic inscriptions found in the graves of high officials working in close relation to the king. This could be the case for Metjen, whose tomb (Mastaba L6) excavated at Saggara, north to the Djoser's pyramid, dates to the IVth Dynasty (twenty-seventh century, Strudwick 2005:192-194). The biography of Metjen, one of the earliest written on a wall of a tomb, gives a good insight into his life and position in the society (Allen 2013: 208-213). Metjen presumably started as a simple scribe, overseer of a provision bureau, a position inherited from his father, himself an honorary scribe. He became a high official of the Old Kingdom and provincial governor, ruling the Eastern part of the Fayyum. For all his services to the pharaoh, he received by royal decree twelve domains with their peasants in three provinces of the Nile Delta as well as a large orchard land from the royal domain and a huge, well-equipped house with its personnel. The IVth decree gives the areas of the fields suggesting the ability, at that time, to compute areas of geometrical figures (Imhausen 2016: 48). Among his titles, Metjen held the office of "Crier", in charge of evaluating the quantity and value of produce and cry the figures to the recording scribes (Clagett 1999: 146). Metjen was a scribe, able to compute, and who, in the service of the king, has risen above his father's social level.

Another example comes from the tomb of Menna at Thebes dating fourteenth century. This high official, married to a noble woman, was both referred to as a scribe and the overseer of the palace and temple agricultural estate. A painting represents him, with a group of people, measuring the size of a field with ropes and computing the respective number of products to deliver (*fig. 2.1*, Hartwig 2013).

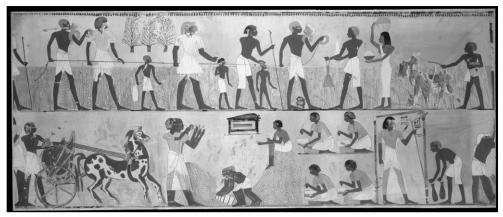


Fig. 2.1: Wall painting from the tomb of Menna in the Valley of the Kings. Rogers Fund, 1930, The Metropolitan Museum of Art.

A wisdom literary composition (BM 10474), *Instructions of Amenemope*, dating to the last centuries of the second millennium, contains advice to his son on how to behave in time of crisis to have a successful life. In the introduction, we learn that Amenemope was responsible for the borders of fields and thus measured land. For this, he used the measuring rope. He also had to control the standard capacity vessel and the measuring of grain (Imhausen 2006: 158-160).

Calculating the surface of geometrical areas is well-documented in the mathematical problems proposed for example by the Rhind Papyrus. The different geometrical shapes considered, rectangles, triangles, trapezoids – except the round one – could correspond to fields.

Accounts of an Egyptian ka-servant

Computations in the form accounts could also be carried by Egyptians of the lower levels of the landed gentry. A group of papyri found in a tomb complex at Thebes documents the life and activities of a middle-class man, Heqanakht, a *ka*-servant (or *ka*-priest) during the first half of the twentieth century (XII Dynasty; Allen 2002). The *ka*-servant was paid by the vizier Ipi to perform the daily offerings at his tomb. Heqanakht, whose household of some twenty people, including a few servants, was in another city, had religious and administrative duties: besides his own lands, he had to manage lands and personnel located in the Thinite nome providing income for the funerary cult. Heqanakht left letters and accounts dealing with land cultivation and loans; as a *ka*-servant, he was literate and numerate, and has presumably written himself several of these texts. Heqanakht had to estimate the yield of the land, proceed to grain measurements, compute, and distribute rations according to rates between different types of grains. He computed values in grain of

different commodities as oil, textiles, or copper. Some of his accounting activities, using operations on fractions, recall problems presented in the mathematical papyri as for example the quantity of grain necessary to produce bread.

Old Assyrian Merchants

Besides officials working for the administration, some merchants, acting privately, had to perform computations on a daily basis as part of their activities. The earliest and best documented private merchants are the inhabitants of Aššur (Iraq) who developed a long-distance trade in Central Anatolia and settled in some forty different towns there during first centuries of the second millennium. The center of the Assyrians' operations was located at Kaneš (Kültepe, near Kayseri), where 23,000 cuneiform tablets have been unearthed (*fig. 2.2*; Larsen 2015).

During their transactions, from the purchase of merchandise in Aššur until their sale in Anatolia, or during their commercial partnerships, the merchants had to realize various computations (Michel 2008). They were using the Mesopotamian weight system to quantify the metals, objects of their commercial exchanges. Weighed silver was used as a mean of payment. They carried out weighing taking into account deficient weights. In order to compute the value of a given metal into silver, they had to multiply or to divide the quantity of metal by the specified conversion ratio (Text 2.1; Michel 2020b). They built up various types of contractual relationships to gather important amounts of silver or to increase the capitals already obtained. They were involved into commercial loans and needed to know how to calculate the due interest. Their trade was built on join-stock companies in which one or several members put an amount of silver or gold at the disposal of an agent to perform commercial transactions. At the end of the operations, the partners set their accounts, paid their debts, and shared the profits according to certain rules, all of which led to complex computations. A large part of Aššur's population seems to have been involved in this long-distance trade, including the king and his family, the high officials, priests and temples, and the merchants' families, both men and women. School tablets show exercises which correspond perfectly to the knowledge merchant needed to master (see below).



Text 2.1: Extract of a letter concerning the transactions in tin Fig. 2.2: Photo of TC2, 24 an extract of a letter concerning the transactions in tin. Photo courtesy of C. Michel -Dept of Antiquités orientales at the Louvre

Egyptian and Mesopotamian Architects

Egyptian monumental buildings, and especially pyramids and tombs as early as the first half of the third millennium (III-IV Dynasties), have acquired a universal reputation for their main building material, the stone that has stood the test of time. These came from carriers in which inscriptions give an idea of the number of workers who were employed to extract a specific quantity of blocs. Some of the Hieratic mathematical papyri propose problems linked to construction planning, with the computation of the number of workers needed to complete a task linked to an architectural project (Imhausen 2016: 112-124; Monnier 2019). Egyptian architects of this period invented both new construction techniques and new geometrical concepts. To the 2 gu 18 mana of tin of your consignment 13 5/6 mana of tin accrued for you as road-tax; 3 mana 15 gin: food for 2 donkeys and the harnessor; 1 donkey died and imposed an expense of 17 gin of silver; 7 gin of silver cost the hire of a donkey driver from Hahhum to Kaneš; 12 gin loss on the tin for current expenses was deducted from the 12 mana of tin for current expenses; you will have to balance 5 mana 15 gin of tin on the tin for current expenses, at 8 gin each (gin of silver), its price in silver: 1/2 mana 9 1/3 gin; I have added 17 gin the price of a donkey, and 7 gin, the hire of a donkey driver, so in all: you shall balance 1 mana 3 1/3 gin of silver; 2 1/4 gin of silver, which they will balance for your, has been credited, so I have balanced only 1 mana 1 gin 15 še of silver with Aššur-malik Weights: $gu \leftarrow \times 60-mana \leftarrow \times 60-gin \leftarrow \times 180- še 1 gin \approx 8.3g$

Among the very few written testimonies of these innovations is a limestone ostracon found in the complex of pharaoh Djoser, at Saqqara, showing the profile of an elliptic vault in red ink with some measures written Hieratic (*fig. 2.3*, Rossi 2003: 115-117; Lightbody and Monnier 2017). The author of this diagram is anonymous. By coincidence, the most famous Egyptian architect who would have designed and supervised the construction of the funerary complex of Djoser, with its step pyramid, at Saqqara, Imhotep, is contemporaneous of this ostracon. As for the heroes, between legendary figure and historical celebrity, a lot of talent has been attributed to Imhotep, first as sculptor and architect, but also as vizier of the king, medical doctor, etc. Egyptians revered him as a son of divinities and praised his talents for more than two millennia.

Another high-ranking architect from this early period is Hemiunu (twentysixth century), the grandson of the IVth Dynasty's founder. He built the great Giza step pyramid. An inscription gives him as head of the judges and vizier of the pharaoh. A statue showing a wealthy man, representing Hemiunu, was found in his mastaba in the Giza necropolis. There is no doubt that many scribes were working for these architects, and presumably produced some of the calculations, but they remain anonymous.

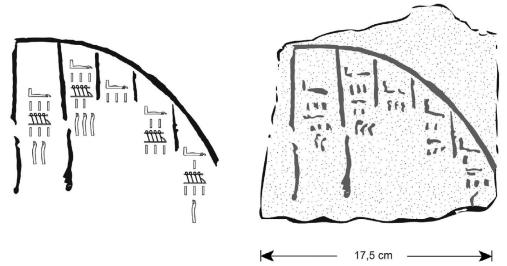


Fig. 2.3: Drawing of the ostracon JE 50036 (Lightbody and Monnier 2017). Photo courtesy of Franck Monnier.

Mesopotamia also provided annotated building plans, some of them are detailed enough to represent each brick of a paved courtyard. The only one that gives both the names of the building's rooms and their dimensions was excavated at Tello, ancient Girsu, and dates from the end of the third millennium (*fig. 2.4*). If the measurements are correct, the plan does not respect the proportions (Gruber and Roaf 2016). Unfortunately, such plans are never signed and so we do not know the identity of their authors.

Most of the buildings in ancient Mesopotamia were made in mud bricks, including the great ziggurats, multi-story towers with a religious function. Administrative tablets from the late third millennium provide important details about the construction of the ziggurats and the number of workers to employ. These activities were based on standard workdays setting the quantities of bricks shaped and transported. Thus, to build only the first terrace of the Ur ziggurat a thousand of workers would have work for more than five months just to prepare and transport the 7,6 million bricks needed (Sauvage 1998). Workers were grouped in teams under the responsibility of supervisors themselves under the direction of a mason in charge of the reception of the material and brick laying. As the construction was officially the great work of kings, the names of their architects who draw the plans and directed the work remain unknown. Tablets give the names of the intermediaries in charge of the organization of the work.

Many problems presented in the Babylonian mathematical texts deal with bricks and brick works; they address some questions an administrator would need to answer as the quantity of earth transported and the number of workers to employ (Nemet-Nejat 1993: 27-35, 43-55; Robson 1999: 57-92). The dimensions of the bricks used in mathematical texts correspond to those found in texts of practices as well as in the excavations of Mesopotamian sites (Sauvage 2020).



Fig. 2.4a: Photo of the tablet AO 338, Louvre Museum. Photo courtesy of C. Michel -Dept of Antiquités orientales at the Louvre.

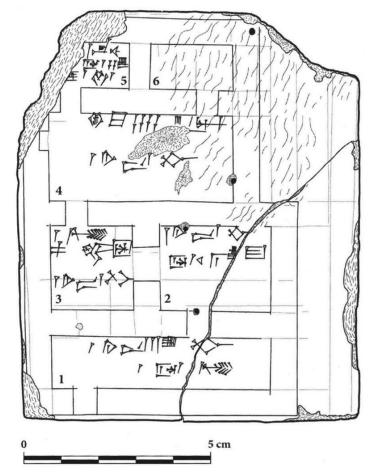


Fig. 2.4b: Drawing of AO 338 by Gruber and Roaf 2016. Photo courtesy of Martin Gruber.

AUTHORS AND OWNERS OF MATHEMATICAL TEXTS

As already noted, land surveying, architecture, management, and trade are among the themes which inspired the problems presented in the Old Babylonian mathematical texts. The quantities mentioned in mathematical texts were often concrete, thus the mathematics taught was often rooted in daily life. The issue at stake now concerns the writers and the authors of these mathematical texts. While authorship is never neglected in Greek mathematics, as we shall see in Chapter 3 below, mathematical texts from Mesopotamia and Egypt are, for the most part, anonymous. When, by chance, the name of the writer is given, there are rarely indications as to his status and biography. Moreover, the writer is not necessarily the author of the text, he may have copied it from another manuscript. Some mathematical texts have been found in context, and it is possible to shape the profile of their owner. However, there is no proof that the owner of a mathematical text and its author are the same person. Despite all these difficulties, this section intends to draw a portrait of the writers, authors, and owners of mathematical texts in Mesopotamia and in Egypt and to show that they emanate from different communities of scribes.

Notables and Scholars of third millennium Mesopotamia

There are relatively few texts for the third millennium that may be considered as 'mathematical texts' by contrast to texts of practice involving mathematical computations. They are usually connected to large scale works and the control of land by high ranked officials. Among these, a group of texts from Southern Mesopotamia dating to middle of the third millennium are linked to the quantification of surfaces. These texts present tables linking the sides and the surfaces of rectangles or squares. Even though they may refer to landsurveying practices, the arrangement of data which separate numerical and metrological ones testify to the introduction of new mathematical concepts (Chapter 3). The earliest examples have been produced by the scholarly milieu of Šuruppak together with lexical lists and Sumerian literary texts. Their authors are learned scribes who use to add their name in a colophon at the end of the tablet; but few are complete. Another example, from an unknown context in Adab, contains a table of increasing sides of squares and their corresponding surfaces. This tablet has been written by the scribe Nammah, whose name is very popular at that time (Edzard 1968). Another text from Zabalam, which shows an attempt to compute small surfaces by creating sub-units of the smaller area unit sar, is authored by Lugal-hegal-su. Once more, there is no data about this scribe. According to Proust (2020), these tablets emanate from an erudite scribal milieu; the mathematical methods developed there have not affected the contemporary administration practices.

For the Sargonic period which saw the first centralized state in Mesopotamia (twenty-fourth-twenty-third centuries), there are a dozen mathematical texts presenting problems, all of these coming from the the South, and not from the Akkadian-speaking north. Some of these, focusing on the computation of areas of squares and rectangles, have been found in a provincial town, nearby Lagaš. They could have belonged to a notable, perhaps the governor of that town, who would have kept his tablets produced during his scribal education (Foster 2016: 216-219). Other mathematical texts deal with irregular figures, presumably parcels of land, for which the scribe has to find the area or one side of the geometrical shape. The resolution of these problems suggests the conversion of metrological measurement units into numbers using a positional system and computations carried on an abacus. Two of these problem tablets have been written by Ur-Ištaran, perhaps administrator of the Girsu palace (Foster 2018, L. 4552). If not dealing with a homonym, then it would confirm the link between mathematical texts and high officials.

Students and masters in early second millennium Mesopotamia

The largest number of mathematical texts date to the Old Babylonian period. They have been found in a context of education in Nippur, Sippar and Ur (South of Iraq), in several sites of the Diyala Valley, in Mari (Syria), and in Susa (Iran). They reflect different traditions with local specific features. Hundreds of such texts found in several houses at Nippur, allowed a reconstitution of the scribal curriculum in this main cultural center. Unfortunately, there is too few data about the owners of these houses.

A house excavated at Tell ed-Dêr is one of the few to allow drawing the portrait of one or two authors of mathematical school texts. It belonged successively to two different persons and contained some 2,000 cuneiform tablets, including eighty linked to a teaching activity (Tanret 2002). The main archive belonged to the family of the great lament priest Inanna-mansum of the city of Sippar-Amnānum. He chose his eldest son, Bēlānum, as successor in his religious function. The sixty-eight school tablets found in his house were all written by this son who became lament priest under the name Ur-Utu in 1642 after his father's death. Among these were six metrological tables for capacities and weights and two mathematical exercises focusing on the computation of field areas (Tanret 2002, no. 42-47, 55-56). Bēlānum learned basic mathematics allowing him to carry mathematical operations for daily economic life. His teacher, Šumum-liși, was a professional scribe working for the family. He wrote administrative documents, contracts and letters and authored 151 texts found in the house. Belanum was not destined to be a scribe, but to have a high responsibility in the temple with an important social status. He however had to be able to check what was written for him by scribes, and this explains his basic education; his master provided him a passive knowledge.

Before the house was rebuilt and occupied by Inanna-mansum family, it belonged to Lamassāni, daughter of Rīš-Šamaš, a *nadītum*-priestess of the Sun God. These priestesses, who belong to the urban elite, were forbidden to marry and reproduce, but they could take care of a niece also destined to become a priestess. In the house of Lamassāni twelve school tablets were discovered, two of them linked to a mathematical education: a metrological list and a multiplication exercise with reciprocals. These could have been written by the niece of Lamassāni during her education (Tanret 2002, no. 57 and 59: 140).

This interesting remark raises the issue of literate and numerate women. The archives of the *nadītum* of Sippar have proven the existence of some twenty female scribes working for the priestesses, some of them being themselves *nadītum*. However, education was not limited to the religious sphere. One of them, Inanna-ama-mu, was scribe of the judges of the Sun God temple. These female scribes did not work exclusively for women and some male scribes

also wrote the same types of documents for the priestesses: field purchases, leases of land, inheritances, donations, etc. To confirm the existence of female scribes at Sippar, four school exercises containing a colophon were written by a female apprentice scribe; they followed the same curriculum as schoolboys (Lion 2011: 99-100). One of these women is munus-dumu-é-dub-ba-a, a Sumerian title referring to a female scribe who would have been through the whole curriculum and became a female Master (Michalowski 2012). There are, up to now, no mathematical texts authored by a woman, however it is very likely that female scribes had received the same education in mathematics as male scribes.

The mathematical school tablets found at Sippar belonged to the elementary level. By contrast, advanced problems may have been written by high level students, by masters or by scholars. These texts were not all written for the purpose of education. Several procedure texts from South Mesopotamia present lists of solved problems of progressive difficulty linked to practical problems, their resolution is based on the arithmetical tools scribes learned during the elementary level. These texts are drawn by masters for advanced teaching. Linked to these procedure texts are catalogues compiling the problems without giving the procedure, thus archiving educational texts (Proust 2014: 87). Other long lists of problem statements presented in an arborescent structure, were not intended to be solved. They are the result of a reflection on mathematic writing techniques by scholars not linked to education but simply interested by developing new mathematics. The houses in which they were presumably preserved, together with elementary mathematical exercises and tables, were both educational and intellectual centers. Some of these mathematical texts could end with a colophon including the date, a proper name, and data linked to the text itself as keyword, and sections. Unfortunately, as they come from illicit excavations, the profile of the scholars who produced these mathematical catalogues series and series remain unknown.

Scholars of late first millennium Babylonia

A few mathematical texts date to the Achaemenid (539-330) and Hellenistic (330-140) Babylonia, but many more astronomical tablets testify to the use of advanced mathematics (Ossendrijver 2012). At that time, the main spoken languages were Aramaic and Greek which were written with ink on papyri, only scholarly works were still written in cuneiform on fresh clay. The authors of the mathematical tablets found at Uruk and Babylon, are known thanks to the colophons in which the scribes mentioned the writer of the text, the place it was composed, its content, eventually the original document from which it was copied, and the owner of the tablet. They belong to important lineages of scribes that we can reconstruct over several generations, the most

learned were referred to as "masters of knowledge" or "experts" (*ummānu*). These men with an encyclopedic culture had students, among whom their own sons, who were writing tablets for them. These mathematical tablets belonged to families involved in medicine, astrology and astronomy.

A dozen mathematical texts dating to the Achaemenid period and showing different methods to evaluate areas were discovered in the house of the exorcist-doctors, *āšipu*, at Uruk (Proust 2019), together with forty astrological and astronomical tablets (Steele 2019), contracts and a great variety of scholarly texts: literary compositions, hymns, omens, exorcisms, medical recipes, etc. (Clancier 2014). Many belonged to two families over respectively four and three generations: the Ekur-zakir descendants succeeded to those of Šangu-Ninurta after rebuilding the house (Proust and Steel 2019: 8-34). These tablets are linked to the education of the younger members of the family, to the practice of the profession of exorcist-doctor, to the encyclopedic knowledge of the senior family members as well as their duties toward the temple. They confirm that important professions as well as scholarly knowledge, including mathematics, was transmitted from father to sons over several generations.

Another group of mathematical texts originates from the library of the Reš temple at Uruk and belonged to priests and astronomers. As well, these texts were found together with a great variety of scholarly tablets including astronomical texts using advance mathematics. Among the writers of these mathematical tablets we find both experts and younger scholars copying texts for the former. As he was a student, Anu-aba-uter wrote four astronomical tablets for Šamaš-ēțir. The latter was the great priest of the Reš temple and an astronomer. Anu-aba-uter, also known as astronomer, was the owner of a mathematical text containing seventeen problems on arithmetical, geometrical and metrological topics (Ossendrijver 2019). He wrote seven tablets for his father, including a Sumerian cult song, and several computational astronomic texts (Robson 2007). At that time, a young scribe studied within his family, having learned from his father or uncle and copying texts for him, but he also completed his education with a master of another literate and numerate family. Contracts found together with the scholarly tablets suggest that some of these families were linked by marriage. Most of these scholars were priests or working for the temple; they put their astronomical knowledge to the service of the temple, predicting astral phenomena based on the compilation of observations, data quantification and development of mathematical procedures in order to build the astronomical tables with the recurrence and periodicity of the observed phenomena (Ossendrijver 2012).

During the middle of the fourth century the Esagil temple of Babylon also employed astronomers, a profession transmitted from father to son and for which they receive a salary of one *mana* of silver (500 g) per year and a cultivable land. They cumulated this office of astronomer with that of priest or exorcist-doctor. During the last centuries of the first millennium, under the Greek domination, the Babylonian temples, with their libraries and experts, represented the centers of knowledge where all innovations in mathematics, astronomy and medicine were made (Beaulieu 2006).

Writers of the Egyptian Mathematical Texts

The question of authorship of the Egyptian mathematical texts is extremely difficult to answer because of their fragmentary state. Written on a perishable medium, they survived in small pieces that had to be reassembled; their colophon with the name of their writer may be missing. Moreover, several mathematical papyri come from the antiquity market and their context of discovery is lost. When the name of the writer is known, his life and social status might not be documented; moreover, the writer may have simply copied the mathematical texts from an earlier manuscript.

This is the case for example of the Rhind Papyrus $(32 \times 500 \text{ cm})$, written in Hieratic, presumably the most well-known sample of Egyptian mathematics. It was bought by A. Henry Rhind and is supposed to come from a small building nearby the Ramesseum (Mortuary temple of Ramses II) at Thebes (Clagett 1999: 113). The mathematical text, on the recto, dating to the Second Intermediate Period of Egypt (ca. 1550) was copied by the scribe Ahmes from a lost original presumably dating to the second half of the nineteenth century. The text starts as follows (*text 2.2*):

Rules for reckoning, for inquiring into things, and the knowledge of all things, mysteries to all secrets. This book was copied in regnal year 33, month 4 of Akhet, under the majesty of the King of Upper and Lower Egypt, Awserre, given life, from an ancient copy made in the time of the King of Upper and Lower Egypt, Nymatre. The scribe Ahmes writes this copy.

Text 2.2: Beginning of the Rhind Papyrus (translation Clagett 1999: 122)

It gives then tables and series of problems supposed to be designed for educational matter. One third of the papyrus contains tables for the division by 2 of integers, and various problems involving divisions and multiplications of fractional expressions. Then only, problems are linked to applied mathematics with divisions of a measure, of beer and bread for rations, computing volumes of cylindrical and rectangular granaries, areas of different shapes and ending with the slopes of pyramids, and miscellaneous algebraic and geometric problems. As suggested by the introduction of the papyrus, an Egyptian official scribe was expected to be able to cope with such problems in the performance of his activities (Imhausen 2016: 65-67).

A quite atypical text on the verso of the papyrus is a historical and political chronicle written in the Hyksos context (Oriental Nile Delta), a kind of marginal milieu in Egypt. It gives objective and factual dates and events that led to the fall of Avaris in an exceptional way as it expresses the point of view of the defeated party (Barbotin 2008: 180-181). The papyrus was later taken to Thebes after the victory of this city. The texts on both sides seem to have been written in a short time; it is not impossible that Ahmes himself wrote or copied this chronicle.

The Rhind Papyrus is the only mathematical Hieratic text bearing the name of its writer. It has been suggested that most the mathematical papyri have been composed during the XIIth or early XIIIth Dynasties, *i.e.* during the first half of the second millennium (Clagett 1999: 113-280). Thus, they are contemporaneous to the bulk of the cuneiform mathematical texts. They originate from different sites but their exact location remains most of the time unknown; when it is known, they were found in a tomb:

- Thebes for both the mathematical leather roll (Imhausen 2016: 69-70) and the Moscow Papyrus (Clagett 1999: 205-206),
- tomb N 408 of This necropolis for the Reisner Papyrus (Clagett 1999: 261-262),
- the workers' town Ptolemais Hormou (El-Lahun), nearby the pyramid of Sesostris II for the Lahun fragments (Clagett 1999: 239-240),
- Akhmim for the Cairo wooden board (Imhausen 2016: 68).

The tomb N 408 in the This necropolis belonged to a high official of the city. Together with the Reisner Papyrus were four rolls of papyri with administrative texts linked to the management of resources at the royal dockyards, the construction of a building, and a list of distribution of rations (Hagen 2018: 75).

The tables used to compute with fractions and the most complete texts with series of problems and their solution suggest that they were used as school text-books in the same manner as the tables and problems addressed to Babylonian apprentice scribes, but the resolutions used different methods (Couchoud 1993: 4-5; Bernard, Proust and Ross 2014). According to A. Imhausen (2003 article), half of the proposed problems refer to practical matters as computation of rations, volume of granaries, field areas, and other problems linked to boats, brewery, breeding, etc.; they might have been part of the education of scribes working for the administration.

The group of Demotic mathematical papyri has been less studied (third century BCE - second century CE; Parker 1972). Some contain also other types of texts, written at about the same time or showing traces of reuse. Several of them come from the antiquity market, others have been found in temple libraries at Tebtynis and Soknopaiou Nesos, and the last two in the Temple of Tutu at Kellis.⁶ The longest of these papyri, which includes mathematics from simple exercises to the computation of the volume of a

pyramid, is written on the verso of the *Code of Hermopolis* (first half of the third century) which deals with the relationships between owner and tenant, problems of property, marriage, etc. with some private commentaries. M. Depauw (1997: 103-105) suggested that the text is a legal and mathematical reference book. There is no indication about its writer who might have been a scholar knowledgeable in several disciplines.

MATHEMATICAL PREPARATION FOR PROFESSIONS

Both in Egypt and Mesopotamia, mathematical training is part of the education of scribes. This profession, considered as the best among all, is reserved to privileged individuals; the intellectual work of the scribe is regularly opposed to tiring manual professions. However, to become a scribe, one had to study hard for several years. Literary compositions about school survived from both ancient Mesopotamia and ancient Egypt. They depict the organization of the education, the topics that were studied, including metrology and mathematics. Texts produced in the context of education provide data about the content of the lessons from elementary to advanced level. They show a great variety in the practice of mathematics and in its transmission according to the milieus considered. In Mesopotamia, the mathematics taught at Nippur (eighteenth century) to the students of the upper society was completely different from that studied by the future Assyrian merchants in Aššur and Kaneš. For ancient Egypt, the sources are scarcer. The advanced level of education, which could last up to twelve years, depended on the future career of the students, whether in administration, priesthood (in the House of Life) or military.

The Edubba compositions and the social profile of the scribes

For the twenty-first century, hundreds of thousands of texts document the management of royal goods which caused multiple controls and an increased production of tablets. The number of scribes dedicated to administrative tasks increases dramatically: 1,600 of them are known by their names. There is a great need of well-trained scribes, and the literate king Šulgi claimed in a hymn to have setup schools in many cities of his country (*text 2.3*):

When I was small, I was at the Edubba, where I learned the scribal art from the tablets of Sumer and Akkad. None of the nobles could write on clay as I could. There where people regularly went for tutelage in the scribal art, I qualified fully in subtraction, addition, reckoning and accounting. (...) In the south, in Urim, I caused a 'House of the Wisdom' of Nisaba to spring up in sacrosanct ground for the writing of my hymns; up country in Nippur I established another. May the scribe be on duty there and transcribe with his hand the prayers which I instituted in the E-kur; and may the singer perform, reciting from the text. The Edubba are never to be altered; the places of learning shall never cease to exist.

Text 2.3: Hymn Šulgi B. Extracts from lines 10-20, 308-319, translation: <u>http://etcsl.orinst.ox.ac.uk/section2/tr24202.htm</u> consulted September 20, 2019

The 'House of the Wisdom' refers to schools which were part of temples, especially those dedicated to Nisaba, the goddess of the scribes during the Ur III empire. Such a tradition could have survived during the Old Babylonian period in parallel to the education in private houses (Charpin 2017: 110-115). In this hymn, the Edubba, the 'House of the tablets', *i.e.* 'the school', appears as a place where nobles and princes learned writing, computing and accounting.

The organization of scribal education in the late third millennium southern Mesopotamia might be reflected by the Sumerian 'Edubba compositions' written and copied during the first centuries of the second millennium. These texts provide a social profile of the students going to school: most of them came from wealthy families of the social elite. At home, the schoolboy recited his tablet to his father assuming that the father was literate and able to control homework, if not a professional scribe, he might had an intellectual activity (Attinger 2017). In another composition, a father recommended his son to be attentive at school in order to be able to succeed him in his activity; this is a good example of social reproduction (*text 2.4*).

Of all the human professions which exist on earth and of which the god Enlil has named, he has not named any profession more difficult than the art of the scribe. In accordance with the prescriptions of the god Enlil, the son must succeed his father in his profession.

Text 2.4: Edubba B, Sjöberg, 1973

The Edubba compositions depict the education of the young scribe in schools: the life of the schoolboy, his relations with his master, his friends and his

parents. They advocate the learning of the Sumerian language and give interesting insights into the curriculum, including mathematics (*text 2.5*).

Now I've been here for the stipulated time, I can cope with Sumerian, scribal work, archiving, accounting, calculation! (...) I want to write tablets: The tablet of measures from 1 *gur* of barely to 600 *gur*, The tablet of weights from 1 shekel to 20 minas of silver, With the marriage contracts that people will bring me, The partnership contracts, I can choose controlled weights of one talent, The sale of houses, of fields, of slaves, The guaranties in silver, contracts of field rental, Contracts of cultivation of palm date grove, Even adoption contracts, I know how to write all this.

Text 2.5: Edubba D, translation Civil 1985

The education was based on the copy and the memorization of long lists of items: signs, words, personal names, professions, etc. The apprentice-scribe also learned the metrological systems by copying metrological lists, in order to be able to control the weights and proceed to the computation of balance sheets: all knowledge important for goods management and administrative tasks.

Curriculum at Nippur

Because they are literary compositions meant to entertain the reader, the data given in the Edubba compositions must be treated with caution, but they give hints about the main disciplines taught, Sumerian and mathematics, in a presumably late third millennium context. There are great changes in the educational system at the very beginning of the second millennium, with a growing emphasis on writing. Before, lists and tables were perhaps transmitted orally (Proust 2015).

The lists mentioned in the Edubba texts correspond to those found in several Old Babylonian sites, especially at Nippur in some houses where a master taught a few students. In the 45 m² house F archaeologists found 1,300 school tablets dating to the first half of the eighteenth century, ninety per cent being school material recycled as construction material in the walls, and in a clay basin. The house served as a place of education, but the tablets were later discarded (Robson 2001).

School tablets show different shapes and layout that have been arranged in four groups corresponding to different functions (Civil, Green and Lambert 1979). Type II tablets have different exercises on the obverse and on the reverse and can bear two hands: the master's and the pupil's copy. N.

Veldhuis (1997) observed that one side presents a new lesson while the other contains a lesson learned previously; he could thus reconstruct the progression of the items taught and the curriculum of the scribal education at Nippur, distinguishing different phases. Applied to mathematics, this method allowed establishing the general structure of the mathematical training curriculum (Proust 2007; Robson 2009); it was articulated with that of writing and Sumerian (Proust 2014).

During the elementary level, the pupil learned how to draw signs and learned the vocabulary organized by themes. In parallel, he learned by rote metrological lists enumerating measurements of capacity, weight, surface and length. At the end of the elementary level, the list to be memorized were more complex with signs arranged by their shape, metrological tables giving the correspondence between the measurement units and abstract numbers written in sexagesimal place value notation (SPVN, Proust 2010), and numerical tables: reciprocals, multiplications, squares, square roots and cube roots.

During the intermediary level, the pupils copied proverbs, contract models in Sumerian, and had to solve small mathematical exercises based on operations with SPVN numbers. These were linked to the computation of areas of various geometrical figures. During the advanced level, they copied Sumerian literary compositions and solved mathematical problems using all what they had learned before (Proust 2014; Bernard, Proust and Ross 2014). The school tablets found at Nippur present a great homogeneity by contrast to what was found in many other cities where the typology of tablets varies a lot, suggesting a different organization.

Mathematical training through practice: the case of the Assyrian merchants

This is the case for many of the 'school tablets' found in the houses of the Assyrian merchants at Aššur and Kaneš. Besides large rectangular tablets with lists of weights given in an increasing order, words, personal names and sentences extracted from letters, a dozen round small tablets concern exclusively arithmetic exercises. Most of these consist in converting a quantity of a metal into another metal according to a given conversion ratio.

2 gu 14 mana of copper the top-pack-elitum at 1/2 mana 5 gin. Its (value in) silver: 2 mana 6 1/2 gin 15 še [The correct result is 2 1/2 mana 6 1/3 gin] Text 2.5: Old Assyrian school tablet from Aššur Comments: Michel 2020b, text 2.1F In *Text 2.5*, the scribe had to calculate the weight of silver necessary for the purchase of copper which is quantified by a top-pack-*elitum*, knowing that a top-pack-*elitum* weighs 30 *mana*; he failed since the result is not correct. The computation was presumably done by a rule of three, which might have been part of the arithmetic training of the apprentice merchant.

To solve such exercises, the Assyrians did not convert their quantities in the SPVN system in use at Nippur, but carried their computations on the metrological values, including fractions, starting with the largest measurement units (Michel 2020b). These merchants received their mathematical education through practical professional tasks. They were taught by a master or by a family member. Their wives and daughters were fully involved in the long-distance trade, earning money from the sale of their textiles in Anatolia and investing their revenues in financial operations. Many were numerate, keeping their accounts and claiming the amounts due to them. Some women too have left tablets. Like men, they presumably learned the basics of writing and computing from their parents (Michel 2020a).

The transmission of mathematics in Egypt

The transmission of mathematical knowledge in Egypt is less documented due to the lack of school exercises by apprentice scribes.⁴ However, as for Mesopotamia, literary compositions document the education of scribes, a profession which is also considered as one of the most important in the Egyptian society. In a letter to his son who left school to work in a field a man depicts the hard manual work he will have to face and how he will be put under pressure by the scribes who calculate the taxes he will have to pay at the time of harvest. By contrast, the "scribe he is the taskmaster of everyone. There is no taxing of the work of the scribe. He does not have dues." (Anastasi V, 17, 2-3, Ramesside period, Simpson 2003: 439). The scribe is the one to compute the taxes due by the others.

In another section of the papyrus, a father explains to his son the benefits of going to school and the topics that will be taught to him (*text 2.7*):

I have placed you at the school along with the children of magistrates in order to instruct and to teach you concerning this aggrandizing calling (...) You bring your book daily with a purpose, be not idle. They say, "Three plus three'. You... another happy occasion, in which you grasp the meaning of a papyrus book (...) You will make calculations quietly: let no sound coming forth from your mouth be heard (...) Fall in with the ways of your instructor and hear his teachings. Be a scribe.

Text 2.7: *Papyrus Anastasi V*, 22, 6-7, 23,1-2, 4-6 (*Imhausen 2016:* 143-144)

The school mentioned here could allude to the 'House of instruction', a royal school already known from late third and early second millennia texts, thus contemporaneous to the bulk of the mathematical papyri. In the temples, scribes were linked to the 'House of Life' where they learned religion, but also mathematics, astronomy, and medicine (Bernard, Proust and Ross 2014: 36-38). The text gives hints on the mathematics learned, additions and computations based on a book, presumably those presenting problems to solve which have been referred to as mathematical papyri above and were addressed to male students only. These texts reflect an advanced level in mathematics; there are almost no text documenting an elementary level of mathematical education.

A satirical letter, Papyrus Anastasi I, dating to the 19th Dynasty (ca. 1250), was sent by a military scribe, Hori, to his colleague scribe Amenemope (Mapu), accusing him of not meeting the standards of the profession (Couchoud 1993: 183-184; Imhausen 2016: 148-156). He blames him for being incompetent in terms of mathematical and technical matters despite his high rank in military administration. To illustrate his accusations, he gives examples which concern the resolution of mathematical problems that he considers Mapu unable to solve (Anastasi I, 13,8-18,2): computing the number of bricks needed to build a ramp of given dimensions, the number of workers needed to move an obelisk which dimensions are specified, the number of workers to erect a colossal statue using a box system filled with sand, or the rations of a military group of a given number of soldiers. Three of these problems focus on the number of workers necessary to complete a task and their cost in terms of wages. This echoes the many mathematical problems found both in Egypt and Mesopotamia.

These both scribes worked for the army and had different military ranks. More generally, scribes had a variety of social positions according to the domain in which they work and their tasks. The ordinary scribes who produced the administrative and economic documents might have been in charge of daily accounting, using the skills they learned at school. Working for temples or for the state, they did not belong to the upper class (Imhausen 2003). By contrast, royal scribes, as high officials, belonged to the elite and could pay to have their personal decorated tomb.

CONCLUSION:

A DIVERSITY OF USES IN A VARIETY OF MILIEUS

This chapter has identified a variety of practitioners of mathematics and their activities through the texts discovered in Mesopotamia and Egypt. Mathematical texts and texts documenting mathematical practices include different mathematical traditions developed for different goals in different milieus. Mathematical texts appeared together and developed in parallel with

other scientific texts, including those in the fields of divination, astronomy and medicine. At the beginning of the second millennium, both in Egypt and Mesopotamia, we observe a very fertile production of mathematical problems rooted in daily life activities. To solve them, different sophisticated mathematical methods were developed by highly educated Egyptian and Mesopotamian scribes and masters. A variety of mathematical cultures existed also within these two geographical areas. Although this chapter considered the Ancient Near East and more precisely Mesopotamia and Egypt, no claim is being made that these mathematical cultures, which already show a great diversity, are representatives to all others, especially Greek and Roman, not to speak of China and India.

Many of the mathematical texts with problems are composite, they represent a compilation of some earlier texts. They were regularly copied over several centuries, whether for educational purposes or for patrimonial purposes, in order to be kept in libraries. As for texts indicating other kinds of mathematical practices, one must distinguish the many agents involved in producing them, including high officials, royal scribes, heads of bureaus, administrators, and accountants.

In both Mesopotamia and Egypt, there were no specialists in mathematics: there were scholars who went through the complete curriculum and were knowledgeable in various domains including mathematics, astronomy, medicine, literature, etc. Mathematics was a subject taught in most 'schools,' but with variations in its content. For example, the mathematics studied in Nippur schools and in Aššur merchants' houses was quite different in the topics covered and its procedures. In Hellenistic Mesopotamia, by contrast, the general and mathematical knowledge of scholars was much more evident. In Babylon and Uruk, for example, students speaking Aramaic as their mother tongue, and fluent in Greek, had to master Sumerian and Akkadian to access textbooks in mathematics, astronomy, divination and medicine. They were born in highly educated families in which knowledge was passed down from generation to generation. The mathematics they developed was by then closely associated with astronomy. In Egypt as well, scholars assimilated foreign knowledge and developed their mathematics in new directions.

¹ From now on, all the dates are given BCE, except when specified.

² Precise numbers are given by the database Trismegistos (<u>https://www.trismegistos.org/index.php</u>), accessed September 20, 2019. At this date, the database inventory gives 2,027 Hieratic papyri and 4,792 Demotic papyri, 9,307 Demotic ostraca and 4,305 Hieratic ostraca (but this number overlaps the previous one because many of these are in Demotic with a few words or sentences in Hieratic). Note that Hieroglyphic papyri are rare; they are mainly copies of texts engraved or painted on the walls.

³ Graves provide perfect climatic conditions for the preservation of papyri. The important towns were located along the Nile where the atmosphere is more humid, and modern cities are built and excavations are hardly possible.

⁴ The *Late Egyptian Miscellanies*, according to the most recent interpretations, are not school exercises, see Imhausen 2016: 143.

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